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Investigating dynamical complexity in the magnetosphere using various entropy measures

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[1] The complex system of the Earth's magnetosphere corresponds to an open spatially extended nonequilibrium (input-output) dynamical system. The nonextensive Tsallis entropy has been recently introduced as an appropriate information measure to investigate dynamical complexity in the magnetosphere. The method has been employed for analyzing D_{st} time series and gave promising results, detecting the complexity dissimilarity among different physiological and pathological magnetospheric states (i.e., prestorm activity and intense magnetic storms, respectively). This paper explores the applicability and effectiveness of a variety of computable entropy measures (e.g., block entropy, Kolmogorov entropy, T complexity, and approximate entropy) to the investigation of dynamical complexity in the magnetosphere. We show that as the magnetic storm approaches there is clear evidence of significant lower complexity in the magnetosphere. The observed higher degree of organization of the system agrees with that inferred previously, from an independent linear fractal spectral analysis based on wavelet transforms. This convergence between nonlinear and linear analyses provides a more reliable detection of the transition from the quiet time to the storm time magnetosphere, thus showing evidence that the occurrence of an intense magnetic storm is imminent. More precisely, we claim that our results suggest an important principle: significant complexity decrease and accession of persistency in D_{st} time series can be confirmed as the magnetic storm approaches, which can be used as diagnostic tools for the magnetospheric injury (global instability). Overall, approximate entropy and Tsallis entropy yield superior results for detecting dynamical complexity changes in the magnetosphere in comparison to the other entropy measures presented herein. Ultimately, the analysis tools developed in the course of this study for the treatment of D_{st} index can provide convenience for space weather applications.

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1. Introduction

[2] Nonlinearly evolving dynamical systems, such as space plasmas, generate complex fluctuations in their output signals that reflect the underlying dynamics. Accurate detection of the dissimilarity of complexity between normal and abnormal magnetospheric states (e.g., prestorm activity and magnetic storms) can vastly improve space weather diagnosis and, consequently, the mitigation of space weather hazards.

[3] Various complexity measures have been developed during the last 20 years for real-world time series in order to estimate the complexity of the corresponding dynamical system. The main types of complexity measures are entropies, fractal dimensions, and Lyapunov exponents. Fractal dimensions and Lyapunov exponents are both working well, but they generally require long data sets for statistically significant results, resulting in inconvenience in many studies and applications. On the other hand, entropies have the advantages of simplicity, extremely fast calculation, and antinoise ability. Entropy techniques provide convenience for detecting and capturing useful information of time series. Some entropy measures based on symbolic dynamics adopt a range partition to generate a partition in the symbolization transform, but their results may be compromised by the nonstationarity of the time series. The data sets obtained from most space physics studies are usually nonstationary, rather short, and noisy. One of our objectives is to find an effective complexity measure that requires short data sets for statistically significant results, provides the ability to make fast and robust calculations, and can be used to analyze nonstationary and noisy data, which is convenient for the analysis of magnetospheric time series.

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[4] The hourly disturbance storm time (D_{st}) geomagnetic activity index is computed from an average over 4 midlatitude magnetic observatories (http://swdcwww.kugi.kyoto-u.ac.jp/), and hence serves as a proxy for the magnetospheric ring current, and thus magnetic storm occurrence. Magnetic storms are the most prominent global phenomenon of geospace dynamics, interlinking the solar wind, magnetosphere, ionosphere, atmosphere and occasionally the Earth's surface [*Daglis and Kozyra*, 2002; *Daglis et al.*, 2003, 2008]. Magnetic storms occur when the accumulated input power from the solar wind exceeds a certain threshold.

[5] Recently *Balasis et al.* [2008] analyzed D_{st} time series by introducing the nonextensive Tsallis entropy as an effective complexity measure. The Tsallis entropy fluctuations sensitively showed the complexity dissimilarity among different "physiological" (quiet time) and "pathological" states (intense magnetic storms) of the magnetosphere. The Tsallis entropy fluctuations also implied the emergence of two distinct patterns: (1) a pattern associated with intense magnetic storms, which is characterized by a higher degree of organization, and (2) a pattern associated with normal periods, which is characterized by a lower degree of organization. *Balasis et al.* [2006] analyzed time series of the D_{st} index in terms of Hurst exponent, H, based on the use of wavelet transforms. The fractal spectral analysis gave evidence for the existence of two different patterns: (1) a pattern associated with intense magnetic storms, which is characterized by a fractional Brownian persistent behavior, and (2) a pattern associated with normal periods, which is characterized by a fractional Brownian antipersistent behavior.

