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Dynamical complexity in D_{st} time series using non-extensive Tsallis entropy

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[1] Nonlinearly evolving dynamical systems, such as space plasmas, generate complex fluctuations in their output signals that reflect the underlying dynamics. The non-extensive Tsallis entropy has been proposed as a measure to investigate the complexity of system dynamics. We employ this method for analyzing D_{st} time series. The results show that Tsallis entropy can effectively detect the dissimilarity of complexity between the pre-storm activity and intense magnetic storms ($D_{st} < -150$ nT), which is convenient for space weather applications. **Citation:** Balasis, G., I. A. Daglis, C. Papadimitriou, M. Kalimeri, A. Anastasiadis, and K. Eftaxias (2008), Dynamical complexity in D_{st} time series using non-extensive Tsallis entropy, *Geophys. Res. Lett.*, *35*, L14102, doi:10.1029/2008GL034743.

1. Introduction

[2] Dynamical complexity detection for output time series of complex systems is one of the foremost problems in physics, biology, engineering, and economic sciences. Especially in magnetospheric physics, accurate detection of the dissimilarity between normal and abnormal states (e.g. pre-storm activity and magnetic storms) can vastly improve space weather diagnosis and, consequently, the mitigation of space weather hazards.

[3] The uncertainty of an open system state can be quantified by the Boltzmann-Gibbs (B-G) entropy, which is the widest known uncertainty measure in statistical mechanics. B-G entropy (S_{B-G}) cannot, however, describe non-equilibrium physical systems with large variability and multi-fractal structure such as the solar wind [*Burlaga et al.*, 2007]. Inspired by multi-fractal concepts, *Tsallis* [1988, 1998] has proposed a generalization of the B-G statistics, which is briefly described in section 2.

[4] Here we study whether certain signatures of D_{st} time series indicate the transition from pre-storm activity to magnetic storms. Magnetic storms (MSs) are the ultimate result of the interaction between the terrestrial magnetic field and embedded hot plasma and particular magneto-plasma structures that originate at the Sun and propagate to the near-Earth space environment. MSs produce a number of distinct physical effects in near-Earth space environment: acceleration of charged particles in space, intensification of electric currents in space and on the ground, impressive aurora displays, and global magnetic disturbances on the Earth's surface [*Daglis et al.*, 2001, 2003]. The latter serve as the basis for storm monitoring via the hourly D_{st} index, which is computed from an average over 4 mid-latitude magnetic observatories (http://swdcwww.kugi.kyoto-u.ac.jp/).

[5] The D_{st} data used in this study include two intense magnetic storms, which occurred on 31 March 2001 and 6 November 2001 with minimums D_{st} -387 nT and -292 nT, respectively, as well as a number of smaller events (e.g. May and August 2001 with $D_{st} \sim -100$ nT in both cases). The complex system of the Earth's magnetosphere corresponds to an open spatially extended non-equilibrium (input - output) system; therefore we employ the time-dependent Tsallis entropy (S_q) as a measure of dynamics complexity, thus quantifying the degree of predictability in magnetospheric evolution.

[6] Our analysis reveals that Tsallis entropy detects the pattern of alterations in D_{st} time series prior to the intense storm events and is able to discriminate between the different states of the magnetosphere. Furthermore, we compare the results of our analysis with the results of a previously published fractal spectral analysis, performed through wavelets [*Balasis et al.*, 2006]. The results suggest that a significant complexity decrease coupled with appearance of persistency can be confirmed in the D_{st} index at the transition from pre-storm activity to intense magnetic storms. We suggest that this feature may be used as a diagnostic tool for forthcoming extreme events in space plasmas.

2. Principles of Tsallis Entropy

[7] The aim of statistical mechanics is to establish a direct link between the mechanical laws and classical thermodynamics. One of the crucial properties of the S_{B-G} in the context of classical thermodynamics is extensivity, namely proportionality with the number of elements of the system. The S_{B-G} satisfies this prescription if the subsystems are statistically (quasi-) independent, or typically if the correlations within the system are essentially local. In such cases the system is called extensive.

