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Dynamical Complexity in D_{st} Time Series Using Entropy Concepts

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Abstract. The complex system of the Earth's magnetosphere corresponds to an open spatially extended nonequilibrium (input - output) dynamical system. This paper explores the applicability and effectiveness of a variety of computable entropy measures (e.g. Block entropy, T-complexity and Approximate entropy) to the investigation of dynamical complexity in the magnetosphere. We show that as the magnetic storm approaches there is clear evidence of significant lower complexity (higher organization) in the magnetosphere. Approximate entropy yields superior results for detecting dynamical complexity changes in the magnetosphere in comparison to the other entropy measures presented herein. The analysis tools developed in the course of this study for the treatment of D_{st} index can provide convenience for space weather applications.

1. Introduction

Nonlinearly evolving dynamical systems, such as space plasmas, generate complex fluctuations in their output signals that reflect the underlying dynamics. Accurate detection of the dissimilarity of complexity between normal and abnormal magnetospheric states (e.g. pre-storm activity and magnetic storms) can vastly improve space weather diagnosis and, consequently, the mitigation of space weather hazards.

Various complexity measures have been developed during the last 20 years for real-world time series in order to estimate the complexity of the corresponding dynamical system. The main types of complexity measures are entropies, fractal dimensions, and Lyapunov exponents. Fractal dimensions and Lyapunov exponents are both working well, but they generally require long datasets for statistically significant results, resulting in inconvenience in many studies and applications. On the other hand, entropies have the advantages of simplicity, extremely fast calculation, and antinoise ability. Entropy techniques provide convenience for detecting and capturing useful information of time series. Some entropy measures based on symbolic dynamics adopt a range partition to generate a partition in the symbolization transform, but their results may be compromised by the nonstationarity of the time series. The datasets obtained from most space physics studies are usually nonstationary, rather short, and noisy. One of our objectives is to find an effective complexity measure that requires short datasets for statistically significant results, provides the ability to make fast and robust calculations, and can be used to analyze nonstationary and noisy data, which is convenient for the analysis of magnetospheric time series.

The hourly Disturbance storm-time (D_{st}) geomagnetic activity index is computed from an average over 4 mid-latitude magnetic observatories (http://sw dcwww.kugi.kyoto-u.ac.jp/), and hence serves as a proxy for the magnetospheric ring current, and thus magnetic storm occurrence. Magnetic storms are the most prominent global phenomenon of geospace dynamics, interlinking the solar wind, magnetosphere, ionosphere, atmosphere and occasionally the Earth's surface (Daglis & Kozyra, 2002, Daglis et al., 2003, 2009). Magnetic storms occur when the accumulated input power from the solar wind exceeds a certain threshold.

In this paper, we study in terms of nonlinear techniques whether certain characteristic signatures emerged in D_{st} time series indicating the transition from pre-storm activity to magnetic storms. We consider one year of D_{st} data (2001) including two intense magnetic storms, which occurred on 31 March 2001 and 6 November 2001 with minimum D_{st} values -387 nT and -292 nT respectively, as well as a number of weaker events (e.g. May and August 2001 with D_{st} $\sim$ -100 nT in both cases). More precisely, the temporal evolution of nonlinear characteristics is studied by applying a variety of recently proposed entropy techniques: the original D_{st} time series is projected to a symbolic sequence and then analyses in terms of dynamical (Shannon-like) Block entropy and T-complexity follow. For the purpose of comparison we also analyze the original D_{st} data by means of Approximate Entropy ($ApEn$). This analysis verifies the results of symbolic dynamics.

2. Symbolic dynamics

The discovery that simple deterministic systems can show a vast richness of behaviors in response to variations of initial conditions and / or control parameters, has been at the origin of an intense interdisciplinary research activity since the 1950's (Khinchin, 1957; Nicolis, 1991, 1995). One of the outcomes of this work has been the realization that for an appropriate description of such complex systems, one needs to resort to a probabilistic approach (Nicolis & Gaspard, 1994). It is well known since the pioneering work of Gibbs and Einstein that we can describe dynamics from two points of view. On the one hand, we have the individual description in terms of trajectories in classical dynamics, or of wave-functions in quantum theory. On the other hand, we have the description in terms of ensembles described by a probability distribution (called the density matrix in quantum theory) (Prigogine & Driebe, 1997). Now, once one leaves the description in terms of trajectories, a basic question that must be dealt with concerns the amount of information one may have access to on the temporal evolution of the system in the course of time.

