Investigating magnetospheric dynamics using various complexity measures

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Abstract. Dynamical complexity detection for output time series of complex systems is one of the foremost problems in physics, biology, engineering, and economic sciences. Especially in magnetospheric physics, accurate detection of the dissimilarity of complexity between normal and abnormal states (e.g. pre-storm activity and magnetic storms) can vastly improve space weather diagnosis and, consequently, the mitigation of space weather hazards. A variety of complexity measures based on linear and nonlinear analysis techniques (i.e., wavelet transforms and entropies, respectively) is applied to the Dst index time variations in order to detect changes that can play the role of warnings for future magnetic storm occurrence.

INTRODUCTION

Accumulated evidence points to the complex character of magnetosphere dynamics. For instance, Baker et al. [1], Vassiliadis et al. [2] and Sharma et al. [3] studied the occurrence of low dimensional chaos in magnetospheric activity, while Klimas et al. [4] discussed the nonlinear character of magnetosphere dynamics. Other studies [5, 6] witnessed several aspects of the complex character of the plasma sheet. Thus, recent advances in the study of complexity and complex systems open new research perspectives to the investigation of the magnetospheric dynamics [7].

In the context of complex system theory, we are aiming to develop a quantitative identification of magnetic storm warnings. Therefore, we have used concepts of entropy and tools of information theory in order to identify statistical patterns in the Dst index time variations. It is expected that a significant change in the statistical pattern represents a deviation from normal (background) behavior, revealing the presence of an abnormal state related to a magnetic storm.

The present study is focused on one year Dst data (2001), characterized by two intense magnetic storms, in the first and last trimester of the considered interval (i.e., 31/3/2001 and 6/11/2001 with minimum Dst -387 nT and -292 nT, respectively). The data were retrieved from the World Data Center for Geomagnetism, Kyoto (http://swdcwww.kugi.kyoto-u.ac.jp/) and are presented in Figure 1.

First, fractal spectral analysis of the Dst time series offers information concerning normal/abnormal magnetospheric state discrimination by showing that the abnormal state follows the fractional Brownian motion (fBm) model and has persistent behavior while, on the contrary, the normal state follows the same model but with anti-persistent behavior.

Then, we attempt a symbolic analysis of Dst index in terms of non-extensive Tsallis entropy. It is well-known that the traditional Shannon entropy works best in dealing with systems composed of subsystems which can access all the available phase space and which are either independent or interact via short-range forces. For systems exhibiting long-range correlations, memory, or fractal properties, non-extensive Tsallis entropy becomes the most appropriate mathematical tool [8, 9]. A central property of magnetic storm emergence is the occurrence of coherent large-scale collective behavior with a very rich structure, resulting from repeated nonlinear interactions among the constituents of the global geospace system. Consequently, Tsallis entropy is an appropriate tool for identifying magnetic storm precursors.

LINEAR TECHNIQUES

To analyze Dst data, we have first used computational algorithms based on wavelet transforms [10, 11, 12, 13]. The wavelet spectral analysis is superior to the Fourier spectral analysis, because it provides excellent decompositions of even transient, non-stationary signals [14]. It has the ability of providing a representation of the signal in both the time and frequency domains. In contrast to the Fourier transform, which provides the description of the overall regularity of signals, the wavelet transform identifies the temporal evolution of various frequencies.
This property is suitable for the signals under investigation, because they are non-stationary by their nature, and have a time-varying frequency content.

Wavelet spectral analysis allows quantitative monitoring of the signal evolution by decomposing a time series into a linear superposition of predefined mathematical waveforms, each with finite duration and narrow frequency content [15]. Thus, the frequency range of the analyzing wavelets corresponds to the spectral content of time series components. Wavelet analysis is becoming a common tool for analyzing localized variations of power within a time series. By decomposing a time series into time-frequency space, one is able to determine both the dominant modes of variability and how those modes vary in time. The advantage of analyzing a signal with wavelets as the analyzing kernel, is that it enables one to study features of the signal locally with a detail matched to their scale. Owing to its unique time-frequency localization, wavelet analysis is especially useful for signals that are non-stationary, have short-lived transient components, have features at different scales, or have singularities.

