CELLULAR AUTOMATA MODELS AND MHD APPROACH IN THE CONTEXT OF SOLAR FLARES [1]

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Abstract: We address in detail the cellular automaton approach, developed for the case of solar flares, and compare it to the magnetohydrodynamic (MHD) theory. We consider solar flares as typical complex dynamical systems and we are interested in their global dynamical behavior, through their statistical properties.

1 Introduction

The behavior of several dynamical systems in nature, is sometimes complex and unpredictable, despite the fact that their evolution is governed by simple physical laws. The main reason for the complex behavior of dynamical systems, is that a local change in a subset of the system can affect the evolution of the whole system. A complex dynamical system can be defined as a system that consists of a large number of different non-linear interacting sub-systems.

Statistical physics and mathematics are the most common tools to explore how a complex system evolve. The usual approach is to limit the degrees of freedom of the system by making several assumptions (e.g. all the sub-systems of the complex system are identical), in this way it is possible to use a set of differential equations. The next step, if these equations are not easily solvable, is to try to construct difference equations and to use numerical computational schemes. Although this kind of an approach is successful in some cases (e.g. description of the motion of ideal gas), in several cases can not be applied.

It is clear that, if we are interested in knowing the global behavior of a complex dynamical system, simple mathematical tools should be used. These types of mathematical tools are the Automata and the Cellular Automata (CA). The advantage in developing them is that we can study nature without the intervenience of the differential and/or the difference equations.

An automaton is an input - output model with an internal state. A very simple differential equation can be used for the description of its evolution. A cellular automaton model on the other hand consists of a number of automata distributed in a specific grid. In other words, each grid site in a CA is an automaton. We must emphasize that CA models are by nature discrete in time and space (for a detailed discussion on cellular automata see Wolfram 1986). For the development of a CA model, one needs just to specify the grid (i.e. to give the dimensions and the type of the grid cells), the boundary conditions, the evolution rules and finally, the interaction rules between the nearest neighbors of a given grid site.

In this paper, we will present CA models that have been developed in order to explain the observed statistical properties of solar flaring activity. In Section 2. the basic results of the statistical studies of the solar flaring activity are presented, together with a discussion on the MHD and CA approaches that can be used for the description of the dynamical evolution of solar flares. A qualitative comparison between

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the approaches is made, addressing their advantages and drawbacks. In Section 3. we review the basic evolution rules and the main results of the classical CA models, given also their physical interpretation. Finally in the last section we present a new type of CA model (extended CA) which is consistent with the MHD approach and is applied to the ”standard” solar flare scenario (see Parker 1993).

2 Solar flares as complex dynamical systems

Solar flares are the manifestation of an energy release process. During solar flares, magnetic energy of 10^{29} to 10^{33} ergs is released in solar chromosphere and corona over a few minutes. It is now widely accepted that the solar activity is mainly due to the dissipation of the magnetic energy stored previously in the solar corona by means of magnetic reconnection processes (for details on magnetic reconnection see Priest and Forbes 2000). Observations of the radiation signatures of solar energetic particles (especially in the hard X-rays) indicate that the energy release process is fragmented into a large number of sub-events (Vilmer 1993; Aschwanden et al. 1995, Vilmer and Trottet 1997). In general, solar flares are considered to be made up by a large number of reconnection events distributed over an active region (see Fig. (1)) and their evolution and global behavior is similar to complex dynamical systems.

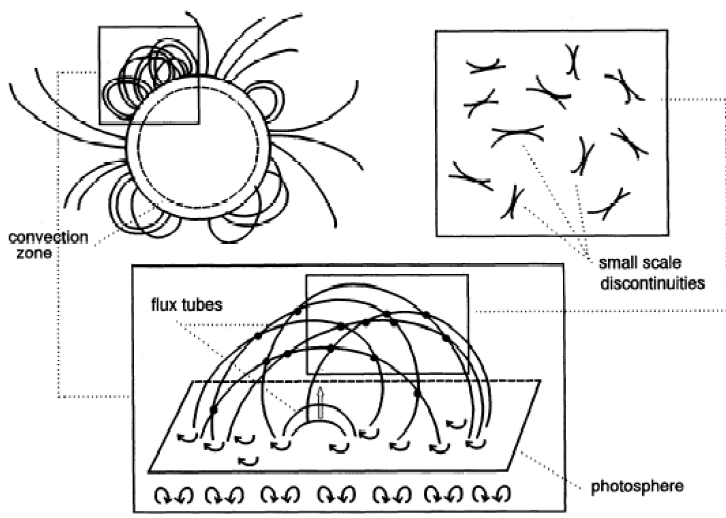

Figure 1: Illustration that shows the complex nature of solar active regions (from Anastasiadis and Vlahos, 1994).

