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Identification of Non-Linear Space Weather Models of the Van Allen Radiation Belts Using Volterra Networks

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Abstract. Many efforts have been made to develop general dynamical models of the Van Allen radiation belts based on data alone. Early linear prediction filter studies focused on the response of daily-averaged relativistic electrons at geostationary altitudes (Nagai (1988); Baker et al. (1990)). Vassiliadis et al (2005) extended this technique spatially by incorporating SAMPEX electron flux data into linear prediction filters for a broad range of L-shells from 1.1 to 10.0 RE. Nonlinear state space models (Rigler & Baker (2008)) have provided useful initial results on the timescales involved in modeling the impulse-reponse of the radiation belts. Here, we show how NARMAX models, in conjunction with nonlinear time-delay FIR neural networks (Volterra networks) hold great promise for the development of accurate and fully data-derived space weather specification and forecast tools.

1. Theory

The overall methodology we have adopted is based on using NARMAX models to then contruct equivalent nonlinear Volterra neural networks that benefit from Takens' Theorem for time-delay embedding combined with the ability of multilayer perceptrons to be universal function approximators.

1.1. STEP 1: Constructing a taxonomy of nonlinear input-output models

We began with a generalisation of the Wold time series decomposition (Wold (1954)) having the form,

$$J_t = \delta + f(\mathbf{\Psi}_t) + \varepsilon_t \equiv \hat{J}_t + \varepsilon_t \tag{1}$$

where J_t is the electron flux time series, $\hat{J}_t = \delta + f(\mathbf{\Psi}_t)$ are the model predictions, f is a general function (linear or nonlinear), δ is a constant (zero in the absence of trend), $\varepsilon_t = J_t - \hat{J}_t$ are the prediction errors and $\mathbf{\Psi}_t$ is an "information matrix" constructed from lag polynomials Φ_p, Θ_q, N_r, lag operators L^i, L^j, L^k and differencing indices d_1, d_2 and which has the general form (Taylor et al. (2009)),

$$\mathbf{\Psi}_t = \Phi_p(1 - L^i)^{d_1} J_t + \Theta_q \varepsilon_t + N_r(1 - L^k)^{d_2} I_t, \tag{2}$$

corresponding to a Nonlinear AutoRegressive Integrated Moving-Average eXogenous input NARIMAX(p, d_1, q, d_2, r) process. Note that in the case of multiple inputs, I_t

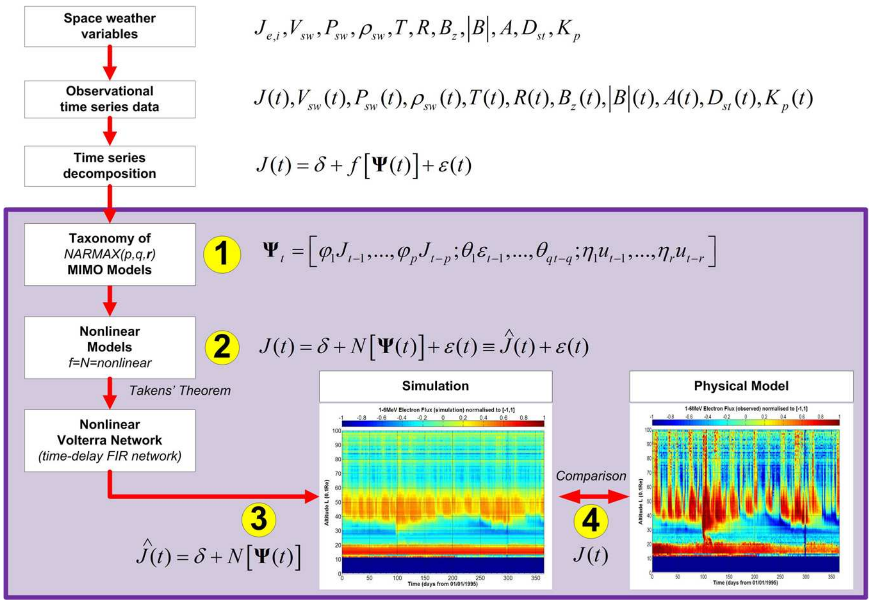

Figure 1.: The overall methodology we have adopted.