We have applied the continuous wavelet transform with the Morlet wavelet as the basis function [16]. Our results have been checked for consistency using the Paul and DOG mother functions [16]. We should also stress that there are several parameters of the wavelet transform (e.g., frequency range, power spectral density amplification factor, etc.), which were needed to be correctly adjusted in order to capture different kinds of magnetospheric signals. This tuning of the wavelet transform is rather time consuming, but it has been an important step of our analysis.

If a time series is a temporal fractal then a power-law of the form $S(f) \propto f^{-\beta}$ is obeyed, with $S(f)$ the power spectral density and $f$ the frequency. The spectral scaling exponent $\beta$ is a measure of the strength of time correlations. The goodness of the power law fit to a time series is represented by a linear correlation coefficient, $r$.

The $\beta$ exponent is related to the Hurst exponent $H$: $\beta = 2H + 1$, with $0 < H < 1$ ($1 < \beta < 3$) for the fBm model [12]. The exponent $H$ characterizes the persistent/anti-persistent properties of the signal.

For the 1-year period considered here (year 2001), applying the wavelet transform to the hourly Dst values leads to a matrix with $65 \times 365 \times 24$ elements, where 65 is the number of frequencies. Then, power spectral densities were estimated in the frequency range from 2 to 128 h, using a moving window of 256 samples. The number of samples by which the moving window sections overlap is 255. Finally, the spectral parameters $\beta$ and consequently Hurst exponents $H$ were calculated for each window.

In Figure 1 the values of the $H$ parameter are shown for the Dst index data. When the corresponding $H$ values are in the interval $(0, 0.5)$ the time series has anti-persistent properties, which means that if the fluctuations increase with time, it is likely to decrease in the interval immediately following and vice versa. Physically, this implies that fluctuations tend to induce stability within the system (negative feedback mechanism).

Figure 1 reveals that the Dst time series exhibits anti-persistent properties during the quiet period (i.e. well before and after 31 March and 6 November 2001 intense magnetic storms). If $H$ takes values in the interval $(0.5, 1)$ the signal exhibits persistent properties, which means that if the amplitude of the fluctuations increases with time, it is likely to continue increasing in the immediately next interval. In other words, the underlying dynamics is governed by a positive feedback mechanism.

Figure 1 shows that Dst exhibits persistent properties $(0.5 < H < 1)$ around 31 March and 6 November 2001 intense magnetic storms (c.f. parts of $H$ plot marked in red represent persistency). Thus, it is evident that the onset and development of the magnetic storms of 31/3/2001 and 6/11/2001 are associated with persistent behavior.

The value $H = 0.5$ suggests that there is no correlation between the repeated increments. Consequently, this particular value takes on a special physical meaning: it marks the transition between anti-persistent and persistent behavior in the time series. To conclude, one can recognize two different regimes. The first refers to epochs of low magnetospheric activity, where the associated Dst time series follows anti-persistent behavior. The second regime refers to the epoch including an intense magnetic storm, where the Dst time series shows persistent behavior.

**NONLINEAR TECHNIQUES**

Recently there has been renewed interest in extreme dynamic events in nature. In the last few years, a new branch of research, named “the physics of complex systems” has emerged within the field of statistical mechanics. An important characteristic of interconnected systems is that there are emerging properties of the systems, which are not properties of the individual constituents. We refer to “a complex system” as one whose phenomenological laws describing the global behavior of the system are not necessarily directly related to the “microscopic” (i.e., elemental) laws that regulate the evolution of its elementary parts. In other words, “complexity” is the emergence of a non-trivial behavior due to the interactions of the subunits that constitute the system.