In the last two decades, the availability of several space-born solar instruments, together with the existence of several ground based telescopes, have given the opportunity to perform a number of statistical studies of the flaring activity of the Sun. These types of observations (Dennis 1985; Vilmer 1987; Pick et al. 1990; Crosby et al. 1993; Crosby et al. 1998) revealed that the frequency-distributions of flares as a function of total energy, peak luminosity, and duration are well-defined power laws, extending over several orders of magnitude. The most important questions addressed in these statistical observational studies of flaring activity were the following:

- What is the frequency distribution of the characteristic flare parameters?

- Is there a correlation between the flare parameters?
- Are there periodicities and/or time evolution of the frequency distributions?
- What is the behavior of high and low energy cut-off?

The main results of these studies can be summarized as following:

- The frequency distributions follow a power law distribution

$$dN(X) = X^{-b} dX \tag{1}$$

  where N is the frequency of appearance of the parameter X over the interval [X,X+dX]

- A more accurate functional form for the statistical data could include an exponential term

$$\frac{dN(E)}{dE} \propto E^{-b} exp\left(-\frac{E}{E_c}\right) \tag{2}$$

  where E_c is the cut-off energy.

- The characteristics of the flare (total energy, maximum brightness, total duration) obey correlations of the form

$$X \propto Y^{\delta} \tag{3}$$

- There is no correlation between the flare energy and the elapsed time between flares.
- It is not clear yet if flares appear randomly in time or there exist a non- scaling-law for the elapsed times between flares.
- Soft X-ray bursts seem to follow statistical behavior similar to the Hard X-ray, but the convolutions from the measured photon flux to the flare energetic depend on the model used.

Following the observational evidence of the fragmentation of energy release process during solar flares, a number of qualitative models have been developed (for reviews see van den Oord 1994; Vlahos 1996; Bastian and Vlahos 1997; Anastasiadis 2002). These models revealed the necessity to study and understand the global behavior of the evolution of the complex active regions.

Traditionally, the study of the energy release process was relying on the magnetohydrodynamic (MHD) theory. In this framework, the most common approach was the development of numerical simulation codes, given the complex nature of the energy release problem in the solar corona, which involves strongly nonlinear effects (e.g. Einaudi et al. 1996; Georgoulis et al. 1998, and references therein). In MHD numerical simulations, the energy release process (i.e. magnetic reconnection) is simulated in detail (see Fig. (2)), but these simulations are time consuming and can only treat a small number of reconnection events (bursts) and relatively small volumes, leading thus to poor statistics for comparison to solar flare observations.

Alternatively, several cellular automaton (CA) models, based on the approach used in complex dynamical system theory, have been developed in order to explain the solar flare statistics derived from the observations (Lu and Hamilton 1991; Lu et al. 1993; Vlahos et al. 1995; Georgoulis and Vlahos 1996;

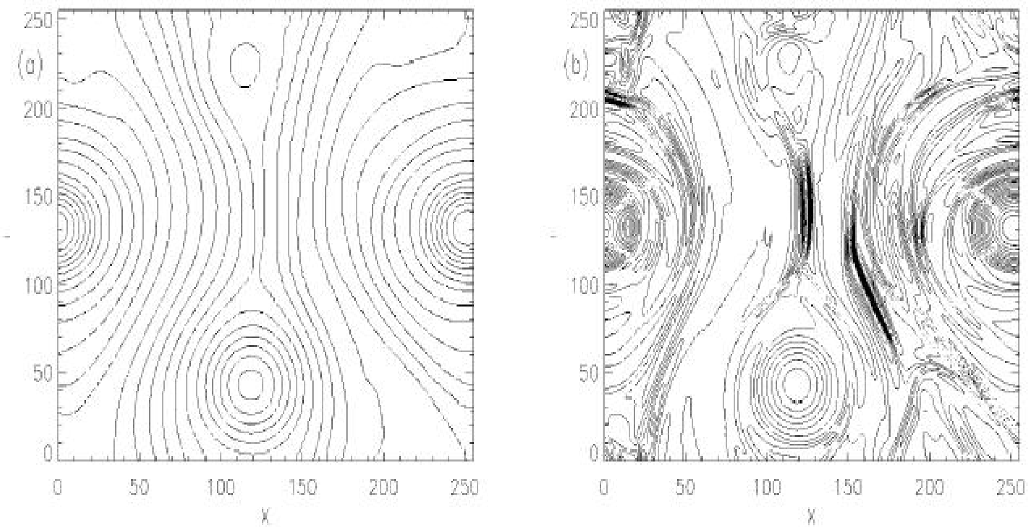

Figure 2: MHD numerical simulations showing a high-resolution spatial configuration of (a) the magnetic field magnitude and (b) the current density magnitude (from Georgoulis et al. 1998).