[8] In general, however, the situation is not of this type and correlations may be far from negligible at all scales. In such cases the S_{B-G} is non-extensive. *Tsallis* [1988, 1998] introduced an entropic expression characterized by an index q which leads to a non-extensive statistics, $S_q = k\frac{1}{q-1}(1 - \sum_{i=1}^{W} p_i^q)$, where p_i are the probabilities associated with the microscopic configurations, W is their total number, q is a real number, and k is Boltzmann's constant. The value of q

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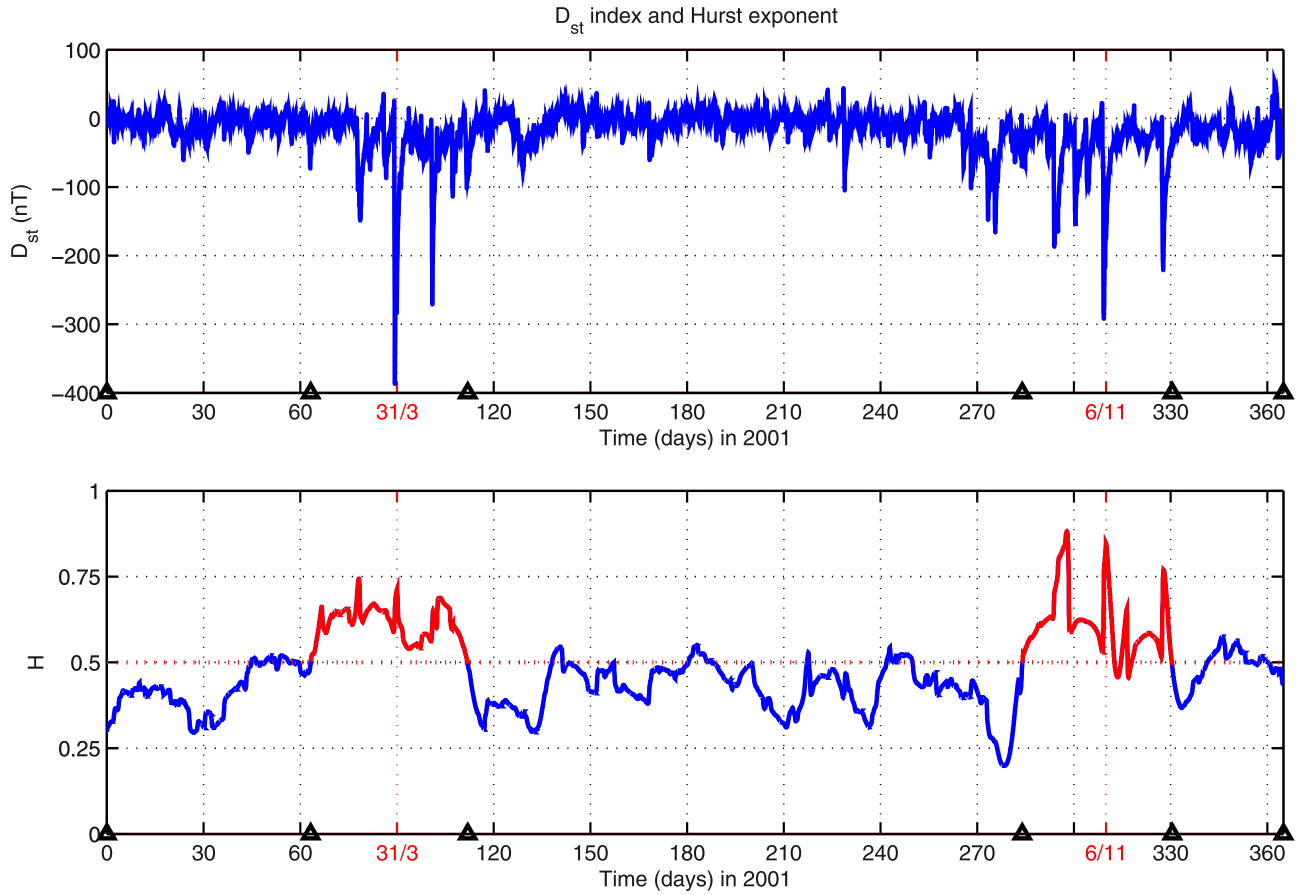

Figure 1. (top) D_{st} time series and (bottom) Hurst exponents H. The 31 March and 6 November 2001 MSs are marked with red. The red dashed line in H plot marks the transition between anti-persistent and persistent behavior. The triangles denote the time intervals corresponding to the 5 time windows discussed in the text.

is a measure of the non-extensivity of the system: $q \rightarrow 1$ corresponds to the standard extensive B-G statistics.