will be a vector $\mathbf{I}_t$. The information matrix then contains operators acting on time-delayed (lagged) time series of electron flux J_{t-p} (autoregression AR), lagged equation errors ε_{t-q} (moving-average MA), and lagged inputs I_{t-r} (eXogenous). The particular class of model chosen depends on how exactly $\mathbf{\Psi}_t$ is defined and on the functional form of f. For example, the nonlinear autoregressive, moving-average, exogenous input NARMAX(p, q, r) process has $\mathbf{\Psi}_t = \Phi_p J_t + \Theta_q \varepsilon_t + N_r I_t$ and represents the time series decomposition,

$$J_t = \delta + f\Big[\quad \varphi_1 \quad J_{t-1}, \cdots, \varphi_p J_{t-p}, \phi_1 \varepsilon_{t-1}, \cdots, \phi_q \varepsilon_{t-q}, \tag{3}$$

$$\eta_1 \quad I_{t-1}, \cdots, \eta_r I_{t-r}\Big] + \varepsilon_t. \tag{4}$$

The table below shows the complete taxonomy of input-output models we have identified with this class. Armed with input-output relations, we now construct neural networks to model f.

Function	Autoregression order	Moving-Average order	Inputs	Model
1	1	0	0	AR(1)=Random walk
1	p	0	0	AR(p)
1	p	0	r	ARX(p, r)
1	0	q	0	MA(q)
1	0	∞	0	MA(∞)=Wold Decomposition
1	0	q	r	MAX(q, r)
1	p	q	0	ARMA(p, q)
1	p	q	r	ARMAX(p, q, r)
f	p	0	0	NAR(p)
f	p	0	r	NARX(p, r)
f	0	q	0	NMA(q)
f	0	q	r	NMAX(q, r)
f	p	q	0	NARMA(p, q)
f	p	q	r	NARMAX(p, q, r)

1.2. STEP 2: Equivalence between input-output models and Volterra Networks

Equivalence between any NARIMAX(p, d_1, q, d_2, r) process and its Volterra network representation is guaranteed by a special combination of 2 Theorems:

1. Takens' Theorem (Takens (1981)): There is a 1-to-1 mapping between a time series and the underlying dynamical state space
2. Universality Theorem (Horkin et al. (1989)): Nonlinear Multilayer Perceptrons are universal and exact function approximators

1.3. STEP 3: Construction of Volterra networks

Feedforward MLPs with lagged inputs create short-term memory and incorporate nonlinear dynamics into the network state space. In the case that neural activation functions are linear then they operate as finite impulse-response (FIR) networks (Wan (1993)). Linear FIR models already exist in the literature (Vassiliadis et al. (2005)). Here we develop ***nonlinear*** (sigmoidal activation function) time-delay FIR networks (Volterra networks) based on the NARMAX$(p, 0, r)$ process, whose general architecture is shown in Figure 2. In this poster we will show initial results of nonlinear autoregressive modeling of the electron flux based on the NAR(p) process as as example of the network operation and capability.