[6] In this paper, we study in terms of nonlinear and linear techniques whether certain characteristic signatures emerged in D_{st} time series indicating the transition from prestorm activity to magnetic storms. We consider one year of D_{st} data (2001) including two intense magnetic storms, which occurred on 31 March 2001 and 6 November 2001 with minimum D_{st} values -387 nT and -292 nT respectively, as well as a number of weaker events (e.g., May and August 2001 with $D_{st} \sim -100$ nT in both cases). More precisely, first the temporal evolution of nonlinear characteristics is studied by applying a variety of recently proposed entropy techniques: the original D_{st} time series is projected to a symbolic sequence and then analyses in terms of dynamical (Shannon-like) block entropy, T complexity and approximate entropy follow. These analyses suggest that as the magnetic storm approaches, there is a clear transition from higher to lower complexity. We further verify our results in terms of the nonextensive Tsallis entropy, which is based on a statistical approach different than the classical Boltzmann-Gibbs theory.

[7] Although these methods have been studied both within pure mathematics and in a number of science applications, the present study is, to our knowledge, their first application to the magnetospheric physics case. It would be highly desirable to confirm the above mentioned conclusion based on an independent linear fractal spectral analysis. By monitoring the temporal evolution of the fractal spectral characteristics of D_{st} time series using wavelet techniques we show that significant alterations in associated scaling parameters occur (e.g., transition from antipersistent to persistent behavior) as an intense magnetic storm approaches. The observed convergence between nonlinear and linear analyses warns that the onset of an intense magnetic storm is imminent.

2. Symbolic Dynamics

[8] The discovery that simple deterministic systems can show a vast richness of behaviors in response to variations of initial conditions and/or control parameters, has been at the origin of an intense interdisciplinary research activity since the 1950s [*Khinchin*, 1957; *Nicolis*, 1991, 1995]. One of the outcomes of this work has been the realization that for an appropriate description of such complex systems, one needs to resort to a probabilistic approach [*Nicolis and Gaspard*, 1994]. It is well known since the pioneering work of Gibbs and Einstein that we can describe dynamics from two points of view. On the one hand, we have the individual description in terms of trajectories in classical dynamics, or of wave functions in quantum theory. On the other hand, we have the description in terms of ensembles described by a probability distribution (called the density matrix in quantum theory) [*Prigogine and Driebe*, 1997]. Now, once one leaves the description in terms of trajectories, a basic question that must be dealt with concerns the amount of information one may have access to on the temporal evolution of the system in the course of time.

[9] One of the approaches developed in this context is "coarse graining," whereby a complex system is viewed as an "information generator" producing messages consisting of a discrete set of symbols defined by partitioning the full continuous phase space into a finite number of cells. We refer to such a description as "symbolic dynamics" [*Nicolis et al.*, 1989; *Nicolis*, 1991, 1995; *Nicolis and Gaspard*, 1994]. One of its merits is to provide a link between dynamical systems and information theory [*Nicolis*, 1991; *Ebeling and Nicolis*, 1992].

[10] From the initial dynamical system we can generate a sequence of symbols, where the dynamics of the original (under analysis) system has been projected. This symbolic sequence can be analyzed by terms of information theory such as entropy estimations, information loss, automaticity and other prominent properties.

[11] There exist some canonical ways for generating symbolic dynamics out of a given dynamical system [*Nicolis et al.*, 1988, 1989; *Nicolis*, 1991, 1995; *Ebeling and Nicolis*, 1992]. To produce symbolic dynamics out of the evolution of a given system, we set up a coarse-grained description incorporating from the very beginning the idea that a physically accessible state corresponds to a finite region rather than to a single point of phase space. Let C_i ($i = 1,2,\ldots K$) be the set of cells in phase space constituted by these regions, assumed to be connected and nonoverlapping. As time goes on, the phase space trajectory performs transitions between cells thereby generating sequences of K symbols, which may be regarded as the letters of an alphabet. We shall require that, in the course of these transitions, each element of the partition is mapped by the law of evolution on a union of elements.