[9] This is the basis of the so called non-extensive statistical mechanics, which generalizes the B-G theory. The entropic index q characterizes the degree of non-additivity reflected in the following pseudo-additivity rule: $S_q(A + B) = S_q(A) + S_q(B) + (1 - q)S_q(A)S_q(B)$.

[10] The cases $q > 1$ and $q < 1$, correspond to sub-additivity, or super-additivity, respectively. For subsystems that have special probability correlations, extensivity is not valid for S_{B-G}, but may occur for S_q with a particular value of the index q. Such systems are sometimes referred to as non-extensive [*Boon and Tsallis*, 2005]. The parameter q itself is not a measure of the complexity of the system but measures the degree of non-extensivity of the system. It is the time variations of the Tsallis entropy for a given q (S_q) that quantify the dynamic changes of the complexity of the system. Lower S_q values characterize the portions of the signal with lower complexity.

3. Application to Data: Estimation of Tsallis Entropy Through Symbolic Dynamics

3.1. Tsallis Entropy in Terms of Symbolic Dynamics

[11] Symbolic dynamics provides a rigorous way of looking at the invariant, robust properties of the dynamics [*Hao*, 1989]. New methods of nonlinear dynamics derived from the symbolic dynamics have been introduced to distinguish between different states of the system interactions. These methods provide a detailed description and classification of dynamic changes of various real-world time series [*Schwarz et al.*, 1993; *Wanliss et al.*, 2005].

[12] Herein, we estimate S_q based on the concept of symbolic dynamics: from the initial measurements we generate a sequence of symbols, where the dynamics of the original (under analysis) system has been projected [*Hao*, 1989]. More precisely, the original D_{st} time series of length N, $(X_1, X_2, \ldots, X_N)$, is projected to a symbolic time series $(A_1, A_2, \ldots, A_N)$ with A_n from a finite alphabet of λ letters $(0, \ldots, \lambda - 1)$ [see, e.g., *Kalimeri et al.*, 2008].

[13] After symbolization, the next step in identification of temporal patterns is the construction of symbol sequences with size L. We use the technique of lumping. Thus, we stipulate that the symbolic sequence is to be read in terms of distinct successive "blocks" of length L, $A_1, A_2, \ldots, A_L/A_{L+1}, \ldots, A_{2L}/A_{jL+1}, \ldots, A_{(j+1)L}$.

[14] The number of all possible blocks of length L in a λ-letter alphabet is $N_\lambda = \lambda^L$. We determine the probabilities of occurrence of each of N_λ different kind of blocks, $p(L)_{A_1,A_2,\ldots,A_L} = \frac{\text{Number of blocks of the form } A_1,A_2,\ldots,A_L \text{ encounter by lumping}}{\text{Total number of blocks encountered by lumping}}$.

[15] To be more concrete, the simplest possible coarse graining of the D_{st} index is given by choosing a threshold C (usually the mean value of the data considered) and assigning the symbols "1" and "0" to the signal, depending on