1998). These models based on the Self Organized Criticality (SOC), a well known property of externally driven complex dynamical systems (for details on SOC see Bak 1996). We will term these models *classical* CA models. The classical CA models simulate the energy storage/release process using simple evolution rules, neglecting the details of the processes. They allow the global modeling of solar active regions, at the expense of understanding in details the underlying physics. The main advantage of them is that they can treat a large number of elementary energy-release events and relatively large volumes, yielding thus results on a good statistical ground, and that they can explain the power-law frequency distributions of the solar flare parameters. We must emphasize here that, in addition to the classical CA models, several different types of CA models have been constructed (for recent review see Charbonneau et al. 2001).

In the following table we outline the main differences between the two approaches

	MHD Approach	**Cellular Automata**
storage/release of energy	detailed	simple evolution rules
number of events	small/(1 or 2)	large number of events
numerical codes	complicate and time consuming	simple and fast
observations	poor statistics	in good agreement

Summarizing, we must mention that the CA approach does not explain what happens locally or on a short time intervals, but it allows to understand the statistics of the global behavior. The MHD approach, on the other hand, may reveal insights into the local processes, but coupling this understanding to a global description is not completely feasible, mainly due to a large number of boundary conditions. In this sense, the two approaches can be considered as complementary, and a description of a complex system, such as solar flares, should ideally combine them. Thus, it is obvious to address the question if the CA approach can be interpreted as a simplified and/or modified MHD approach (for detailed discussion see Isliker et al. 1998; Vassiliadis et al. 1998).

3 Classical cellular automata models and their interpretation

In this section we will present in detail, the general aspects of the classical CA models. These models have shown that the energy release inside active regions may well be a result of an internal self-organization process. The continuous loading of the active region with new magnetic flux can produce several magnetic discontinuities. Simple rules were applied for the redistribution of magnetic fields and the release of magnetic energy at these discontinuities. The basic rules are the following (see Lu et al. 1991; 1993; Vlahos et al. 1995; Georgoulis and Vlahos 1996; 1998):

1. **Initial Loading.** A 3-D cubical grid is constructed, and to each grid point a scalar "field" is associated. The scalar quantity stands for the magnitude of the "magnetic field strength" at a certain location. The three-dimensional cubical simulation box models a limited part of the solar atmospheric, mainly coronal, layers. The initial configuration is random and stable, corresponding to a quiescent initial stage.

   We emphasize at this point that the physical importance of the operational field (which we call "magnetic field strength") has triggered several discussions. In Lu et al. (1991), where vector fields were introduced, this field, say $\vec{F}$, was associated with the magnetic field $\vec{B}$. This notation is apparently problematic, however, because in this case the divergence of $\vec{B}$ should vanish ($\nabla \cdot \vec{B} = 0$). Such a property was not clear in those models.

2. **Loading.** Randomly selected grid points receive scalar increments with a rate equal to one point per iteration. The perturbation applied locally is by no means weak or constant; it is the outcome of a power-law probability function of the form

$$P(\delta B) = A(\delta B)^{-\alpha} \tag{4}$$

   where $P(\delta B)$ is the probability of an increment δB to be accumulated to the ambient field at a randomly chosen lattice site. A is an arbitrary constant of the order unity. The index α of the power law determines the value of the increment δB which corresponds to a certain probability $P(\delta B)$. We normalize $P(\delta B)$ to vary between zero and one and randomly selected numbers lying in this range, give rise to different increments δB. Increments δB obtained may vary from very small values to multiples of the critical threshold (see below). In case a lattice site gains an increment δB, its first- and second-order neighbors gain an additional small quantity of magnetic field, set to 25% and 10% of δB, respectively. In this sense the loading, and, as we will see later, the relaxation of magnetic discontinuities is not a local process.

