CELLULAR AUTOMATA MODELS: A SANDPILE MODEL APPLIED IN FUSION *

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SUMMARY

We present the basic properties of a simple cellular automaton (CA) model proposed for the study of turbulence and turbulent transport in magnetically confined plasmas. The CA is a running sandpile model, based on the Self Organized Criticality (SOC). Such a model, despite its simplicity, provides useful insights regarding the dynamics of magnetically confined plasmas.

1 CELLULAR AUTOMATA AND SELF ORGANIZED CRITICALITY

The purpose of this contribution is to introduce the key elements of a sandpile model, based on the Self Organized Criticality (SOC), that could provide the computational investigation of the dynamics for the case of magnetically confined plasmas. In this section, it is important to give several useful definitions, such as what is a spatially extended dynamical system, how we can study it, what is the Self Organized Critical (SOC) state and what are the basic properties that a system has at the SOC state.

A spatially extended dynamical system is a system with both temporal and spatial degrees of freedom. Here the term degrees of freedom corresponds to the independent field variables which are necessary to describe the state of the system. Such a system is a non-linear complex dynamical one (i.e. a system that consists of a large number of different non-linear interacting sub-systems).

In order to study any spatially extended dynamical system, usually we try to limit the degrees of freedom, by making several assumptions (i.e. all the sub-systems of the complex dynamical system are identical). Our goal is to construct a set of differential equations which can describe the evolution of the system. There is always the possibility that these differential equations are not easy to solve and thus we use several numerical computational schemes. The use of numerical schemes requires the construction of a set of difference equations from the set of differential equations. This is the most common approach, but in several cases it can not be applied. In these cases the construction of Cellular Automata (CA) models, can provide the unswear to the question, what is the spatio-temporal evolution of spatially extended dynamical systems.

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Cellular Automata (CA) models can be considered as a set of simple rules applied in a specific grid. These models are by nature discrete in time and space and for their development one needs to specify the grid (i.e. to give the dimensions and its type), the boundary conditions, the evolution rules and the interactions of the nearest neighbors of a given grid site. The important advantage in developing CA models is that we can study nature without the need of constructing and solving differential equations (for detailed discussion on CA models see Wolfram (1986)).

An interesting class of spatially extended dynamical systems is the ones that can reach a self organized critical state. We must emphasize that Self Organized Criticality (SOC), despite the various efforts done in the past (see Hwa and Kardar 1992; Gil and Sornette 1996), is a just a concept and not a theory. When a spatially extended dynamical system is in a self organized critical state it exhibits self organization and critical behavior. We must emphasize here that any system which has either the property of self organization and/or critical behavior is not necessary in SOC state.

The self organization is the property that a system has, during its evolution, to minimize its number of degrees of freedom, when a local disturbance appears due to an external driver. This system then exhibits self similarity. In other words, the system has a scale invariant behavior and its statistical studies will show power law types of frequency distributions (see Bak et al. 1988; Kadanoff et al. 1989).

On the other hand a dynamical system exhibits critical behavior when a disturbance, that exceeds a critical value, can be absorbed by the system or might cause catastrophic events in a cascading way (i.e. in a form of avalanches). A typical property of such systems is that the Fourier power spectra of the time series of the system parameters show power laws with spectral index close to 1. This property is usually called $1/f$ flicker noise (Bak et al. 1987).

In the next section we try to address the problem of fusion and the role of turbulence and turbulent transport. In section 3, we are going to present the key elements of a sandpile CA model that was developed to study the turbulence and the turbulent transport in fusion plasmas. Finally, in the last section, we present some selective results. Note that the detail and extensive presentation of the model and its results is not the main goal of this contribution, so the interested reader should refer to the original papers (see the reference list).

2 TURBULENCE AND TRANSPORT IN FUSION

It is well known that one of the most important parameters in controlling fusion is the role of magnetic fields. The plasma in fusion devices is confined by large magnetic fields which are created by driving currents through coils usually wrapped around a torus. The basic problem in fusion is that we need the core plasma to be hot enough for fusion to happen, but on the other hand the edges must be cold enough not to melt the torus walls (see Fig.1). It is clear that such a set up creates temperature and density gradients in the magnetically confined plasmas.