[12] In this paper, we restrict ourselves to the simplest possible coarse graining of the magnetospheric signal. This is given by choosing a threshold C and assigning the symbols "1" and "0" to the signal, depending on whether it is above or below the threshold (binary partition). The

threshold is usually the mean value of the data considered. In this way, each time window of the original D_{st} time series for a given threshold is transformed into symbolic sequences, which contains "linguistic" or "symbolic dynamics" characteristics. The selection of a two-symbol alphabet satisfies terms of simplicity and computational convenience.

3. Concepts of Block Entropy, *T* Complexity, and Approximate Entropy

[13] The term "entropy" is used in both physics and information theory to describe the amount of uncertainty or information inherent in an object or system. Clausius introduced the notion of entropy into thermodynamics in order to explain the irreversibility of certain physical processes in thermodynamics. In statistical thermodynamics the most general formula for the thermodynamic entropy S of a thermodynamic system is the Boltzmann-Gibbs entropy,

$$S_{B-G} = -k \sum p_i \ln p_i$$

where k is the Boltzmann constant and p_i are the probabilities associated with the microscopic configurations.

[14] The Boltzmann-Gibbs entropy translates over almost unchanged into the world of quantum physics to give the von Neumann entropy,

$$S = -k \ \mathrm{Tr}(\rho \ln \rho)$$

where ρ is the density matrix of the quantum mechanical system.

[15] Shannon recognized that a similar approach to Boltzmann-Gibbs entropy could be applied to information theory. In his famous 1948 paper [*Shannon*, 1948], he introduced a probabilistic entropy measure H_S:

$$H_S(X) = -\sum_{i=1}^{n} p(x_i) \log_b p(x_i),$$

where b is the base of the logarithm used and p denotes the probability mass function of a discrete random variable X with possible values $\{x_1, \ldots, x_n\}$.

3.1. Dynamical (Shannon-Like) Block Entropy

[16] Block entropies, depending on the word-frequency distribution, are of special interest, extending Shannon's classical definition of the entropy of a single state to the entropy of a succession of states [*Nicolis and Gaspard*, 1994; *Karamanos and Nicolis*, 1999]. Each entropy takes a large (small) value if there are many (few) kinds of patterns, therefore, it decreases while the organization of patterns is increasing. In this way, the block entropy can measure the complexity of a signal.

[17] In particular, we estimate the block entropies by lumping. Lumping is the reading of the symbolic sequence by "taking portions," as opposed to gliding where one has essentially a "moving frame." In general, the basic novelty of the entropy analysis by lumping is that, unlike the Fourier transform or the conventional entropy by gliding, it gives results that can be related to algorithmic aspects of the sequences.

[18] It is useful to transform the initial raw data of the magnetospheric signal into symbolic sequences taking values in the alphabet $\{0,1\}$, according to the rules $A_i = 1$ if $A(t_i) > E[A(t_i)]$ and $A_i = 0$ if $A(t_i) < E[A(t_i)]$, where $A(t_i)$ are the values of the measured field at time t_i and $E[A(t_i)] = \langle A(t_i)\rangle$ is the mean value in the particular time windows, as it is nicely stated by *Schwarz et al.* [1993].

[19] Consider a subsequence of length N selected out of a very long (theoretically infinite) symbolic sequence. We stipulate that this subsequence is to be read in terms of distinct "blocks" of length n,

$$\ldots \underbrace{A_1 \ldots A_n}_{B_1} \underbrace{A_{n+1} \ldots A_{2n}}_{B_2} \ldots \underbrace{A_{jn+1} \ldots A_{(j+1)n}}_{B_{j+1}} \ldots \quad (1)$$

We call this reading procedure "lumping."