3. **Instability Criteria.** Instabilities occur if "large field gradients" exceed a critical threshold. The units of the model are arbitrary, so a critical "gradient" B_{cr} (critical threshold) is arbitrarily introduced. We use two ways of defining the "magnetic gradient" or the "slope" at a given lattice site. The slope at a grid point i can be either the field difference S_i between that site and the average of its first-order vicinity

$$S_i = B_i - \frac{1}{6}\sum_j B_{i,j} \tag{5}$$

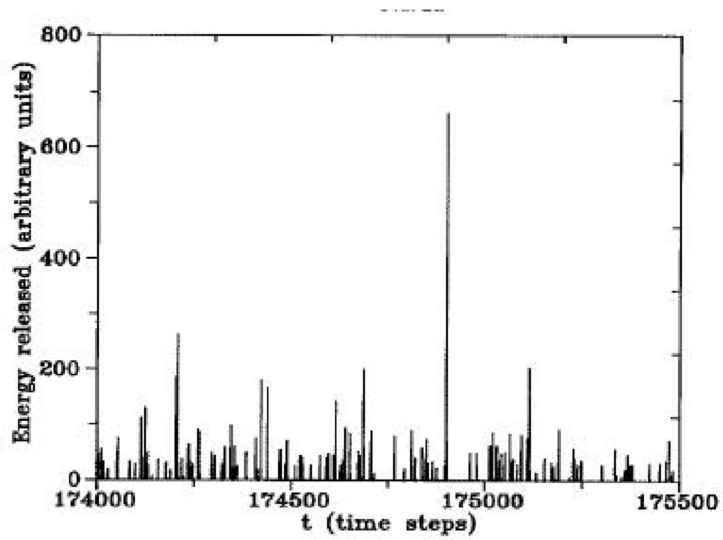

Figure 3: Energy release time series, calculated using Eq. (15) (from Georgoulis and Vlahos, 1996).

or the field differences $S_{i,j}$ between the site i and each one of its first-order neighbors independently

$$S_{i,j} = B_i - B_{i,j} \tag{6}$$

where B_i is the value of the local magnetic field at the i-location and $B_{i,j}$ ($j = 1, 6$) are the magnetic field strengths of its six first-order neighbors.

Instability criteria are defined by the inequalities

$$S_i > B_{cr} \tag{7}$$

$$S_{i,j} > B_{cr} \tag{8}$$

The onset of an instability appears when either of the Inequalities (7-8) is satisfied. The instability detection routine starts from checking Inequality (7) and proceeds to checking Inequality (8), in case Inequality (7) is not satisfied anywhere in the system. Relaxation process is the consequence of two mechanisms: $i)$ Redistribution of a portion of the excess field to the vicinity of an unstable site, and $ii)$ energy release.

4. **Field Restructuring.** If the first of the Inequalities (7-8) is satisfied, the first-order vicinity of the unstable location undergoes a global field restructuring ruled by the relations

$$B_i \longrightarrow B_i - \frac{6}{7} B_{cr} \tag{9}$$

$$B_{i,j} \longrightarrow B_{i,j} + \frac{1}{7} B_{cr} \quad , \qquad j = 1, 6 \tag{10}$$

Similarly, if one (or more) slope(s) $S_{i,j}$ satisfy Inequality (8), the redistribution of the field is given by the relations

$$B_i \longrightarrow B_i - \frac{6}{7} B_{cr} \tag{11}$$

$$B_{i,j} \longrightarrow B_{i,j} + \zeta_j \tag{12}$$

provided that

$$\zeta_j = \frac{6}{7} B_{cr} \frac{S_{i,j}}{\sum_j S_{i,j}} \tag{13}$$

$$\sum \zeta_j = \frac{6}{7} B_{cr} \tag{14}$$

We need to emphasize at this point that the sum in the denominator of Eq. (13) stands only for those slopes $S_{i,j}$ that satisfy Inequality (8).

5. **Secondary Instabilities.** Under suitable conditions which depend on the nature of the local magnetic configurations in the box, the onset and relaxation of an initial, seed instability, may trigger the emergence of a cascade of similar events, in an avalanche-type manner. Apart from the obvious implication that the redistribution of magnetic fields may result to new instabilities surrounding the neighboring locations, we have introduced a simple mechanism due to which secondary relaxation events may be triggered (events that occur when the gradients do not exceed the critical threshold). The first- and second-order neighbors of the primary unstable site may participate in this process. In this way, we attempt to examine the possibility of a flare implicitly affecting the field configurations of its closest vicinity and beyond.

6. **Energy Release.** For both cases of field restructuring (relations (11) -(12) or relations (13) - (14)), we associate an energy release E_R

$$E_R = (B_i - \frac{6}{7} B_{cr})^2 \tag{15}$$

with each event. The form of Eq. (15) implies that this energy should be viewed as ”magnetic” energy release.

The above methodology provides, for the first time, an opportunity to study the possible uniqueness of flaring frequency-distribution indices, by examining the intrinsic behavior of the external driver. It is also clear that, if the power-law index α in Eq. (4) increases, the probability of obtaining large energetic events decreases and thus the mean activity in the simulated active region is decreased. On the other hand, decreasing α, we obtain larger δBs, which trigger more energetic events, thus increasing the mean activity. The main result of the classical CA models is that energy-release time series (see Fig. (3)) obeys a double power-law frequency distribution (see Fig. (4)). It also exhibits a scale-invariant behavior and encloses a self - similar nature.