One of the approaches developed in this context is "coarse-graining", whereby a complex system is viewed as an "information generator" producing messages consisting of a discrete set of symbols defined by partitioning the full continuous phase space into a finite number of cells. We refer to such a description as "symbolic dynamics" (Nicolis et al., 1989; Nicolis, 1991, 1995; Nicolis & Gaspard, 1994). One of its merits is to provide a link between dynamical systems and information theory (Nicolis, 1991; Ebeling & Nicolis, 1992).

From the initial dynamical system we can generate a sequence of symbols, where the dynamics of the original (under analysis) system has been projected. This symbolic sequence can be analyzed by terms of information theory such as entropy estimations, information loss, automaticity and other prominent properties.

There exist some canonical ways for generating symbolic dynamics out of a given dynamical system (Nicolis et al., 1988, 1989; Nicolis, 1991, 1995; Ebeling & Nicolis, 1992). To produce symbolic dynamics out of the evolution of a given system, we set up a coarse-grained description incorporating from the very beginning the idea that a physically accessible state corresponds to a finite region rather than to a single point of phase space. Let C_i $(i = 1, 2, ...K)$ be the set of cells in phase space constituted by these regions, assumed to be connected and nonoverlapping. As time goes on, the phase space trajectory performs transitions between cells thereby generating sequences of K-symbols, which may be regarded as the letters of an alphabet. We shall require that, in the course of these transitions, each element of the partition is mapped by the law of evolution on a union of elements.

In this paper, we restrict ourselves to the simplest possible coarse graining of the magnetospheric signal. This is given by choosing a threshold C and assigning the symbols "1" and "0" to the signal, depending on whether it is above or below the threshold (binary partition). The threshold is usually the mean value of the data considered. In this way, each time window of the original D_{st} time series for a given threshold is transformed into symbolic sequences, which contains "linguistic" or "symbolic dynamics" characteristics. The selection of a two-symbol alphabet satisfies terms of simplicity and computational convenience.

3. The concepts of Block entropy, T-complexity and Approximate entropy

The term "entropy" is used in both physics and information theory to describe the amount of uncertainty or information inherent in an object or system. Clausius introduced the notion of entropy into thermodynamics in order to explain the irreversibility of certain physical processes in thermodynamics. In statistical thermodynamics the most general formula for the thermodynamic entropy S of a thermodynamic system is the Boltzmann-Gibbs entropy,

$$S_{B-G} = -k \sum p_i \ln p_i$$

where k is the Boltzmann constant and p_i are the probabilities associated with the microscopic configurations.

The Boltzmann-Gibbs entropy translates over almost unchanged into the world of quantum physics to give the von Neumann entropy,

$$S = -k \ \mathrm{Tr}(\rho \ln \rho)$$

where ρ is the density matrix of the quantum mechanical system.

Shannon recognized that a similar approach to Boltzmann-Gibbs entropy could be applied to information theory. In his famous 1948 paper (Shannon, 1948), he introduced a probabilistic entropy measure H_S:

$$H_S(X) = -\sum_{i=1}^{n} p(x_i) \log_b p(x_i),$$

where b is the base of the logarithm used and p denotes the probability mass function of a discrete random variable X with possible values $\{x_1, ..., x_n\}$.

3.1. Dynamical (Shannon-like) Block entropy

Block entropies, depending on the word-frequency distribution, are of special interest, extending Shannon's classical definition of the entropy of a single state to the entropy of a succession of states (Nicolis & Gaspard, 1994; Karamanos & Nicolis, 1999). Each entropy takes a large (small) value if there are many (few) kinds of patterns, therefore, it decreases while the organization of patterns is increasing. In this way, the block entropy can measure the complexity of a signal.

In particular, we estimate the block entropies by lumping. Lumping is the reading of the symbolic sequence by "taking portions", as opposed to gliding where one has essentially a "moving frame". In general, the basic novelty of the entropy analysis by lumping is that, unlike the Fourier transform or the conventional entropy by gliding, it gives results that can be related to algorithmic aspects of the sequences.

It is useful to transform the initial raw data of the magnetospheric signal into symbolic sequences taking values in the alphabet $\{0, 1\}$, according to the rules $A_i = 1$ if $A(t_i) > E[A(t_i)]$ and $A_i = 0$ if $A(t_i) < E[A(t_i)]$, where $A(t_i)$ are the values of the measured field at time t_i and $E[A(t_i)] =< A(t_i) >$ is the mean value in the particular time windows, as it is nicely stated in Schwarz et al. (1993).

