Ion acceleration through an auroral potential structure: Numerical simulations

A. Anastasiadis, I. A. Daglis and I. D. Kontodinas National Observatory of Athens, Institute for Space Applications and Remote Sensing, Penteli, GR-15236, Greece

Abstract

We perform numerical simulations for the acceleration of O^+ ions in a potential well which has an exponential form in x-direction with a characteristic length L_x . The oxygen ions are moving in the presence of constant magnetic field B_z and a constant electric field E_y . We trace the orbits of individual particles for different initial conditions (phase angle and kinetic energy). We found that depending upon the initial conditions ions can be either accelerated or decelerated. Furthermore we perform a parametric study for the interactions of monoenergetic and Maxwellian type of ion distributions, using random phase angle injection of the particles, with respect to our model parameter- the characteristic length of the potential (L_x) .

1 Introduction - Motivation

It is well known that ions are drifting under the action of the $\mathbf{E} \times \mathbf{B}$ force from the magnetotail towards Earth. During their orbit, ions are interacting with an auroral arc, which is elongated in the east-west direction and might has a Gaussian (U-shape) potential structure (e.g. Chen & Palmadesso 1986; Büchner & Zelenyi 1991). The general case of particle motion in a homogeneous magnetic field \mathbf{B} with an orthogonal electric field \mathbf{E} , which has a gradient $\nabla \mathbf{E}$ parallel to \mathbf{E} , was studied by Cole (1976). By examining the analytical solutions of the equation of motion, he argued that particle acceleration is possible when the scale length of the potential becomes less than the particle's gyroradius. Rothwell et al. (1995) examined the behavior of the analytical solutions for a general type of potential. They found that the solutions are very sensitive to the initial conditions and display chaotic behavior on phase angle variations and on changes of the second derivative of the electric filed $(d^2 E_x/dx^2)$.

Here, we re-examine the particle motion in a potential structure of an exponential type that was first proposed by Rotwell et al. in 1992. In their work, they argued that due to the presence of a hyperbolic fixed point in the phase space some particles are trapped by the potential and some are not. This effect leads to the stochastic acceleration of the ions. Moreover they claimed that stochastic heating of a particle population can occur due to the interaction with the well. In this paper, we present preliminary results of numerical simulations, indicating that particles are accelerated or decelerated depending only upon their initial conditions (phase angle and initial kinetic energy) and not due to any presence of a fixed point in the phase space. Furthermore we follow the evolution of Maxwellian type initial particle distributions in order to deduce if there is any heating of the particles.

2 Numerical simulations - Results

We assume the presence of a potential structure along the x-direction with a form:

$$\phi(x) = \phi_o \, \exp\left[-\left(\frac{x}{L_x}\right)\right]^2 \tag{1}$$



Figure 1. Ion orbits with the same initial kinetic energy $(E_i = 200 \text{ eV})$ and for the same model parameters but with a different phase angle. *Top*: ion with phase angle 2° . *Bottom*: ion with phase angle 70° .

the corresponding electric field will then be given by the relation:

$$E_x(x) = -\nabla\phi(x) \tag{2}$$

We consider that ions are moving under the presence of a constant background magnetic field in the z- direction (B) and a constant electric field along the y- direction (E_y) . This background electric field provides that the particle will drift in x- direction, towards the potential well. The motion of an ion with mass M and charge e, in the assumed configuration, is described by the equations:

$$M\dot{V}_x = e[E_x + V_y B] \tag{3}$$

$$M\dot{V}_u = e[E_u - V_x B] \tag{4}$$

where $V = (V_x + V_y)^{1/2}$ is the ion's total velocity. We numerically integrate the above equations of motion using a stepwise adaptive technique with a fourth order Runge-Kutta method (see Press et al. 1986). The total energy :

$$E_t = \frac{1}{2}MV^2 + e\phi(x) - eyE_y \tag{5}$$

is constant during each time step of the integration procedure, assuming that the second order derivative of E_x is zero. For an accuracy criterion of our numerical simulations, we keep the maximum error in calculating the total energy to be less than 10^{-3} of the initial total energy.

In Figure 1. the orbits of two oxygen ions (M = 16) with the same initial energy $(E_i = 200 \text{ keV})$, starting at the same point in space $((x, y) = (-10L_x, 0))$, but with



Figure 2. Kinetic energy variation (E_{final}/E_i) of 10^3 ions with initial kinetic energy of $E_i = 200$ keV and random phase angles (monoenergetic distributions), as a function of the characteristic potential length L_x . Note the strong dependence of the acceleration process on the scale length.

a different phase angles (2° and 70°) are presented, for the case of $E_y = 1 \text{ mV/m}$, B = 144 nT, $\phi_o = 3 \text{ kV}$ and $L_x = 50 \text{ km}$. It is clear that the ion with the small phase angle is decelerated while the one with the large phase angle is accelerated.

In addition to individual particles we trace monoenergetic distributions of 10^3 particles, following the same numerical scheme. Using the same parameters for E_y , ϕ_o and B, but with random phase angles, in Figure 2. we present our results concerning the energy variation - final kinetic energy versus initial kinetic energy - of monoenergetic ion distributions ($E_i = 200 \text{ eV}$) with respect to the characteristic length of the potential well L_x . We found that for scale lengths $L_x \approx 110 \text{ km}$, which is comparable to the ion gyroradius (R_i), a large fraction (up to 70%) of the injected particle distribution is accelerated, reaching even to energies up to 6 times the injection energy. In the case where the scale length of the potential is not comparable to the R_i , only a small fraction (around 45%) is accelerated while the remaining is decelerated. From our study, it is not clear if there will be a net energy gain and whether that can be interpreted as an increase of the bulk kinetic energy.

Furthermore, we examine what happens to a group of ions injected with a Maxwellian velocity distribution towards the potential well. Firure 3. shows the variation of the assumed initial velocity distribution of 10^4 oxygen ions, with thermal kinetic energy $E_i = 200$ keV and random phase angles, interacting with the potential structure of $L_x = 10$ km and $\phi_o = 3$ kV, with a background electric filed $E_y = 1$ mV/m. Note that only the high velocity part of the initial distribution function has not been altered significantly. This is due to the fact that ions with high energies do not get affected considerably by the potential, as their gyroradii are too big compared to the characteristic scale length (L_x) of the potential well.

3 Conclusions

Here we have applied a numerical integration scheme to examine the interaction of oxygen ions with a potential well of an exponential type, with characteristic length (L_x) . Based on our preliminary results, we can conclude that the acceleration process of ions exhibits a stochastic nature, only due to its strong dependence upon the initial conditions (initial kinetic energy and phase angle of the ions). As we followed the evolution of particle distributions (monoenergetic and Maxwellian types), we can claim that there is no clear evidence for the heating of the bulk ion population, due to the interaction of the particles with the potential well. A further and more detailed investigation of this problem is currently under study.



Figure 3. Ion velocity distributions of 10^4 particles. *Top*: Injected Maxwellian type with thermal energy $E_i = 200$ keV and random phase angles. *Bottom*: Final distribution after the interaction with the potential well of $L_x = 10$ km and $\phi_o = 3$ kV, with $E_y = 1$ mV/m.

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