[20] The following quantities characterize the information content of the sequence [*Khinchin*, 1957; *Ebeling and Nicolis*, 1992]

[21] 1. The dynamical (Shannon-like) block entropy for blocks of length n

$$H(n) = -\sum_{(A_1,\ldots,A_n)} p^{(n)}(A_1,\ldots,A_n) \cdot \ \ln p^{(n)}(A_1,\ldots,A_n) \quad (2)$$

where the probability of occurrence of a block $A_1 \ldots A_n$, denoted $p^{(n)}(A_1,\ldots,A_n)$, is defined by the fraction (when it exists) in the statistical limit as

$$\frac{\text{No. of blocks,} \quad A_1 \ldots A_n, \quad \text{encountered when lumping}}{\text{total No. of blocks}} \quad (3)$$

starting from the beginning of the sequence, and the associate entropy per letter

$$h^{(n)} = \frac{H(n)}{n}. \quad (4)$$

[22] 2. The conditional entropy or entropy excess associated with the addition of a symbol to the right of an n block

$$h_{(n)} = H(n+1) - H(n). \quad (5)$$

[23] 3. The entropy of the source (a topological invariant), defined as the limit (if it exists)

$$h = \lim_{n\to\infty} h_{(n)} = \lim_{n\to\infty} h^{(n)} \quad (6)$$

which is the discrete analog of metric or Kolmogorov entropy.

[24] We now turn to the selection problem that is to the possibility of emergence of some preferred configurations (blocks) out of the complete set of different possibilities. The number of all possible symbolic sequences of length n (complexions in the sense of Boltzmann) in a K letter alphabet is [*Karamanos and Nicolis*, 1999]

$$N_K = K^n. \quad (7)$$

Yet not all of these configurations are necessarily realized by the dynamics, nor they are equiprobable. A remarkable theorem due to McMillan [*Khinchin*, 1957; *Nicolis and Gaspard*, 1994], gives a partial answer to the selection problem asserting that for a block $(A_1,\ldots,A_n)$ the following holds

$$p_n(A_1,\ldots,A_n) \sim e^{-H(n)} \tag{8}$$

for almost all blocks $(A_1,\ldots,A_n)$. In order to determine the abundance of long blocks one is thus led to examine the scaling properties of $H(n)$ as a function of n.

[25] As we have already mentioned, the Fourier spectrum or the standard convention of the entropy analysis by gliding, do not help us to distinguish between symbolic sequences with completely different levels of complexity and spectra [*Karamanos*, 2001]. Unlike the previous methods, the novelty of the entropy analysis by lumping gives results, which can be connected with algorithmic aspects of the sequences, in particular with the property of the sequence to be generated by deterministic or stochastic automata [see *Karamanos*, 2001]. Also, the entropy analysis by lumping of some weakly chaotic systems, gives a rather characteristic entropy spectrum, as explained by *Karamanos* [2001]. This shows that the entropy analysis by lumping is much more sensitive in algorithmic and ergodic properties of (weakly) chaotic systems than the classical conventional entropy analysis by gliding, or the correlation functions.

3.2. *T* Complexity

[26] In this section we introduce the grammar-based complexity measure referred here as the T complexity or T entropy. T entropy is a different grammar-based complexity/information measure defined for infinite, as well as finite strings of symbols [*Titchener*, 1998, 2000; *Ebeling et al.*, 2001; *Steuer et al.*, 2001]. It is a weighted count of the number of production steps required to construct the string from its alphabet. Briefly, it is based on the intellectual economy one makes when rewriting a string according to some rules. The basic fact for the T complexity is that it puts the problem of the algorithmic compressibility in a well understandable basis (and also in a firm mathematical basis).

[27] Let us note again that the method of T entropy is based on the rewriting of a word according to some basic rules. This way of rewriting is unique and therefore leads to a unique characterization by the corresponding T complexity measure. Before analyzing in some depth the results coming from the application of the notion of T complexity in real-world problems, we would like to describe how the T complexity is computed, at least for finite strings.

[28] The T complexity of a string is defined by the use of one recursive hierarchical pattern copying (RHPC) algorithm [*Titchener*, 2000]. It computes the effective number of T augmentation steps required to generate the string.

[29] The T complexity may be thus computed effectively from any string and the resultant value is unique.

[30] We shall denote by $\mathbb{N}$ the set of natural numbers, and let $\mathbb{N}^+ = \mathbb{N} \setminus \{0\}$. Let the set $A = \{a_1,\ldots,a_l\}$, $l > 1$, be a finite alphabet. The elements of A are called *symbols* or *characters* and the cardinality of A is denoted by $\#A$, i.e., $\#A = l$. A^* denotes the free monoid generated by A under concatenation. The elements of the set A^* are called *strings*; λ denotes the empty string. We further denote the set $A^* \setminus \{\lambda\}$ by A^+.