Recently using a classical CA the acceleration of high energy electrons in solar flares was studied (see Anastasiadis et al. 1997), connecting, for the first time, the energy release process with the acceleration mechanism. In addition, the explanation of the observed frequency distributions of the solar flare parameters (i.e. total energy, peak-luminosity, total duration) has been accomplished, using the classical CA models (see Georgoulis et al. 2001).

Despite the benefits that one has of running a CA model for solar flares, several questions and inconsistencies with the MHD approach have been revealed (for details see Isliker et al. 1998; Vassiliadis et al. 1998). The most important problems are:

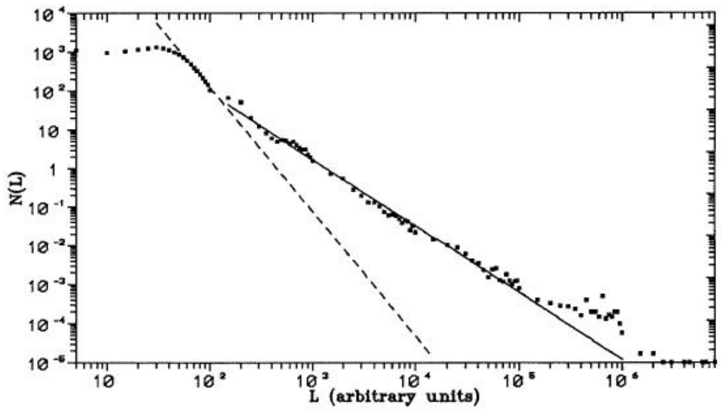

Figure 4: A double power-law frequency distribution of the peak-luminosity (from Georgoulis and Vlahos, 1996).

- What is the physical interpretation of the basic rules of the classical CA models?
- How can we calculate the other physical variables, avoiding the several inconsistencies that the classical CAs exhibit with MHD and Maxwell equations (i.e. the fact that the $\nabla\vec{B} \neq 0$ in the CA models).

In Isliker et al. (1998), we found that, for the classical CAs, the magnetic field evolves according to the induction equation of the form:

$$\frac{\partial \vec{B}(\vec{x},t)}{\partial t} \;=\; \eta \nabla^2 \vec{B}(\vec{x},t) \;+\; \vec{S}(\vec{x},t) \tag{16}$$

where η is the diffusivity and $\vec{S}(\vec{x},t)$ is a random in space and time source function, mimicking the $\nabla \times (\vec{v} \times \vec{B})$ term of the induction equation, where $\vec{v}$ is the velocity field. The $\eta = l^2/\tau$, connected with the characteristic size l and the relaxation time τ.

In the stable mode of the classical CAs, when the inequalities (7 - 8) are not satisfied, the Eq. (16) reduces to

$$\frac{\partial \vec{B}(\vec{x},t)}{\partial t} \;=\; \vec{S}(\vec{x},t) \;=\; \nabla \times (\vec{v} \times \vec{B}) \tag{17}$$

describing in simplifying way the convection due to the turbulent motion in active regions and the plasma flow, through a simple random function. In the unstable mode, when the inequalities (7 - 8) are satisfied, the Eq. (16) reduces to

$$\frac{\partial \vec{B}(\vec{x},t)}{\partial t} \;=\; \eta \nabla^2 \vec{B}(\vec{x},t) \tag{18}$$

describing the induction equation in the diffusive regime, i.e. there where the η is so large that the convective term can be neglected. This equation is just a diffusive equation of the magnetic field. The restructure of the magnetic field in the classical CAs (Eq. (9- 12)) can be explained as the asymptotic solution of the Eq. (18) (see Fig. (5)) with the boundary condition