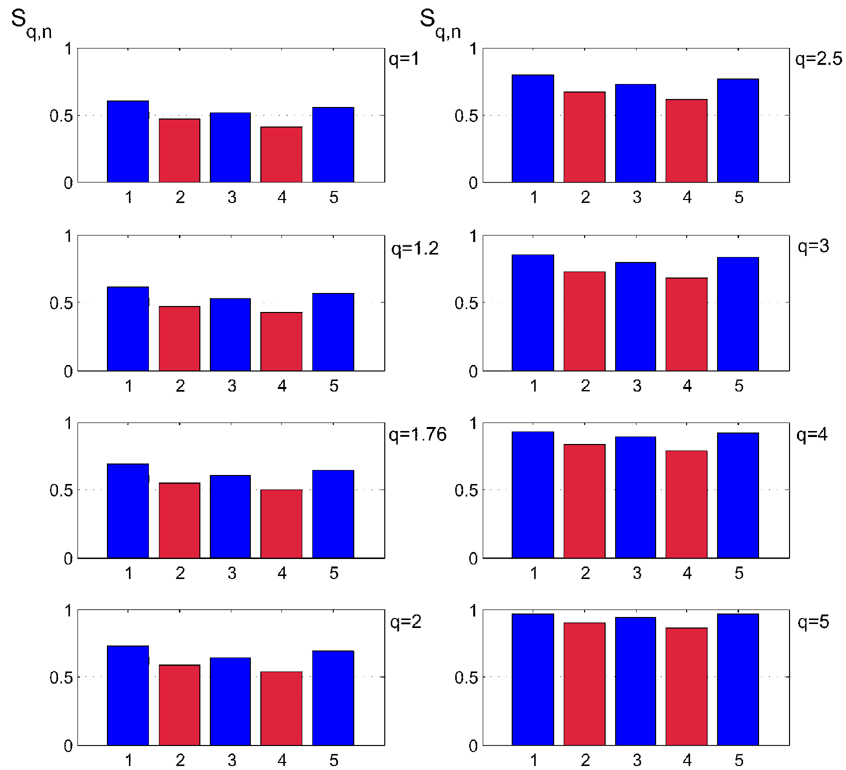

Figure 2. The normalized Tsallis entropies S_q calculated at the 5 time windows (derived after the initial D_{st} time series was divided into 5 shorter time intervals as shown in Figure 1) for various values of the entropic index q.

whether it is above or below the threshold (binary partition). Thus, we generate a symbolic time series from a 2-letter (λ = 2) alphabet (0,1), e.g. 0110100110010110…. Reading the sequence by lumping of length L = 2 one obtains 01/10/10/01/10/01/01/10/…. The number of all possible kinds of blocks is $\lambda^L = 2^2 = 4$, namely 00, 01, 10, 11. Thus, the required probabilities for the estimation of the Tsallis entropy p_{00}, p_{01}, p_{10}, p_{11} are the fractions of the blocks 00, 01, 10, 11 in the symbolic time series.

[16] The S_q for the word length L is

$$S_q(L) = k\frac{1}{q-1}\left(1 - \sum_{(A_1,A_2,\ldots,A_L)}\left[p(L)_{A_1,A_2,\ldots,A_L}\right]^q\right). \quad (1)$$

[17] Broad symbol-sequence frequency distributions produce high entropy values, indicating a low degree of organization. Conversely, when certain sequences exhibit high frequencies, lower entropy values are produced, indicating a high degree of organization.

3.2. Tsallis Entropy in Its Time-Dependent Fashion

[18] A way to examine transient phenomena is to divide their outputting time series into shorter time intervals, related with different activity levels of the corresponding natural systems, and consequently analyze these time windows separately. If this analysis yields different results for time windows associated to an intense magnetic storm, for instance, in comparison to time windows associated to the regular state of the magnetosphere, then a transient behavior can be extracted.

[19] In Figure 1 the D_{st} time series is presented. The one year D_{st} data (2001) are divided into 5 shorter time series (see triangles denoting 5 distinct time windows in Figure 1). The second and fourth time windows include the D_{st} variations associated to the 2 intense MSs of 31/3/2001 and 6/11/2001, respectively. Within each of the 5 time windows, the Tsallis entropy S_q is calculated for different values of the entropic index q (1, 1.2, 1.76, 2, 2.5, 3, 4, 5) using the technique of lumping.

[20] Figure 2 shows the normalized S_q for the 5 different windows. The entropies have been normalized with respect to the entropies given in equation (1) for a uniform distribution of probabilities. We study the temporal evolution of the S_q as the global instability is approaching. The blue time windows are referred to the normal state of magnetosphere. Their S_q values are lower in respect to that given in equation (1) for a uniform distribution of probabilities. This evidence indicates the existence of an organization in the magnetosphere even in this normal state. The entropies in the red windows drop to rather significantly lower values suggesting the appearance of a new distinct state in the magnetosphere, which is characterized by a lower complexity in comparison to that of the blue (normal) epoch of the magnetosphere.

[21] As expected, our results depend upon the Tsallis q value. Figure 2 clearly illustrates the superiority of the q values restricted in the range $1 < q < 2$ to magnify differences of the S_q and thus of the complexity as the global instability is approaching. It is worth mentioning that the non-extensive q parameters that clearly quantify the temporal evolution of the complexity in the D_{st} time series are in full