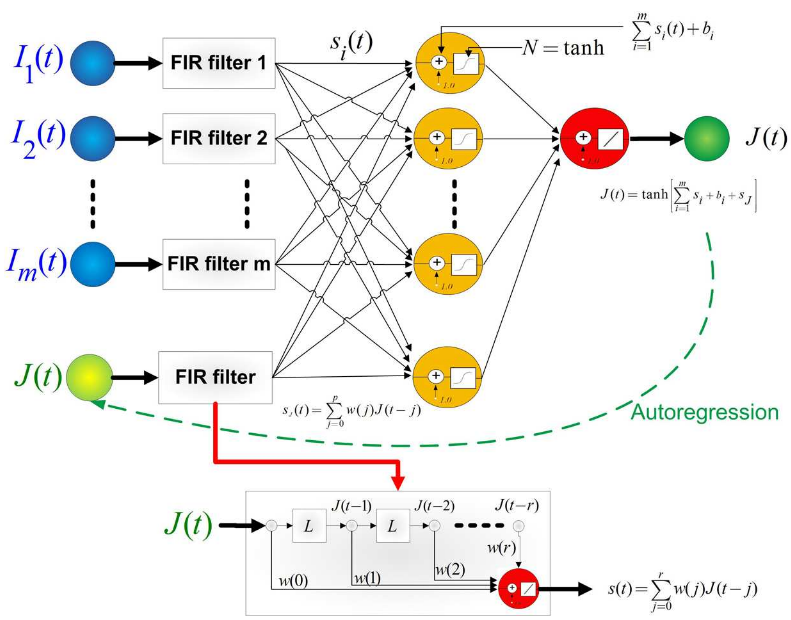

Figure 2.: A schematic diagram of the nonlinear time-delay (Volterra) network used in this work.

In order to measure the degree of success in reproducing observed values $J(t)$ from the network model $\hat{J}(t)$, we used the data-model correlation coefficient C:

$$C = \frac{1}{T}\frac{1}{\sigma_J \sigma_{\hat{J}}} \int_0^T \left(\hat{J}(t) - \left\langle \hat{J}(t) \right\rangle\right)\left(J(t) - \langle J(t) \rangle\right) dt \tag{5}$$

where $\langle J(t) \rangle$ and σ_J are the mean and standard deviation of $J(t)$.

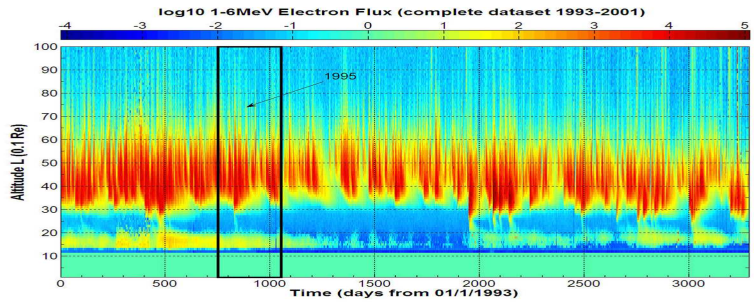

Figure 3.: The raw dataset comprises daily-averaged values of 1-6MeV electron flux J_e over the time interval 01/01/1993-31/12/2001. The data for 1995 used in this study is indicated. Note that SAMPEX-PET data is available only above L=1.1.

1.4. STEP 4: Identification of the physical model

Since the neuron activation function and weights (connections) in Volterra networks are extractable and therefore explicit, the network architecture can be converted into equations with known AR and MA coefficients thus providing model equations. In the Results section, we present a spatio-temporal model of the electron flux calculated with a NAR(30) process. The 30 lag steps in the FIR filter makes it impractical to write down the resulting model equations here, but Figures 3b), 4) and 5b) suggest that the simulated model is reproducing fairly well, the physics.

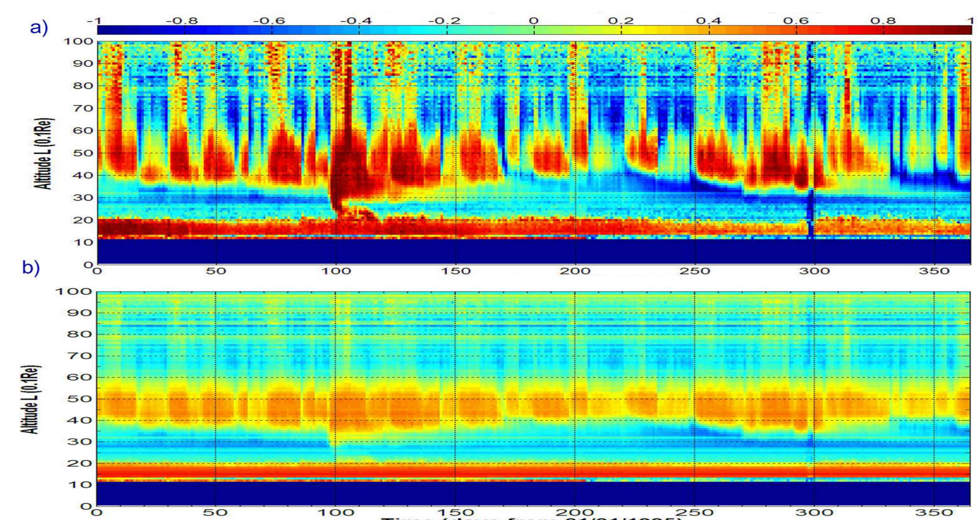

Figure 4.: a) The raw electron flux data normalised to the interval [-1,1] and b) initial results from the nonlinear autoregression $NAR(30)$ Volterra network revealing fairly successful modeling of the salient features in this time interval.