$$(\vec{n}\nabla)\vec{B}(\vec{x},t) \;=\; 0 \tag{19}$$

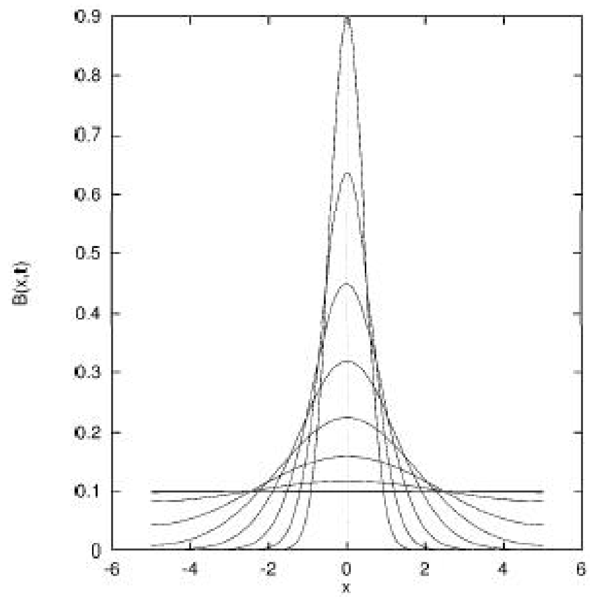

Figure 5: The temporal evolution in 1D space of the magnetic field $B(x,t)$, undergoing the diffusive process (i.e. Eqs. (18 - 19)) (from Isliker et al. 1998).

where $\vec{n}$ is the nornal unit vector.

As the CA models are by their nature discrete in time and space, we found that (Isliker et al. 1988) the most important problem, arising in calculating the other physical variables and avoiding the inconsistency of $\nabla\vec{B} \neq 0$, is to find a way to calculate derivatives on the CA grid. This fact revealed the necessity to construct a new type of CA models that will be in consistent with the MHD approach.

4 MHD consistent CA models - The extended CA (X-CA)

There are two basically different ways of developing CA models for flares further: (i) Either one considers CA models *per se*, tries to change the existing models further or invent new ones, with the only aim of adjusting them to reproduce still better the observations, i.e. one makes them a tool the results of which explain and maybe predict observed properties of flares. In this approach, one has not to care about possible inconsistencies with MHD or even Maxwell's equations. (ii) On the other hand, one may care about the physical identification and interpretation of the various components of the model, not just of its results, and one may want the CA model to become consistent with the other approach to solar flares, namely MHD. We followed the second approach by constructing the *extended* CA (X-CA) model.

The X-CA model consists in the combination of a classical CA model with a set-up which is superimposed onto the classical CA (for details see Isliker et al. 2000, 2001). We use a 3-D cubic grid ($30 \times 30 \times 30$ in the following) with the vector potential $\vec{A}$ as the primary grid variable. The central problem is how to calculate derivatives, since CAs are by their nature discrete models, as demonstrated in Isliker et al. (1998). We differentiate $\vec{A}$ and the magnetic field $\vec{B}$ by using 3-D cubic spline interpolation (evaluating several alternative methods, it turned out that calculating the derivatives in this way has remarkable advantages over other methods). The magnetic filed $\vec{B}$ and the current density $\vec{J}$ are