Consider a subsequence of length N selected out of a very long (theoretically infinite) symbolic sequence. We stipulate that this subsequence is to be read in terms of distinct "blocks" of length n,

$$\ldots \underbrace{A_1 \ldots A_n}_{B_1} \underbrace{A_{n+1} \ldots A_{2n}}_{B_2} \ldots \underbrace{A_{jn+1} \ldots A_{(j+1)n}}_{B_{j+1}} \ldots \tag{1}$$

We call this reading procedure "lumping".

The following quantities characterize the information content of the sequence (Khinchin, 1957; Ebeling & Nicolis, 1992)

i) The dynamical (Shannon-like) block-entropy for blocks of length n

$$H(n) = - \sum_{(A_1,\ldots,A_n)} p^{(n)}(A_1, \ldots, A_n) \cdot \ln p^{(n)}(A_1, \ldots, A_n) \tag{2}$$

where the probability of occurrence of a block $A_1 \ldots A_n$, denoted $p^{(n)}(A_1, \ldots, A_n)$, is defined by the fraction (when it exists) in the statistical limit as

$$\frac{\text{No. of blocks,} \quad A_1 \ldots A_n, \quad \text{encountered when lumping}}{\text{total No. of blocks}} \tag{3}$$

starting from the beginning of the sequence, and the associate entropy per letter

$$h^{(n)} = \frac{H(n)}{n}. \tag{4}$$

ii) The conditional entropy or entropy excess associated with the addition of a symbol to the right of an n-block

$$h_{(n)} = H(n+1) - H(n). \tag{5}$$

iii) The entropy of the source (a topological invariant), defined as the limit (if it exists)

$$h = \lim_{n\to\infty} h_{(n)} = \lim_{n\to\infty} h^{(n)} \tag{6}$$

which is the discrete analog of metric or Kolmogorov entropy.

We now turn to the selection problem that is to the possibility of emergence of some preferred configurations (blocks) out of the complete set of different possibilities. The number of all possible symbolic sequences of length n (complexions in the sense of Boltzmann) in a K-letter alphabet is (Karamanos & Nicolis, 1999)

$$N_K = K^n. \tag{7}$$

Yet not all of these configurations are necessarily realized by the dynamics, nor they are equiprobable. A remarkable theorem due to McMillan (Khinchin, 1957; Nicolis & Gaspard, 1994), gives a partial answer to the selection problem asserting that for a block $(A_1, ..., A_n)$ the following holds

$$p_n(A_1, ..., A_n) \sim e^{-H(n)} \tag{8}$$

for almost all blocks $(A_1, ..., A_n)$. In order to determine the abundance of long blocks one is thus led to examine the scaling properties of $H(n)$ as a function of n.

As we have already mentioned, the Fourier spectrum or the standard convention of the entropy analysis by gliding, do not help us to distinguish between symbolic sequences with completely different levels of complexity and spectra (Karamanos, 2001). Unlike the previous methods, the novelty of the entropy analysis by lumping gives results, which can be connected with algorithmic aspects of the sequences, in particular with the property of the sequence to be generated by deterministic or stochastic automata (see Karamanos, 2001). Also, the entropy analysis by lumping of some weakly chaotic systems, gives a rather characteristic entropy spectrum, as explained in (Karamanos, 2001). This shows that the entropy analysis by lumping is much more sensitive in algorithmic and ergodic properties of (weakly) chaotic systems than the classical conventional entropy analysis by gliding, or the correlation functions.

3.2. T-complexity

In this Section we introduce the grammar-based complexity measure referred here as the T-complexity or T-entropy. T-entropy is a different grammar-based complexity / information measure defined for infinite, as well as finite strings of symbols (Titchener, 1998, 2000; Ebeling et al., 2001; Steuer et al. 2001). It is a weighted count of the number of production steps required to construct the string from its alphabet. Briefly, it is based on the intellectual economy one makes when rewriting a string according to some rules. The basic fact for the T-complexity is that it puts the problem of the algorithmic compressibility in a well understandable basis (and also in a firm mathematical basis).

Let us note again that the method of T-entropy is based on the *rewriting* of a word according to some basic rules. This way of rewriting is unique and therefore leads to a unique characterization by the corresponding T-complexity measure. Before analyzing in some depth the results coming from the application of the notion of T-complexity in real-world problems, we would like to describe how the T-complexity is computed, at least for finite strings.