One basic property of nature is that it hates gradients. Whenever a gradient (slope) gets too steep, nature finds a way to flatten it out. Typical examples are that moun-

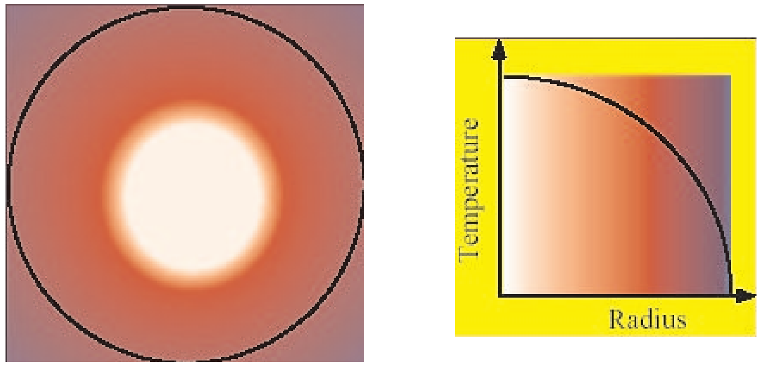

Figure 1: Illustration of the density and temperature variation inside a fusion torus.

tains get eroded, sand and show avalanches. The same happens when gradients are appearing in magnetically confined plasmas inside fusion machines. In that case, turbulence starts to develop in order to eliminate the gradients.

It is obvious that in order to control fusion, we need to control turbulence. From this point of view, the problem of fusion can be described by the following circle: The magnetically confined plasma is driven externally (i.e. the external driver are the necessary fuels to heat the plasma) leading to the generation of gradients. In this way, free energy is accumulated in the developing gradients which drive the turbulence. The turbulent transport can remove the free energy by relaxing the gradients and thus turning off the turbulence. The circle starts from the beginning as long as we continue to drive the system externally.

In addition to the above general description of the fusion problem, we must emphasize that there is a strong coupling between the bulk plasma flows (external or internal) and the turbulence in plasmas. If the bulk flow is uniform the imbedded turbulence is advected. In this way the developing eddies of turbulence in a fusion plasma can move the heat and the density towards the edge (see Fig. 2 top). On the other hand if the bulk flow is spatially dependent (sheared flow), the dynamics can be completely different, as can be seen in Fig.2 (bottom).

The magnetically confined plasmas which are driven externally are spatially extended dynamical systems which exhibit SOC. Their complicated dynamics observed in simulations and experiments shows that relaxation and transport processes occur through avalanches of all sizes. The dynamics of such systems can be computationally investigated with a cellular automaton model of a running sandpile. This model allows us to investigate the major dynamical scales and the effect of an applied sheared flow on these dominant scales.

3 THE SANDPILE MODEL FOR FUSION

In order to understand the complex dynamics of turbulence and the transport process in magnetic confinement devices a simple Self Organized Criticality (SOC) model based on the dynamics of a running sandpile has been used (see for details Newman et