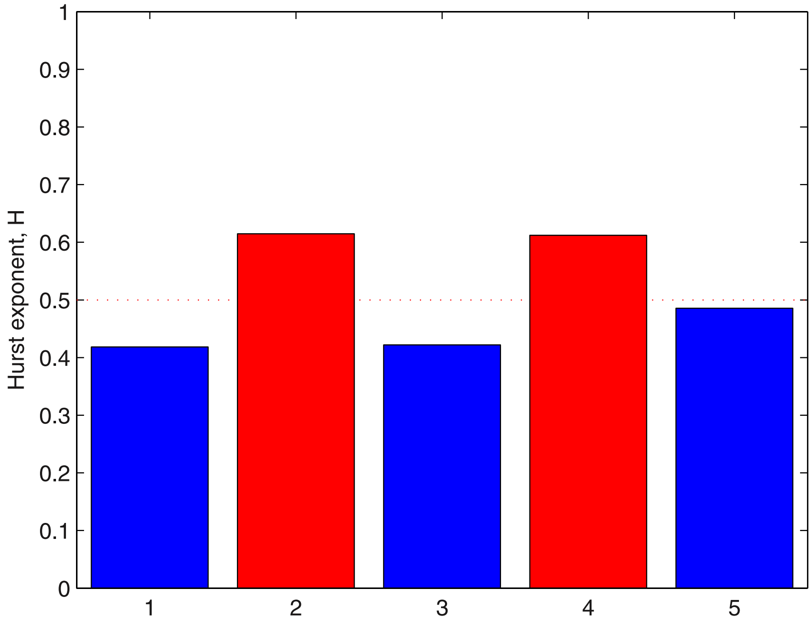

Figure 3. Average values of the Hurst exponents H calculated at the same 5 time windows as the Tsallis entropies S_q were in Figure 2. The red dashed line marks the transition between anti-persistent and persistent behavior.

agreement with the upper limit $q < 2$ obtained from several studies involving the Tsallis non-extensive framework [*Vilar et al.*, 2007, and references therein]. Moreover, they are in harmony with an underlying sub-extensive system, $q > 1$, verifying the emergence of strong interactions in the magnetosphere, especially during the occurrence of an intense MS.

[22] It is worth mentioning that the case $q = 1$ (Figure 2) seems to provide a hint for two different patterns in the evolution of the system under study. However, the magnetosphere, due to the appearance of strong interactions across its system, especially immediate before the MS occurrence, clearly violates the B-G statistics. Therefore the result for this particular case has absolutely no physical meaning.

[23] As mentioned, the results depend upon the entropic index q. Therefore the appropriate choice of the q index is significant and needs to be examined [*Naudts*, 2002]. It is expected that, for every specific system, better discrimination will be achieved with appropriates ranges of q values [*Tsallis*, 1998]. Thus, a challenge will be to estimate the appropriate value of q which is associated with the generation of magnetic storms. We will attempt an estimation of the appropriate choice of the q index based on ideas rooted in the areas of scale invariance and universality at a subsequent publication.

[24] Various tests have been performed with different lengths of word (L) for the purposes of symbolic dynamics analysis (see section 3.1), as well as with different candidate lengths of time windows of the initial time series. We managed to achieve, by gradually varying both kinds of length, the optimal values in order to best describe and resolve the transition from the normal magnetospheric state to intense magnetic storms. Because of space limitations, only the results for the optimal length values are given in this paper, which are also the most interesting from the physical point of view.

4. A Comparison of Complexity and Anti-persistency/Persistency in Their Time-Dependent Fashion

[25] It would be highly desirable to confirm the above mentioned emergence of two different patterns in the D_{st} time series based on an independent analysis. For this purpose, we compare the time-dependent fashion of the S_q with that of anti-persistency/persistency extracted by a fractal spectral analysis in terms of the Hurst exponent, H (for details on the calculation of H the reader is referred to *Balasis et al.* [2006]). The spectral analysis was based on wavelet tools previously developed by *Mandea and Balasis* [2006] (see report on http://www.sciencemag.org/content/vol314/issue5798/twil.dtl) and *Balasis and Mandea* [2007]. In Figure 1 the values of the H parameter are shown for the D_{st} index data. When the corresponding H values are in the interval (0 0.5) the time series has anti-persistent properties, which means that if the fluctuations increase with time, it is likely to decrease in the interval immediately following and vice versa. Physically, this implies that fluctuations tend to induce stability within the system (negative feedback mechanism). Figure 1 reveals that the D_{st} time series exhibits anti-persistent properties during the quiet period (i.e. well before and after 31 March and 6 November 2001 MSs). If H takes values in the interval (0.5 1) the signal exhibits persistent properties, which means that if the amplitude of the fluctuations increases with time, it is likely to continue increasing in the immediately next interval. In other words, the underlying dynamics is governed by a positive feedback mechanism. Figure 1 shows that D_{st} exhibits persistent properties ($0.5 < H < 1$) around 31 March and 6 November 2001 MSs (c.f. parts of H plot marked in red represent

persistency). Thus, it is evident that the onset and development of the MSs of 31/3/2001 and 6/11/2001 are associated with persistent behavior.