A basic reason for our interest in “complexity” is the striking similarity in behavior near the critical point among systems that are otherwise quite different in nature in Nature. Perhaps one of the main reasons for the growing interest in complex networks is that many sys-
tems in the real world, either naturally evolved or artificially designed, are indeed organized in a networked fashion. The main feature of collective behavior is that an individual unit’s action is dominated by the influence of its neighbors; the unit behaves differently from the way it would behave on its own. Finally, ordering phenomena emerge as the units simultaneously change their behavior to a common pattern [17, 18]. The emphasis in the structure formation in complex systems is bridging the gap between what one element does and what many of them do when they function cooperatively. Consequently, it follows that the science of complexity is about revealing the principles that govern the ways in which these new properties appear. Indeed, an important challenge in this field of research is to distinguish characteristic epochs in the evolution of precursory activity and identify them with the equivalent last stages in the extreme event preparation process. We attempt to approach this challenge bringing together experimental precursory data and aspects having their roots in statistical physics.

Complexity is a measure of off-equilibrium “order”. The entropy provides a way of quantifying the order/disorder of a time series.

A way to examine transient phenomena is to analyze the pre-storm Dst time series into a sequence of distinct time windows. The aim is to discover a clear difference of dynamical characteristic as the extreme event is approaching. It is expected that as a magnetic storm approaches, there is a clear transition from higher to lower complexity. If the analysis yields different complexity values for a number of consecutive time-windows, then a different pre-storm epoch in the time series, probably corresponding to a characteristic stage of the magnetic storm preparation process, can be recognized. In summary, a time-dependent complexity is employed to characterize the level of precursory “injury”.

Shannon entropy

The term “entropy” is used in both physics and information theory to describe the amount of uncertainty or information inherent in an object or system. Clausius introduced the notion of entropy into thermodynamics in order to explain the irreversibility of certain physical processes in thermodynamics. In statistical thermodynamics the most general formula for the thermodynamic entropy $S$ of a thermodynamic system is the Boltzmann-Gibbs entropy,

$$S_{B-G} = -k \sum p_i \ln p_i$$

where $k$ is the Boltzmann constant and $p_i$ are the probabilities associated with the microscopic configurations.

The Boltzmann-Gibbs entropy translates over almost unchanged into the world of quantum physics to give the von Neumann entropy,

$$S = -k \text{Tr}(\rho \ln \rho)$$

where $\rho$ is the density matrix of the quantum mechanical system.

Shannon recognized that a similar approach to Boltzmann-Gibbs entropy could be applied to information theory. In his famous 1948 paper [19], he introduced a probabilistic entropy measure $H_S$:

$$H_S(X) = -\sum_{i=1}^{n} p(x_i) \log_b p(x_i),$$

where $b$ is the base of the logarithm used and $p$ denotes the probability mass function of a discrete random variable $X$ with possible values $\{x_1, \ldots, x_n\}$.

Symbolic dynamics

The basic idea of symbolic dynamics is quite simple. One divides the phase space into a finite number of partitions and labels each partition with a symbol (e.g. a letter from some alphabet). Instead of representing the trajectories by infinite sequences of numbers-iterates from a discrete map or sampled points along the trajectories of a continuous flow, one watches the alteration of symbols. Of course, in so doing one loses an amount of detailed information, but some of the invariant, robust properties of the dynamics may be kept, e.g. periodicity, symmetry, or the chaotic nature of an orbit [20].

In the framework of symbolic dynamics, time series are transformed into a series of symbols by using an appropriate partition which results in relatively few symbols. After symbolization, the next step is the construction of “symbol sequences” (“words” in the language symbolic dynamics) from the symbol series by collecting groups of symbols together in temporal order.

To be more precise, the simplest possible coarsening of a time series is given by choosing a threshold $C$ (usually the mean value of the data considered) and assigning the symbols “1” and “0” to the signal, depending on whether it is above or below the threshold (binary partition).

Non-extensive Tsallis entropy

The uncertainty of an open system state can be quantified by the Boltzmann-Gibbs entropy, which is the widest known uncertainty measure in statistical mechanics. Boltzmann-Gibbs entropy cannot, however, describe
nonequilibrium physical systems with large variability and multi-fractal structure such as the solar wind. Inspired by multi-fractal concepts, Tsallis [8, 9] has proposed a generalization of the Boltzmann-Gibbs statistics, which is briefly described here.