The T-*complexity* of a string is defined by the use of one *recursive hierarchical pattern copying (RHPC) algorithm* (Titchener, 2000). It computes the *effective number of T-augmentation steps* required to generate the string.

The T-complexity may be thus computed effectively from any string and the resultant value is unique.

We shall denote by $\mathbb{N}$ the set of natural numbers, and let $\mathbb{N}^+ = \mathbb{N} \setminus \{0\}$. Let the set $A = \{a_1, ..., a_l\}$, $l > 1$, be a finite alphabet. The elements of A are called *symbols* or *characters* and the cardinality of A is denoted by $\#A$, i.e., $\#A = l$. A^* denotes the free monoid generated by A under concatenation. The elements of the set A^* are called *strings*; λ denotes the empty string. We further denote the set $A^* \setminus \{\lambda\}$ by A^+.

The string $x(n)$ is parsed to derive constituent patterns $p_i \in A^+$ and associated copy-exponents $k_i \in \mathbb{N}^+, i = 1, 2, ..., q$, where $q \in \mathbb{N}^+$ satisfying:

$$x = p_q^{k_q} p_{q-1}^{k_{q-1}} ... p_i^{k_i} ... p_1^{k_1} \alpha_0, \quad \alpha_0 \in A. \tag{9}$$

Each pattern p_i is further constrained to satisfy:

$$p_i = p_{i-1}^{m_{i,i-1}} p_{i-2}^{m_{i,i-2}} ... p_j^{m_{i,j}} ... p_1^{m_{i,1}} \alpha_i, \tag{10}$$

$$\alpha_i \in A \quad and \quad 0 \leq m_{i,j} \leq k_j.$$

The T-*complexity* $C_T(x(n))$ is defined in terms of the copy-exponents k_i:

$$C_T(x(n)) = \sum_i^q \ln(k_i + 1). \tag{11}$$

One may verify that $C_T(x(n))$ is minimal for a string comprising a single repeating character. From equation (11) we have:

$$\ln n \leq C_T(x(n)). \tag{12}$$

The upper bound is more difficult to derive. However, for $n > n_0$ (Ebeling et al., 2001)

$$C_T(x(n)) \leq li(\ln 2 \ln(\#A^n)), \tag{13}$$

where $li(z) = \int_z^0 du/\ln u$ is the logarithmic integral function. For a binary alphabet $n_0 \simeq 15$, i.e. small enough to be of no consequence as we are typically concerned with strings in the range of $n = 10^2 - 10^4$ bits. In practice we parse the string repeatedly from *left-to-right* but select the *patterns from right to left*.

The T-*information* $I_T(x(n))$ of the string $x(n)$ is defined as the *inverse logarithmic integral* of the T-complexity divided by a scaling constant $\ln 2$ (Ebeling et al., 2001):

$$I_T(x(n)) = li^{-1}\left(\frac{C_T(x(n))}{\ln 2}\right). \tag{14}$$

In the limit $n \to \infty$ we have that $I_T(x(n)) \leq \ln(\#A^n)$. The form of the right-hand side may be recognizable as the maximum possible n-block entropy of Shannon's definition (see Section 3.1). The neperian logarithm implicitly gives to the T-information the units of *n*ats (nat is a logarithmic unit of information or entropy, based on natural logarithms and powers of e, rather than the powers of 2 and base 2 logarithms which define the bit; the nat is the natural unit for information measures). $I_T(x(n))$ is the total T-information for $x(n)$. The *average T-information rate per symbol*, referred here as the average T-entropy of $x(n)$ and denote by $h_T(x(n))$, is defined along similar lines,

$$h_T(x(n)) = \frac{I_T(x(n))}{n}(nats/symbol). \tag{15}$$

Clearly we note that in the limit of $n \to \infty$, $h_T(x(n)) \leq \ln(\#A) = K$. The correspondence between the T-information and T-entropy on the one hand and Shannon's entropy definitions on the other hand, is reinforced in subsequent investigations (Titchener, 1998, 2000; Ebeling et al., 2001). An example of an actual calculation of the T-complexity for a finite string is given in (Titchener, 1998, 2000; Ebeling et al., 2001).

3.3. Approximate entropy

Related to time series analysis, approximate entropy ($ApEn$) provides a measure of the degree of irregularity or randomness within a series of data (of length N). $ApEn$ was pioneered by Pincus as a measure of system complexity (Pincus, 1991). It is closely related to Kolmogorov entropy, which is a measure of the rate of generation of new information. This family of statistics is rooted in the work of Grassberger & Procaccia (1983) and has been widely applied in biological systems (Pincus & Goldberger, 1994; Pincus & Singer, 1996 and references therein).