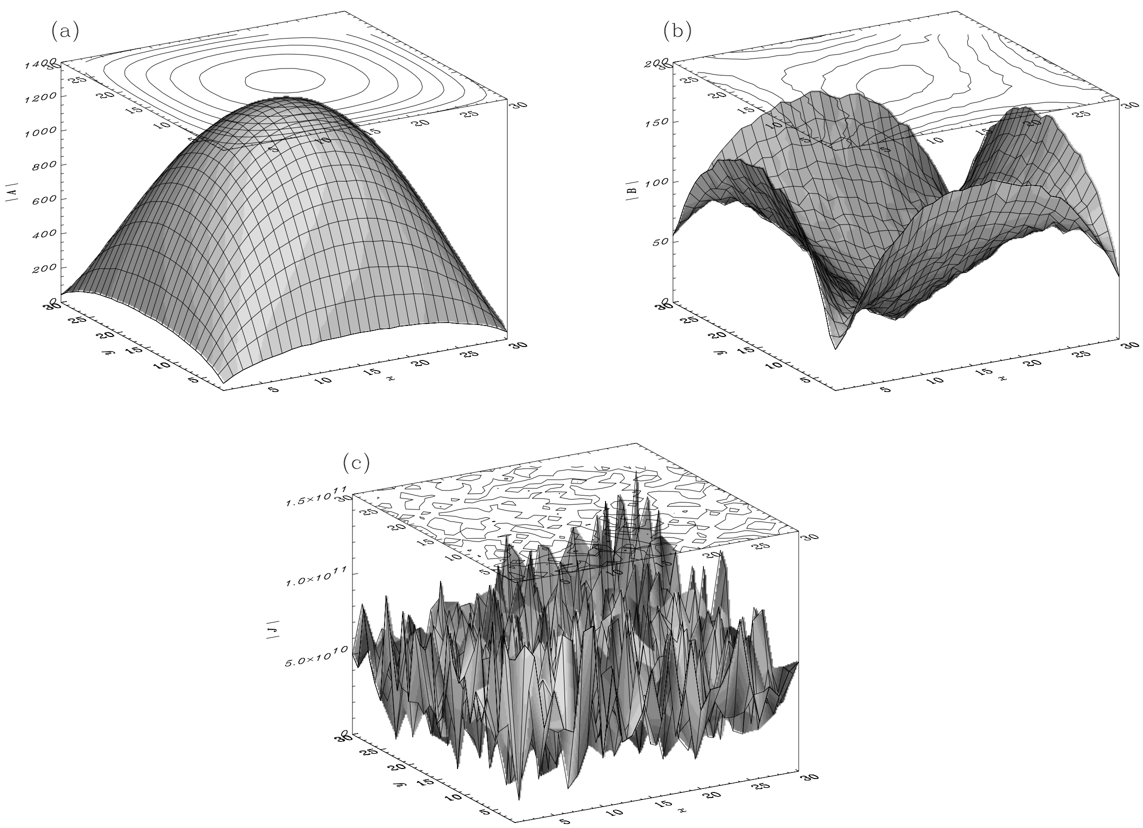

Figure 6: Surface and contour plots of the magnitudes of (a) the vector potential, (b) the magnetic field, and (c) the current density, as a function of x and y, for z= 15 fixed (from Isliker et al. 2000)

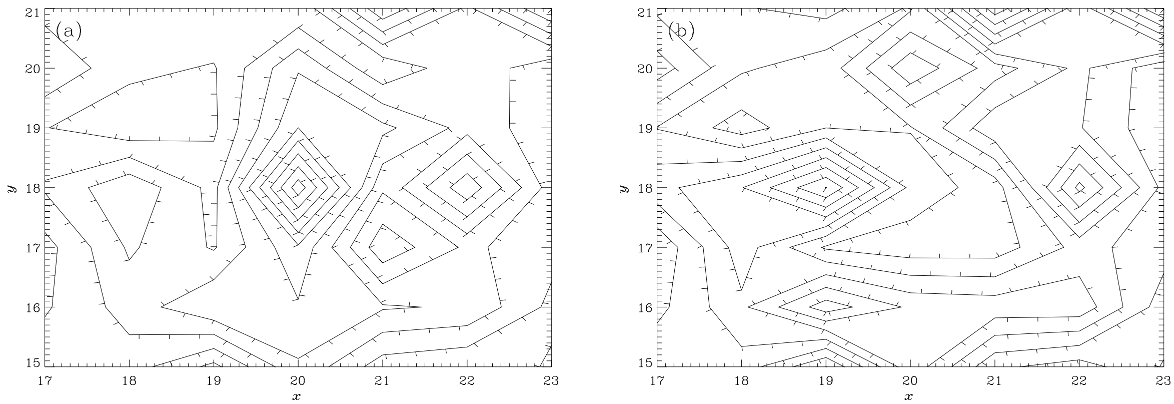

Figure 7: Contour-plot of the current-density, before (a), and after (b) a burst, which occurs in the middle of the plot (the z-coordinate is fixed) (from Isliker et al. 2000).

then given as secondary variables (see Fig. (6)), as in MHD, and determined according to Maxwell's equations,

$$\vec{B} = \nabla \times \vec{A} \tag{20}$$

(which ensures that $\nabla \vec{B} = 0$), and

$$\vec{J} = \frac{c}{4\pi} \nabla \times \vec{B}. \tag{21}$$

The electric field is determined by the simple Ohm's law,

$$\vec{E} = \eta \vec{J}. \tag{22}$$

The dynamic evolution of the X-CA has two distinct phases. In the loading phase, random field increments $\delta\vec{A}$ are dropped at random sites. If a local instability criterion is fulfilled, local bursts (relaxing redistributions of $\vec{A}$) are triggered, during which energy is released, whose amount is determined as Ohmic dissipation ($\sim \eta \vec{J}^2$). As a critical quantity either $d\vec{A}$, the stress in $\vec{A}$, as defined in the classical CAs (e.g. Lu & Hamilton 1991), or, new in the context of CAs, the current density $\vec{J}$ is used.

The X-CA model yields power-law frequency distributions for the diverse flare parameters (total energy, peak flux) which are compatible with the observations, and, unlike the classical CAs, the X-CA yields insight into the physical scenario and the physical processes during flares:

1. The vector-potential, magnetic field, and current exhibit large-scale organization and quasi-symmetries. Depending on the directionality of loading and the boundary conditions, the global topology of the magnetic field has two varieties: Either it forms an arcade of magnetic field lines, or it forms closed magnetic field lines around and along a neutral line.