The aim of statistical mechanics is to establish a direct link between the mechanical laws and classical thermodynamics. One of the crucial properties of the Boltzmann-Gibbs entropy in the context of classical thermodynamics is extensivity, namely proportionality with the number of elements of the system. The Boltzmann-Gibbs entropy satisfies this prescription if the subsystems are statistically (quasi-) independent, or typically if the correlations within the system are essentially local. In such cases the system is called extensive.

In general, however, the situation is not of this type and correlations may be far from negligible at all scales. In such cases the Boltzmann-Gibbs entropy is non-extensive. Tsallis [8, 9] introduced an entropic expression characterized by an index $q$ which leads to a non-extensive statistics,

$$ S_q = k \left( \frac{1}{q-1} \left( 1 - \sum_{i=1}^{W} p_i^q \right) \right), $$

where $p_i$ are the probabilities associated with the microscopic configurations, $W$ is their total number, $q$ is a real number, and $k$ is Boltzmann’s constant. The value of $q$ is a measure of the non-extensivity of the system: $q \to 1$ corresponds to the standard extensive Boltzmann-Gibbs statistics.

This is the basis of the so called non-extensive statistical mechanics, which generalizes the Boltzmann-Gibbs theory. The entropic index $q$ characterizes the degree of non-additivity reflected in the following pseudo-additivity rule:

$$ S_q(A + B) = S_q(A) + S_q(B) + (1 - q)S_q(A)S_q(B). $$

The cases $q > 1$ and $q < 1$, correspond to sub-additivity, or super-additivity, respectively. For subsystems that have special probability correlations, extensivity is not valid for Boltzmann-Gibbs entropy, but may occur for $S_q$ with a particular value of the index $q$. Such systems are sometimes referred to as non-extensive. The parameter $q$ itself is not a measure of the complexity of the system but measures the degree of non-extensivity of the system. It is the time variations of the Tsallis entropy for a given $q$ ($S_q$) that quantify the dynamic changes of the complexity of the system. Lower $S_q$ values characterize the portions of the signal with lower complexity.

Herein, we estimate $S_q$ based on the concept of symbolic dynamics and by using the technique of lumping (for details the reader is referred to [21]). To be more precise, the simplest possible coarse graining of the Dst index is given by choosing a threshold $C$ (usually the mean value of the data considered) and assigning the symbols “1” and “0” to the signal, depending on whether it is above or below the threshold (binary partition). Thus, we generate a symbolic time series from a 2-letter ($\lambda = 2$) alphabet (0,1), e.g. 011010110110110110.

Reading the sequence by lumping of length $L = 2$ one obtains 01/10/01/01/01/01/01/10/... The number of all possible kinds of blocks is $\lambda^L = 2^2 = 4$, namely 00, 01, 10, 11. Thus, the required probabilities for the estimation of the Tsallis entropy $p_{00}, p_{01}, p_{10}, p_{11}$ are the fractions of the blocks 00, 01, 10, 11 in the symbolic time series.

The $S_q$ for the word length $L$ is

$$ S_q(L) = k \frac{1}{q-1} \left( 1 - \sum_{(A_1,A_2,...,A_L)} [p(L)_{A_1,A_2,...,A_L}]^q \right). $$

(4)

Broad symbol-sequence frequency distributions produce high entropy values, indicating a low degree of organization. Conversely, when certain sequences exhibit high frequencies, lower entropy values are produced, indicating a high degree of organization.

Next, we focus on calculations related to the traditional Shannon entropy and the non-extensive Tsallis entropy. Figure 1 gives the temporal evolution of Dst along with corresponding time variations of the Hurst exponent, $H$, Shannon entropy and Tsallis entropy for the whole year of 2001. In terms of entropy measures, we see how nicely Tsallis entropy variations identify the different complexity regimes in the Dst time series (c.f. red part of the corresponding plot). Figure 1 further demonstrates that Tsallis entropy along with Hurst exponent yield superior results in comparison to Shannon entropy regarding the detection of dynamical complexity in the Earth’s magnetosphere (i.e., offer a clearer picture of the transition from normal state to magnetic storms). A possible explanation for this is that Tsallis is an entropy obeying a non-extensive statistical theory, which is different from the usual Boltzmann-Gibbs statistical mechanics obeyed by Shannon entropy. Therefore, it is expected to better describe the dynamics of the magnetosphere, which is a nonequilibrium physical system with large variability.

