# **Estimating Fluxes of SEPs by Unfolding ESA/SREM Data**

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**Abstract.** The Standard Radiation Environment Monitor (SREM) belongs to a second generation of instruments in a program established by the European Research and Technology Centre (ESTEC) of the European Space Agency (ESA) to provide minimum intrusive particle radiation detectors on ESA spacecrafts for space weather applications. SREM detects high-energy electrons and protons and bins the measurements in overlapping energy channels. In order to estimate the particle fluxes associated with Solar Particle Events (SPEs), a method based on the Singular Value Decomposition (SVD) analysis was developed. This method does not require any assumption on the spectral form of the particle fluxes and includes proper schemes treating issues related to several characteristic properties of the detector. As an example, we present results associated to the January 20, 2005 SPE.

# 1. Introduction

The ESA SREM is a silicon solid state detector established by ESA/ESTEC to provide minimum intrusive radiation detectors for space applications, a radiation hazard alarm function for the instruments on-board spacecraft, investigation activities on possible radiation related anomalies observed on spacecraft and in-flight technology demonstration activities. Seven SREM units have been launched (on-board STRV-1C, Proba-1, INTEGRAL, Rosetta, GIOVE-B, Herschel and Planck spacecrafts) providing a valuable set of data that covers both the near-Earth trapped particle radiation belts (RB) and the interplanetary environment.

SREM consists of three silicon detectors and detects electrons and protons with energies 500 keV and 10 MeV respectively. The measurements are binned into 15 channels. The response function of the instrument is such that protons and electron events are mixed and events recorded in high-energy channels appear also in channels with lower energy thresholds. The proton (electron) response matrix of SREM provides the number of the counts  $C_{i,q}^{cal}(E_l)$  at each channel, as it has been calculated from experimental and numerical calibration techniques, using flat integral proton (electron) omni-directional flux profiles  $F_q^{cal}(E_l)$  [ $cm^{-2}$ ] at various energies. Here, *l* defines the calibration energy bin defined by the discrete energy range [ $E_{l,min}, E_{l,max}$ ]. The

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response function,  $RF_{i,q}(E_l)$ , within the energy bin  $E_l$ , of channel *i* for the particle species *q* is defined through:  $RF_{i,q}(E_l) = C_{i,q}^{cal}(E_l)/F_q^{cal}(E_l)$ . It is evident that the measured count-rate  $C_i$  in each channel i = 1..15 will be given by the sum

$$C_i = \sum_{q=p,e} C_{i,q} = \sum_{q=p,e} \left[ \int_0^\infty f_q(E) RF_{i,q}(E) dE \right]$$
(1)

where  $C_{i,q}$  is related to the incident particle fluxes  $f_q(E) [cm^{-2}MeV^{-1}s^{-1}]$ . Considering a step function form for the flux spectra within  $N_p$  ( $N_e$ ) selected proton (electron) Eq. (1) can be written as

$$\mathbf{C} = \mathbf{C}^{\mathbf{CAL}} \cdot \overline{\mathbf{f}} \tag{2}$$

where **C** is a vector containing the measured count-rates of  $N_b$  SREM channels, and  $\mathbf{\bar{f}} = \mathbf{f} \ \Delta \mathbf{E} / \mathbf{F}^{\mathbf{CAL}}$  is a vector containing  $N_x = N_p + N_e$  values of the unknown differential fluxes  $\mathbf{f}$  normalized to  $\mathbf{F}^{\mathbf{CAL}} / \Delta E$ . The matrix  $\mathbf{C}^{\mathbf{CAL}} \in \mathcal{R}^{N_b \times N_x}$  contains the  $C_{i,q}^{cal}$  elements for  $N_b$  channels at the  $N_x$  flux levels.

### 2. Unfolding SREM count-rates

The numerical solution of Eq. (2) is not exact and presents large variations in neighboring energy bins. Following *Höcker and Kartvelishvili* (1996), we solve instead the system of equations:

$$\begin{bmatrix} \mathbf{C}^{\mathbf{C}\mathbf{A}\mathbf{L}}\\ \sqrt{\tau}\mathbf{R} \end{bmatrix} \cdot \mathbf{\bar{f}} = \begin{bmatrix} \mathbf{C}\\ \mathbf{0} \end{bmatrix}$$
(3)

which introduce a proper regularization condition through the inclusion of the matrix  $\mathbf{R} \in \mathcal{R}^{N_x \times N_x}$ :

$\mathbf{R} =$	$\begin{bmatrix} -1\\ 1\\ 0 \end{bmatrix}$	$\begin{array}{c} 1 \\ -2 \\ 1 \end{array}$	$0 \\ 1 \\ -2$	$\begin{array}{c} \dots \\ 0 \\ 1 \end{array}$	  0	0 0 0	0 0 0	· · · · · · ·	 	· · · · · · ·	· · · · · · ·	0 0 0
	: 0 0 0 0	0 0 0 0 0	$\begin{array}{c} \ddots \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$	$\begin{array}{c} \ddots \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$	$\begin{array}{c} \ddots \\ -2 \\ 1 \\ 0 \\ 0 \end{array}$	$\begin{array}{c} \ddots \\ 1 \\ -1 \\ 0 \\ 0 \end{array}$	$egin{array}{c} 0 \\ 0 \\ 0 \\ - au_{ei} \\  au_{ei} \end{array}$	$egin{array}{c} & \dots & 0 \\ & 0 & & \\ &  au_{ei} & & \\ -2 au_{ei} & & \end{array}$	$egin{array}{c} 0 \\ 0 \\ 0 \\  au_{ei} \end{array}$	$\begin{array}{c} 0\\ 0\\ 0\\ 0\\ 0\\ 0 \end{array}$	$\begin{array}{c} \dots \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$	$\begin{array}{c} \vdots \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$
	$\begin{bmatrix} \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix}$	· · · · · · · · · · ·	· · · · · · · · · · · ·	· · · · · · · · · · · ·	· · · · · · · · · · ·	0 0 0 0	$\begin{array}{c} \ddots \\ 0 \\ 0 \\ 0 \end{array}$	· . 0 0	$\begin{array}{c} \ddots \\ \tau_{ei} \\ 0 \\ \cdots \end{array}$	$\begin{array}{c} \ddots \\ -2\tau_{ei} \\ \tau_{ei} \\ 0 \end{array}$	$\begin{array}{c} 0 \\ \tau_{ei} \\ -2\tau_{ei} \\ \tau_{ei} \end{array}$	$\begin{array}{c} \vdots \\ 0 \\ \tau_{ei} \\ -\tau_{ei} \end{array}$