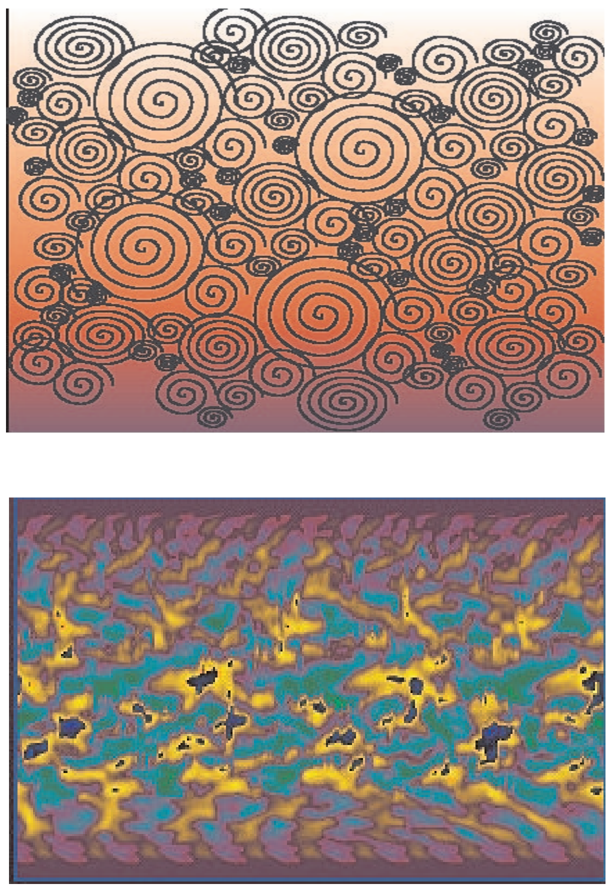

Figure 2: *Top*: Illustration of the turbulent eddies imbedded in a uniform bulk flow. Note that small eddies are moving towards the edges. *Bottom*: Density profile for the case of a sheared flow, located in the middle of the the frame.

al. 1996; Carreras et al. 1999; 2002). We are going to present in this section the key elements of the CA model.

In the following we consider for simplicity, a one dimensional domain divided into L cells of equal length. Let us define by h_n the number of sand grains in a given cell n (i.e. h_n corresponds to the height of the pile). We can then define as Z_n the local gradient (the slope) between two grid sites, given by the relation:

$$Z_n = h_n - h_{n+1} \tag{1}$$

Finally, we consider that N_f is the amount of sand (the number of sand grains) that falls down in an over-turning event which occurs every time that the local gradient Z_n is greater than a critical value Z_{cr}. We can associate the above variables with parameters which can characterize the turbulence as following: Each grid cell is the location of turbulent fluctuation (eddy), Z_{cr} is the gradient at which fluctuations are unstable and grow and N_f is the local eddy induced transport. In this way the average sandpile profile is equivalent to the mean temperature or density profile.

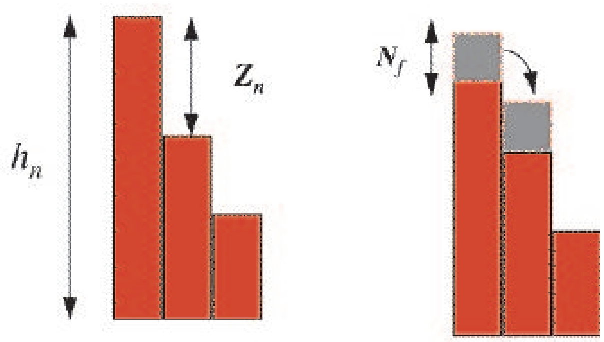

Figure 3: Illustration of the time evolution of the sandpile model. The unstable cell over-turns its excess grains (N_f) to the neighboring cell.

The rules, governing the cellular automaton, are simple and the basic steps are the following:

1. Initialization: Sand grains are added to the cells randomly or with a given probability p_0. Then $h_n = h_n + 1$ otherwise h_n is constant.

2. Instability criterion: For each cell if $Z_n \geq Z_{cr}$ then the cell is unstable.

3. Time evolution: Each cell is advanced in time but only the unstable ones over-turning their excess grains to the next cell (see Fig. 3). That is if the n cell is unstable ($Z_n \geq Z_{cr}$) then:

$$h_n = h_n - N_f \qquad \textit{and} \qquad h_{n+1} = h_{n+1} - N_f \tag{2}$$

4. When all the gradients are relaxed in a time step, we return back to step 1.

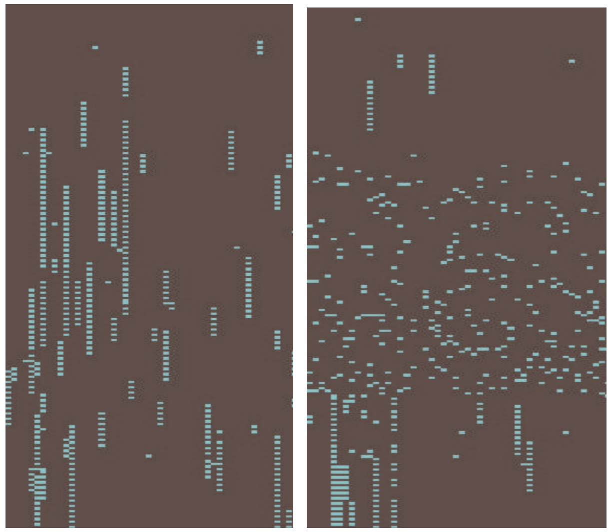

Figure 4: Two different frames of the sandpile model, showing the avalanches of unstable grid site (light colored squares) . *Left:* Avalanches without any sheared flow applied in the CA. *Right:* The applied shear, in the middle, causes the distortion of avalanches in the CA. (adapted from Newman et al. 1996)

In order to apply the above set up for the case of magnetically confined plasmas in a torus, we can easily expand it into a two dimensional domain with the x-axis to be the radial coordinate of the torus and the y-axis the poloidal angle. We can apply in this case, at $x = 0$ closed boundary conditions and open ones at the other end (at $x = L$). Finally periodic boundary conditions can be used in y-direction.