[31] The string $x(n)$ is parsed to derive constituent patterns $p_i \in A^+$ and associated copy exponents $k_i \in \mathbb{N}^+$, $i = 1, 2,\ldots,q$, where $q \in \mathbb{N}^+$ satisfying:

$$x = p_q^{k_q} p_{q-1}^{k_{q-1}} \ldots p_i^{k_i} \ldots p_1^{k_1} \alpha_0, \quad \alpha_0 \in A. \tag{9}$$

[32] Each pattern p_i is further constrained to satisfy:

$$\begin{aligned} p_i &= p_{i-1}^{m_{i,i-1}} p_{i-2}^{m_{i,i-2}} \ldots p_j^{m_{i,j}} \ldots p_1^{m_{i,1}} \alpha_i, \\ &\alpha_i \in A \quad and \quad 0 \le m_{i,j} \le k_j. \end{aligned} \tag{10}$$

[33] The T complexity $C_T(x(n))$ is defined in terms of the copy exponents k_i:

$$C_T(x(n)) = \sum_i^q \ln(k_i + 1). \tag{11}$$

[34] One may verify that $C_T(x(n))$ is minimal for a string comprising a single repeating character. From equation (11) we have:

$$\ln n \le C_T(x(n)). \tag{12}$$

The upper bound is more difficult to derive. However, for $n > n_0$ [*Ebeling et al.*, 2001]

$$C_T(x(n)) \le li(\ln 2 \ln(\#A^n)), \tag{13}$$

where $li(z) = \int_z^0 du/\ln u$ is the logarithmic integral function. For a binary alphabet $n_0 \simeq 15$, i.e., small enough to be of no consequence as we are typically concerned with strings in the range of $n = 10^2$–10^4 bits. In practice we parse the string repeatedly from left to right but select the patterns from right to left.

[35] The T information $I_T(x(n))$ of the string $x(n)$ is defined as the inverse logarithmic integral of the T complexity divided by a scaling constant ln2 [*Ebeling et al.*, 2001]:

$$I_T(x(n)) = li^{-1}\left(\frac{C_T(x(n))}{\ln 2}\right). \tag{14}$$

[36] In the limit $n \to \infty$ we have that $I_T(x(n)) \le \ln(\#A^n)$. The form of the right-hand side may be recognizable as the maximum possible n block entropy of Shannon's definition (see section 3.1). The neperian logarithm implicitly gives to the T information the units of nats (nat is a logarithmic unit of information or entropy, based on natural logarithms and powers of e, rather than the powers of 2 and base 2 logarithms which define the bit; the nat is the natural unit for information measures). $I_T(x(n))$ is the total T information for $x(n)$. The average T information rate per symbol, referred to here as the average T entropy of $x(n)$ and denoted by $h_T(x(n))$, is defined along similar lines,

$$h_T(x(n)) = \frac{I_T(x(n))}{n} (nats/symbol). \tag{15}$$

[37] Clearly we note that in the limit of $n \to \infty$, $h_T(x(n)) \le \ln(\#A) = K$. The correspondence between the T information

and *T* entropy on the one hand and Shannon's entropy definitions on the other hand, is reinforced in subsequent investigations [*Titchener*, 1998, 2000; *Ebeling et al.*, 2001]. An example of an actual calculation of the *T* complexity for a finite string is given by *Titchener* [1998, 2000] and *Ebeling et al.* [2001].

3.3. Approximate Entropy

[38] Related to time series analysis, approximate entropy (*ApEn*) provides a measure of the degree of irregularity or randomness within a series of data (of length *N*). *ApEn* was pioneered by Pincus as a measure of system complexity [*Pincus*, 1991]. It is closely related to Kolmogorov entropy, which is a measure of the rate of generation of new information. This family of statistics is rooted in the work of *Grassberger and Procaccia* [1983] and has been widely applied in biological systems [*Pincus and Goldberger*, 1994; *Pincus and Singer*, 1996; and references therein].