The numerical solution of the regularized Eq. (3) suppresses *a priori* the *curvature* - defined here as the difference in the flux derivatives between two successive energy bins. The solution of system of Eq. (3) can be expressed as a function of the solution of the unregularized one ( $\tau = 0$ ). For this purpose Eq. (3) is rewritten such as the regularization term to be proportional to I,

$$\begin{bmatrix} \mathbf{A} \\ \sqrt{\tau}\mathbf{I} \end{bmatrix} \cdot \mathbf{R}\mathbf{\bar{f}} = \begin{bmatrix} \mathbf{C} \\ \mathbf{0} \end{bmatrix}$$
(4)

where  $\mathbf{A} = \mathbf{C}^{\mathbf{C}\mathbf{A}\mathbf{L}}\mathbf{R}^{-1}$ . The solution of the unregularized form,  $\mathbf{A} \cdot \mathbf{R}\overline{\mathbf{f}} = \mathbf{C}$  can be determined using the standard SVD technique:  $\mathbf{A}$  is expanded into its left  $\mathbf{U} \in \mathcal{R}^{N_b \times N_b}$  and right  $\mathbf{V} \in \mathcal{R}^{N_x \times N_x}$  singular vectors, i.e.  $\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathbf{T}} \cdot \mathbf{S} \in \mathcal{R}^{N_b \times N_x}$  is a diagonal matrix with non-negative diagonal elements in a non-increasing order (the so-called singular values). The solution is  $\overline{\mathbf{f}}^{(0)} = \mathbf{R}^{-1}\mathbf{V}\mathbf{z}^{(0)}$  where  $z_i^{(0)} = d_i^{(0)}/s_i$ and  $\mathbf{d}^{(0)} \equiv \mathbf{U}^{\mathbf{T}}\mathbf{C}$ . The introduction of  $\tau > 0$ , modifies  $d_i^{(\tau)} = d_i^{(0)}/(1 + \tau/s_i^2)$  and consequently the solution of Eq. (4) is given by

$$\overline{\mathbf{f}}^{(\tau)} = \mathbf{R}^{-1} \mathbf{V} \mathbf{z}^{(\tau)}, \text{ where } z_i^{(\tau)} = \frac{d_i s_i}{s_i^2 + \tau}.$$
(5)

Non-zero  $\tau$  regularizes the singularities - small  $s_i$ 's - which appear for large values of index *i* acting as a cut-off low pass Fourier filter which smooths out the solutions. Since, most channels have overlapping energy response it is not needed to consider the measurements of all simultaneously. Instead, we may consider  $N_b < 15$  channels and calculate the  $n_{sol} = 15!/[N_b!(15 - N_b)!]$  solutions which correspond to all the possible combinations of  $N_b$  out of 15 channels. A measure of the goodness of solutions can be given by the value  $\chi_c^2 = \left\langle \left(c_i - c_i^{est}\right)^2 / c_i^{est^2} \right\rangle$  where  $c_i^{est}$  is the reconstructed countrate calculated by Eq. (2). For the selection of regularisation parameter we chose  $\tau$  to be determined by a pre-defined minimum value of  $\chi_c^2$ . We also exclude solutions with large value of  $\chi_c^2$  by setting an upper bound limit. The estimator of the proton and electron fluxes can be simply given by the average of subset of equivalent numerical solutions with  $\chi_{c\ min}^2 \leq \chi_c^2 \propto \chi_c^2 \max$ .

### 3. The solar particle event of January 20, 2005

Here, we present results on proton flux spectra associated with the SPE of January 20, 2005, using mesurements of INTEGRAL/SREM [cf. Figure 1 (upper panel)]. The January 20, 2005 SPE was associated with the largest solar flare that occurred during the descending phase of solar cycle 23. It was an X-class solar flare of X7.1 magnitude, which occurred in active region AR 10720 accompanied by a rather slow Halo CME. This SPE was accompanied by the largest recorded Ground Level Enhancement (GLE) in half a century by neutron monitors all over the world, which was classified in literature as GLE69. In Figure 1 (lower panel) the unfolded proton flux time series are plotted as derived through the application of the presented method for  $N_b = 12$  with  $N_p = 8$  and  $N_e = 4$ . The repeatative sharp peaks on the count rates are due to the crossing of INTEGRAL through outer RB. The developed unfolding method separates succesfully this component and does not attributes these counts to protons. The derived solar proton flux profiles have been validated succesfully through several comparisons with measurements by other instruments on-board different spacecraft (not shown here).

## 4. Conclusions

A method for the unfolding of SREM fluxes based on the SVD analysis is presented. The method does not require any pre-assumption and it properly addresses issues related to the mixing of electron and proton counts in the detector channels. Using the measurements of the INTEGRAL/SREM, we have estimated the proton fluxes during Sandberg et al.



the SPE of January 20, 2005. We conclude that this method is a valuable tool for the analysis of SPEs using SREM data.

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