The approximate entropy examines time series for similar epochs: more similar and more frequent epochs lead to lower values of $ApEn$. In a more qualitative point of view, given N points, the $ApEn$-like statistics is approximately equal to the negative logarithm of the conditional probability that two sequences that are similar for m points remain similar, that is, within a tolerance r, at the next point. Smaller $ApEn$-values indicate a greater chance that a set of data will be followed by similar data (regularity), thus, smaller values indicate greater regularity. Conversely, a greater value for $ApEn$ signifies a lesser chance of similar data being repeated (irregularity), hence, greater values convey more disorder, randomness and system complexity. Thus a low / high value of $ApEn$ reflects a high / low degree of regularity. The following is a description of the calculation of $ApEn$. Given any sequence of data points $u(i)$ from $i = 1$ to N, it is possible to define vector sequences $x(i)$, which consist of length m and are made up of consecutive $u(i)$, specifically defined by the following:

$$x(i) = (u[i], u[i+1], ..., u[i+m-1]). \tag{16}$$

In order to estimate the frequency that vectors $x(i)$ repeat themselves throughout the data set within a tolerance r, the distance $d(x[i], x[j])$ is defined as the maximum difference between the scalar components $x(i)$ and $x(j)$. Explicitly, two vectors $x(i)$ and $x(j)$ are "similar" within the tolerance or filter r, namely $d(x[i], x[j]) \leq r$, if the difference between any two values for $u(i)$ and $u(j)$ within runs of length m are less than r (i.e. $|u(i+k) - u(j+k)| \leq r$ for $0 \leq k \leq m$). Subsequently, $C_i^m(r)$ is defined as the frequency of occurrence of similar runs m within the tolerance r:

$$C_i^m(r) = \frac{[\text{number of} \quad j \quad \text{such that} \quad d(x[i], x[j]) \leq r]}{(N-m-1)},$$

where $j \leq (N-m-1)$.

Taking the natural logarithm of $C_i^m(r)$, the function $\Phi^m(r)$ is defined as the average of $ln(C_i^m(r))$:

$$\Phi^m(r) = \sum_i lnC_i^m(r)/(N-m-1) \tag{17}$$

where $\sum_i$ is a sum from $i = 1$ to $(N - m - 1)$. $\Phi^m(r)$ is a measure of the prevalence of repetitive patterns of length m within the filter r.

Finally, approximate entropy, or $ApEn(m, r, N)$, is defined as the natural logarithm of the relative prevalence of repetitive patterns of length m as compared with those of length $m + 1$:

$$ApEn(m, r, N) = \Phi^m(r) - \Phi^{m+1}(r). \tag{18}$$

Thus, $ApEn(m, r, N)$ measures the logarithmic frequency that similar runs (within the filter r) of length m also remain similar when the length of the run is increased by 1. Thus, small values of $ApEn$ indicate regularity, given that i increasing run length m by 1 does not decrease the value of $\Phi^m(r)$ significantly (i.e., regularity connotes that $\Phi^m[r] \approx \Phi^{m+1}[r]$). $ApEn(m, r, N)$ is expressed as a difference, but in essence it represents a ratio; note that $\Phi^m[r]$ is a logarithm of the averaged $C_i^m(r)$, and the ratio of logarithms is equivalent to their difference. A more comprehensive description of $ApEn$ may be found in (Pincus, 1991; Pincus & Goldberger, 1994; Pincus & Singer, 1996).

In summary, $ApEn$ is a "regularity statistics" that quantifies the unpredictability of fluctuations in a time series. Intuitively, one may reason that the presence of repetitive patterns of fluctuation in a time series renders it more predictable than a time series in which such patterns are absent. $ApEn$ reflects the likelihood that "similar" patterns of observations will not be followed by additional "similar" observations. A time series containing many repetitive patterns has a relatively small $ApEn$; a less predictable (i.e., more complex) process has a higher $ApEn$.

4. Results

Figure 1 gives the temporal evolution of D_{st} along with corresponding time variations of the Block entropy, the T-complexity and, the $ApEn$ for the whole year of 2001. All the relative entropy measures were calculated using a moving time window of 256 hours. We see how nicely the entropy measures identify the different complexity regimes in the D_{st} time series (c.f. red part of the corresponding plots). Figure 1 further demonstrates that the $ApEn$ entropy yields superior results in comparison to the other entropy measures regarding the detection of dynamical complexity in the Earth's magnetosphere (i.e., offer a clearer picture of the transition). A possible explanation for this is that $ApEn$ is more stable when dealing with nonstationary signals of dynamical systems (such the magnetospheric signal) than the other entropy measures presented here.