**CONCLUSIONS AND DISCUSSION**

Non-extensive Tsallis entropy sensitively shows the complexity dissimilarity among different “physiological” (normal) and “pathological” states (intense magnetic storms). Tsallis entropy implies the emergence of two distinct patterns: (i) a pattern associated with the intense
magnetic storms, which is characterized by a higher degree of organization (ii) a pattern associated with normal (quiet-time) periods, which is characterized by a lower degree of organization.

The wavelet spectral analysis in terms of Hurst exponent, $H$, also shows the existence of two different patterns: (i) a pattern associated with the intense magnetic storms, which is characterized by a fractional Brownian persistent behavior (ii) a pattern associated with normal periods, which is characterized by a fractional Brownian anti-persistent behavior.

We stress that the anti-persistent time windows correspond to the time windows of high entropy, while the persistent time windows correspond to the time windows of low entropy.

In summary, a combination of linear and nonlinear analysis techniques proved to be a powerful tool, showing evidence that the occurrence of an intense magnetic storm is imminent, and thus, providing convenience for space weather applications.

It is expected that the results will contribute to a better understanding of magnetic storm mechanism focusing on two fundamental questions that are as yet unanswered: (i) is there a way of estimating the time to global instability? (ii) is the evolution towards global instability irreversible after the appearance of distinguishing features in the Dst time series?

Recently, Balasis et al. [22] demonstrated that other entropy measures such as block entropy, T-complexity, and approximate entropy can also be used to characterize the dynamics of the magnetosphere. Consolini et al. [23] attempted a verification of the magnetospheric nonequilibrium dynamics by investigating the long-term evolution of the Earth’s magnetosphere, as monitored by Dst. They were able to provide a proof of the existence of a steady state far from equilibrium for the Earth’s magnetosphere.

Other studies also indicate the existence of two different regimes in the dynamics of magnetosphere. Sitnov et al. [24] suggest that the substorm dynamics resembles second-order phase transitions, while magnetic storms, are shown to reveal the features of first-order nonequilibrium transitions. The anti-persistency / persistency well meet the second order / first order phase transition correspondingly. Metastability and topological complexity of magnetic field, emerging from Chang’s model [25] also justify the evidence for transition from pre-storm activity to magnetic storms found in our study. Furthermore, Chang et al. [26, 27] and Vörös et al. [28] described intermittent turbulence in space plasmas which is consistent with the ideas derived here. Recently Vörös et al. [29] examined the statistical properties of magnetic fluctuations in the Venusian magnetosheath and wake regions. They found multiscale turbulence at the magnetosheath boundary layer and near the quasi-parallel bow shock.

Additionally, similar behavior to our observations (i.e., reduction of multiscale complexity) was observed in high-latitude geomagnetic activity prior to strong substorms using a different methodology. Uritsky and Puodvin [30] and Uritsky et al. [31] presented cellular automata models which allowed interpretation of the observed effects in terms of transitions between critical, supercritical and subcritical states. In [32] the authors provided evidence for similar behavior in the spatial scaling of the auroral brightness. Wanliss et al. [33] applied symbolic dynamics analysis to Dst time series for modeling magnetic storms. They presented evidence for intermittency and nonGaussianity, which are reflective of large magnetic storms. It was also suggested that the ring current is always out of equilibrium and may undergo state changes via multiplicative cascades.

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**REFERENCES**

FIGURE 1. From top to bottom: Dst time series along with time variations of Hurst exponents, $H$, Shannon entropies and Tsallis entropies, $S_q$. The 31 March and 6 November 2001 magnetic storms are marked with red. The red dashed line in $H$ plot marks the transition between anti-persistent and persistent behavior. The red dashed line in $S_q$ plot marks a possible boundary value for the transition to the lower complexity characterizing the different state of the magnetosphere. The triangles denote 5 time intervals in which: first, third and fifth time windows correspond to anti-persistent ($0 < H < 0.5$) or high Tsallis entropies epochs; second and fourth time windows correspond to persistent ($0.5 < H < 1$) or lower Tsallis entropies epochs.