4 SELECTIVE RESULTS

This simple model, presented in the previous section, displays remarkably rich dynamics which have many characteristics in common with the observed transport dynamics in magnetic confinement devices. In this section, we are going to present only some selective results from such a CA model applied in fusion.

In Fig.4 two different frames of the face of the sandpile are presented. The light colored squares in the figure are unstable (avalanching) sites, while the dark colored squares (the rest) are stable sites. The system is periodic across the pile, closed at the top and open at the bottom. Avalanches of all sizes (a characteristic feature of SOC systems) can be clearly seen on the left frame. The avalanches are extended even at the boundaries, leading to a bad confinement of the plasma. In the right frame, a sheared flow is applied at the middle of the frame. The large transport events (i.e. avalanches) are torn apart in the shear region. This effect reduces the effective transport and improves the confinement of the plasma in the fusion device.

In Fig.5 left, an example of a particle (i.e. sand grain) moving along a sandpile is presented. The particle is allowed to reaches the boundaries of the sandpile, but when that happens, it is placed back inside at the same initial position. It is clear that

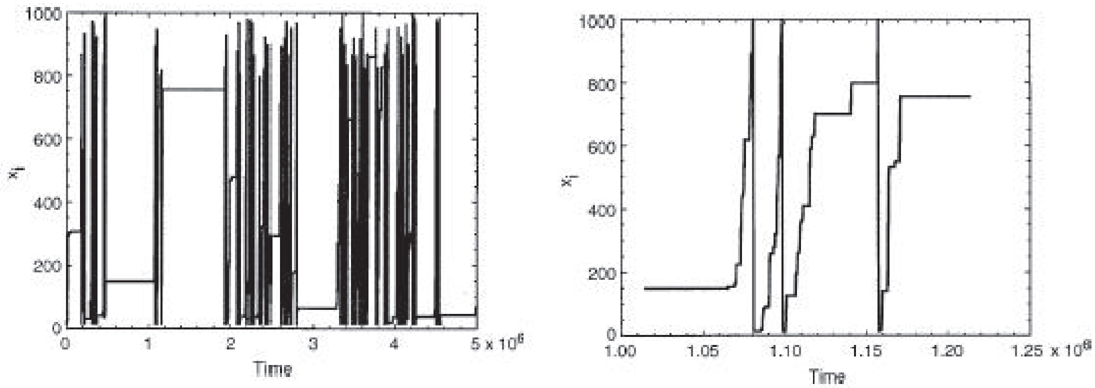

Figure 5: The radial position of a single particle (i.e. sand grain) in a sandpile of length $L = 1000$ is presented. *Left:* The entire trajectory of the particle is shown. *Right:* An expanded view of a particle's trajectory is presented. (adapted from Carreras et al. 1999).

during some time periods the particle moves fast. On the other hand, there are also long waiting time periods. If we look with more detail the orbit by expanding the time scale (see Fig. 5 right), we see that the particle can move out of the pile by flights of all sizes. When an avalanche occurs the fights of the particle depends on the radial extent of the avalanche. On the other hand, in the absence of avalanche the particle remains at a given radial position. It is clear that the transport of grains in the sandpile is not a normal diffusion process. As it was shown by Carreras et al. (1999) by performing numerical simulations of the sandpile model, the underlying transport mechanism for distances less than the sandpile length is super-diffusive.

We tried to present some selective results of this simple CA model. It is clear that with such a model ,we can explore, among other physical processes, the interaction between the avalanche like transport and the sheared flows which are so important in plasma confinement. Finally, the use of the sandpile model can provide useful insights regarding the transport dynamics and the two apparent transport regimes in plasma confinement devices.

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