5. Conclusions and Discussion

Block entropy, T-complexity and Approximate entropy sensitively show the complexity dissimilarity among different "physiological" (quiet-time) and "pathological" states (intense magnetic storms). They imply the emergence of two distinct patterns : (i) a pattern associated with the intense magnetic storms, which is characterized by a higher degree of organization, and (ii) a pattern associated with normal periods, which is characterized by a lower degree of organization.

The present study confirms the conclusions of a previous work based on an independent linear fractal spectral analysis (Hurst exponent) using wavelet transforms (Balasis et al., 2006). The Hurst exponent analysis also shows the existence of two different patterns: (i) a pattern associated with the intense magnetic storms, which is characterized by a fractional Brownian persistent behavior; (ii) a pattern associated with normal periods, which is characterized by a fractional Brownian anti-persistent behavior.

Furthermore, the non-extensive Tsallis entropy has been recently introduced (Balasis et al., 2008), as an effective complexity measure for the analysis of the D_{st} index. Tsallis entropy has been also shown to sensitively detect the complexity dissimilarity between pre-storm activity and intense magnetic storms in the Earth's magnetosphere.

We stress that the anti-persistent time windows correspond to the time windows of higher entropies, while the persistent time windows correspond to the time windows of lower entropies. Importantly, a recent analysis presented by *Carbone and Stanley* [2007] shows that anti-correlated time series, with Hurst exponent $0.5 < H < 1$, are characterized by entropies greater than correlated time series having $0.5 < H < 1$. This suggestion is in agreement with our results.

An important remark is the agreement of the results between the linear analysis in terms of the Hurst exponent and nonlinear entropy analyses. A combination of linear and nonlinear analysis techniques can offer a firm warning that the onset of an intense magnetic storm is imminent.

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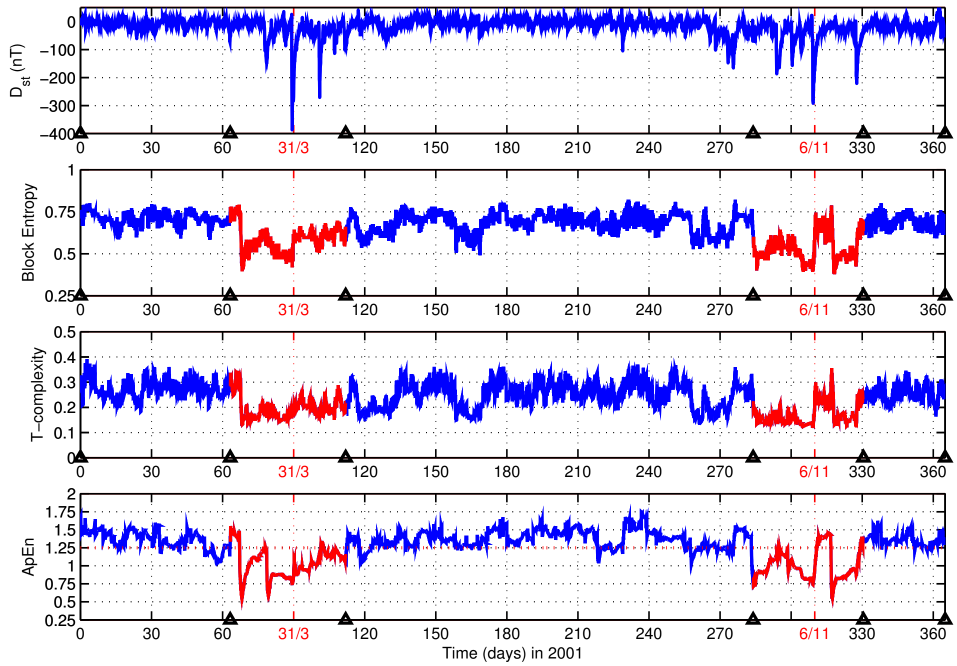

Figure 1.: From top to bottom: D_{st} time series along with time variations of Block entropies, Approximate entopies $ApEn$ and T-complexities. The 31 March and 6 November 2001 magnetic storms are marked with red. The red dashed line in $ApEn$ plot marks a possible boundary value for the transition to the lower complexity characterizing the different state of the magnetosphere.

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