[26] The value $H = 0.5$ suggests that there is no correlation between the repeated increments. Consequently, this particular value takes on a special physical meaning: it marks the transition between anti-persistent and persistent behavior in the time series. To conclude, one can recognize two different regimes. The first refers to quiet epochs of magnetospheric activity. The associated D_{st} time series follows anti-persistent behavior. The second regime refers to the epoch including an intense MS. The D_{st} time series shows persistent behavior.

[27] In Figure 3 we show the average values of the Hurst exponents H calculated at the same 5 time windows as the Tsallis entropies S_q are given in Figure 2. We stress that the anti-persistent epochs ($0 < H < 0.5$) correspond to the epochs of high Tsallis entropies (first, third and fifth time windows given in blue in both Figures 2 and 3), while, the persistent epochs ($0.5 < H < 1$) corresponds to the epochs of lower Tsallis entropies (second and fourth time windows given in red in both Figures 2 and 3). This finding further supports the existence of two different epochs referring to two distinct states of the MS evolution. Anti-persistent behavior and high Tsallis entropy correspond to a regular undisturbed magnetosphere while persistent behavior and lower Tsallis entropy correspond to a disturbed storm-time magnetosphere.

[28] In summary, an ‘‘ordered’’ persistent sequence of D_{st}-values indicates a ‘‘stormy magnetosphere’’, and a ‘‘disordered’’ anti-persistent sequence of D_{st}-values is a clear indication of a ‘‘calm magnetosphere’’.

5. Conclusions and Discussion

[29] Herein, we analyze D_{st} time series by introducing the non-extensive Tsallis entropy, S_q, as an appropriate complexity measure.

[30] The Tsallis entropy sensitively shows the complexity dissimilarity among different ‘‘physiological’’ (normal) and ‘‘pathological’’ states (intense magnetic storms). The Tsallis entropy implies the emergence of two distinct patterns: (i) a pattern associated with the intense magnetic storms, which is characterized by a higher degree of organization, and (ii) a pattern associated with normal periods, which is characterized by a lower degree of organization.

[31] We then analyze the same time windows in terms of Hurst exponent, H, based on the use of wavelet transforms. The wavelet spectral analysis also shows the existence of two different patterns: (i) a pattern associated with the intense magnetic storms, which is characterized by a fractional Brownian persistent behavior; (ii) a pattern associated with normal periods, which is characterized by a fractional Brownian anti-persistent behavior.

[32] We stress that the anti-persistent time windows correspond to the time windows of high Tsallis entropies, while the persistent time windows correspond to the time windows of low Tsallis entropies. Importantly, a recent analysis presented by *Carbone and Stanley* [2007] shows that anti-correlated time series, with Hurst exponent $0.5 < H < 1$, are characterized by entropies greater than correlated time series having $0.5 < H < 1$. This suggestion is in agreement with our results.

[33] In summary, a combination of the Tsallis entropy with the Hurst exponent proved to be a powerful tool, showing evidence that the occurrence of an intense magnetic storm is imminent, and thus, providing convenience for space weather applications. By applying Tsallis entropy and Hurst exponent analysis to small and moderate magnetic storms we conclude that evidence for a similar transition, i.e. from the regular undisturbed magnetosphere to events with $D_{st} > -100$ nT, cannot be found.

[34] Dynamical complexity is a phenomenon expected to be observed in space plasmas. *Chang et al.* [2006] stated that the prerequisite for the onset of the phenomenon is the ability to form multitudes of varieties of large scale coherent structures of different sizes. They made a 2D magnetohydrodynamic (MHD) numerical simulation showing the existence of such structures. Here, we provide evidence for detection of dynamical complexity in the magnetosphere that actually conforms with the expectation for observing the phenomenon.

[35] Reduction of multiscale complexity was also observed in high-latitude geomagnetic activity prior to strong substorms using cellular automata models by *Uritsky et al.* [2001]. Our results favor the idea of intermittent turbulence in magnetosphere and could serve as a starting point for MS forecasting in the future.

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