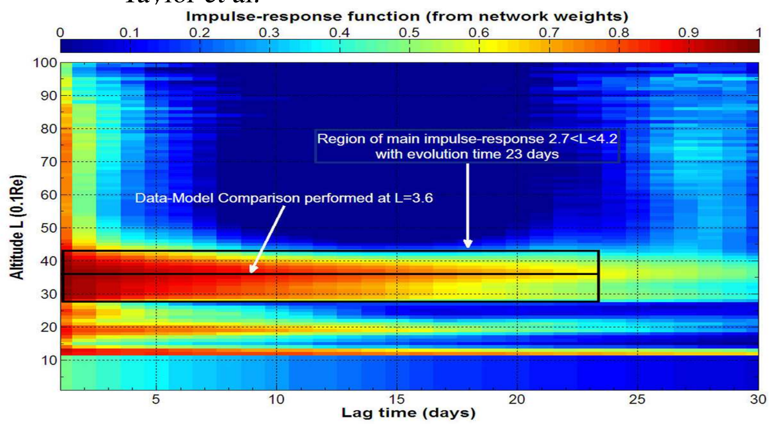

Figure 5.: The impulse-response function obtained directly from the Volterra network neuron weightings. The main impulse-response region is between L=2.7 and L=4.2 and has a duration of approximately 23 days.

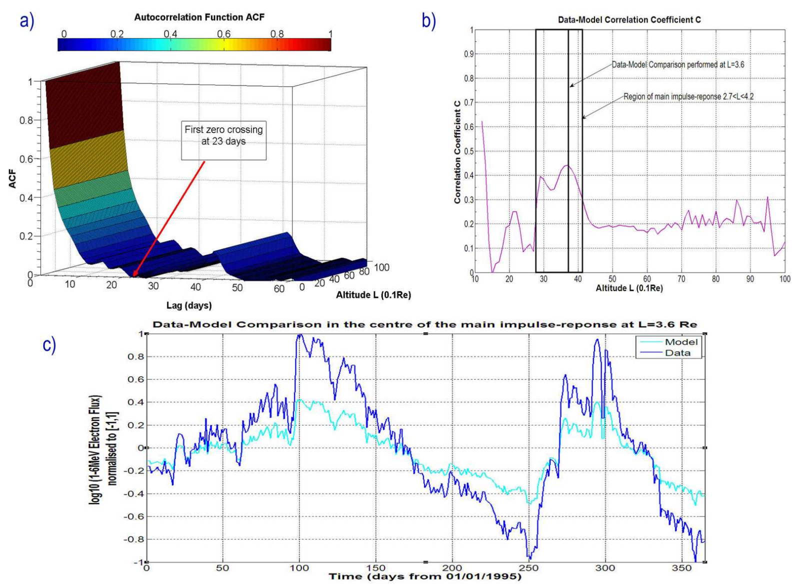

Figure 6.: a) The first zero crossing of the autocorrelation function calculated from the raw data is at a lag of 23 days, b) apart from a numerical artifact at the data edge ($< L = 1.5$), the data-model correlation function C peaks in the main impulse-response region, c) the raw data and the network model at the peak of the impulse response at $L = 3.6$.

2. Discussion

All Volterra networks were trained with the Levenberg-Marquardt backpropagation algorithm (Rumerlhart & McClelland (1986)) for 100 epochs and 10 adaptive passes at each step in L-shell altitude (0.1 Earth radii) and over daily-averaged data covering the whole of 1995. These early results suggest that this particular modeling approach is capable of recovering the nonlinear dynamics implicit in the data.

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