2. The temporal evolution of the model follows the MHD induction equation, which, expressed in terms of the vector potential, writes

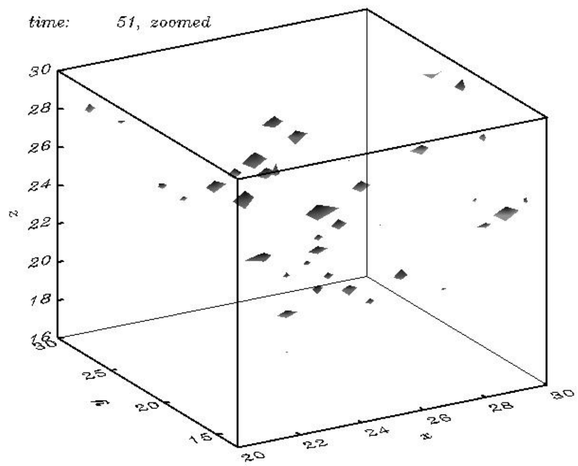

Figure 8: Current dissipation regions (i.e. volumes enclosing regions of super-critical current) at a temporal snap-shot during a flare (from Isliker et al. 2001).

$$\frac{\partial \vec{A}}{\partial t} = \vec{v} \times \vec{B} + \eta \frac{c^2}{4\pi} \nabla^2 \vec{A} - \eta \frac{c^2}{4\pi} \nabla (\nabla \vec{A}) + \nabla \chi, \tag{23}$$

where η is the diffusivity and χ an arbitrary function.

2.a) The loading increments $\delta \vec{A}$ represent perturbations of the form $\delta \vec{A} \sim \vec{v} \times \vec{B}$, with $\vec{v}$ the velocity of an implicitly assumed plasma which flows upwards and expands, so that the loading process implements the action of the convective term in Eq. (23).

2.b) Bursts are localized diffusion processes, accompanied by energy release through current dissipation. Bursts implement the solution to the diffusive part of Eq. (23) in one time-step, with some characteristic simplifications, though.

3.Bursts occur there where the currents are large, and after a burst the local current is relaxed (Fig. (7)).

4. An important innovation of the X-CA model is the direct use of the current in the instability criterion. With this modification, the X-CA directly implements Parker's (1993) flare scenario that an instability is triggered if the current $\vec{J}$ exceeds some threshold, since this implies that, through a current driven instability, the resistivity increases to anomalous values, and diffusion dominates over convection in the time evolution. The X-CA model incorporates thus the kinetic plasma physics which rules the behavior of the resistivity η, simulating the effect of occasionally appearing anomalous resistivities due to current instabilities.

5. The current-dissipation is spatially and temporally fragmented into a large number of practically independent, dispersed, and disconnected dissipation regions with the shape of current-surfaces, which vary in size and are spread over a considerable volume (Fig.(8)). These current-surfaces do not grow in the course of time, but they multiply and are short-lived.

6. Important for future applications to particle acceleration is that the X-CA makes the electric fields explicitly available as a function of time and space. The electric fields are short-lived and spatially distributed, and they spread and travel through space in the course of time (Fig. (9)).

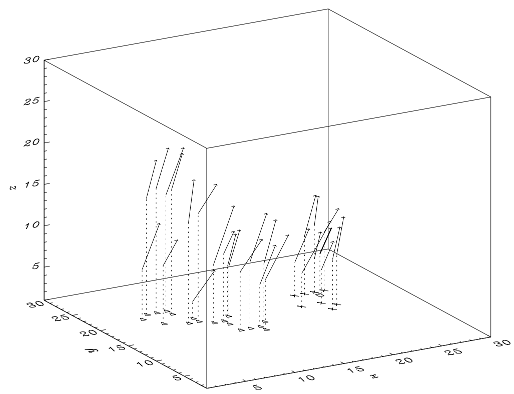

Figure 9: The electric field-vectors during a flare, at three different time-steps: at the beginning of the flare (bold-vector, projected grid-site in x-y–plane marked with a rectangle); after nine time-step (marked with 'x'); after 91 time-steps (marked with triangles). The vectors are shown in 3–D parallel projection, rescaled for visualization purposes, with length proportional to $|\vec{E}|$. Note that the electric fields of three different time-steps are shown together for visualization purposes, in the model actually only one set appears at a time, the fields of the previous time-steps have become zero again, at later times (from Isliker et al. 2001).

In conclusion, the X-CA model is a model for energy release through current dissipation, and it represents a realization of Parker's (1993) flare scenario. It allows interesting future developments, one of which is the introduction of particles into the system, which will allow to model the radiation signatures of the flaring plasma and the accelerated particles. This will put the comparison of CA models to observations on new grounds.

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