[39] The approximate entropy examines time series for similar epochs: more similar and more frequent epochs lead to lower values of *ApEn*. In a more qualitative point of view, given *N* points, the *ApEn*-like statistics is approximately equal to the negative logarithm of the conditional probability that two sequences that are similar for *m* points remain similar, that is, within a tolerance *r*, at the next point. Smaller *ApEn* values indicate a greater chance that a set of data will be followed by similar data (regularity), thus, smaller values indicate greater regularity. Conversely, a greater value for *ApEn* signifies a lesser chance of similar data being repeated (irregularity), hence, greater values convey more disorder, randomness and system complexity. Thus a low/high value of *ApEn* reflects a high/low degree of regularity. The following is a description of the calculation of *ApEn*. Given any sequence of data points $u(i)$ from $i = 1$ to N, it is possible to define vector sequences $x(i)$, which consist of length m and are made up of consecutive $u(i)$, specifically defined by the following:

$$x(i) = (u[i], u[i+1], \ldots, u[i+m-1]). \tag{16}$$

[40] In order to estimate the frequency that vectors $x(i)$ repeat themselves throughout the data set within a tolerance r, the distance $d(x[i], x[j])$ is defined as the maximum difference between the scalar components $x(i)$ and $x(j)$. Explicitly, two vectors $x(i)$ and $x(j)$ are "similar" within the tolerance or filter r, namely $d(x[i], x[j]) \leq r$, if the difference between any two values for $u(i)$ and $u(j)$ within runs of length m are less than r (i.e., $|u(i+k) - u(j+k)| \leq r$ for $0 \leq k \leq m$). Subsequently, $C_i^m(r)$ is defined as the frequency of occurrence of similar runs m within the tolerance r:

$$C_i^m(r) = \frac{[\text{number of} \quad j \quad \text{such that} \quad d(x[i], x[j]) \leq r]}{(N-m-1)},$$

where $j \leq (N - m - 1)$.

[41] Taking the natural logarithm of $C_i^m(r)$, $\Phi^m(r)$ is defined as the average of $ln(C_i^m(r))$:

$$\Phi^m(r) = \sum_i lnC_i^m(r)/(N-m-1) \tag{17}$$

where $\sum_i$ is a sum from $i = 1$ to $(N - m - 1)$. $\Phi^m(r)$ is a measure of the prevalence of repetitive patterns of length m within the filter r.

[42] Finally, approximate entropy, or $ApEn(m, r, N)$, is defined as the natural logarithm of the relative prevalence of repetitive patterns of length m as compared with those of length $m + 1$:

$$ApEn(m, r, N) = \Phi^m(r) - \Phi^{m+1}(r). \tag{18}$$

[43] Thus, $ApEn(m, r, N)$ measures the logarithmic frequency that similar runs (within the filter r) of length m also remain similar when the length of the run is increased by 1. Thus, small values of *ApEn* indicate regularity, given that i increasing run length m by 1 does not decrease the value of $\Phi^m(r)$ significantly (i.e., regularity connotes that $\Phi^m[r] \approx \Phi^{m+1}[r]$). $ApEn(m, r, N)$ is expressed as a difference, but in essence it represents a ratio; note that $\Phi^m[r]$ is a logarithm of the averaged $C_i^m(r)$, and the ratio of logarithms is equivalent to their difference. A more comprehensive description of *ApEn* is given by *Pincus* [1991], *Pincus and Goldberger* [1994], and *Pincus and Singer* [1996].

[44] In summary, *ApEn* is a "regularity statistics" that quantifies the unpredictability of fluctuations in a time series. Intuitively, one may reason that the presence of repetitive patterns of fluctuation in a time series renders it more predictable than a time series in which such patterns are absent. *ApEn* reflects the likelihood that "similar" patterns of observations will not be followed by additional "similar" observations. A time series containing many repetitive patterns has a relatively small *ApEn*; a less predictable (i.e., more complex) process has a higher *ApEn*.

4. Principles of Nonextensive Tsallis Entropy

[45] The uncertainty of an open system state can be quantified by the Boltzmann-Gibbs entropy, which is the widest known uncertainty measure in statistical mechanics. Boltzmann-Gibbs entropy cannot, however, describe non-equilibrium physical systems with large variability and multifractal structure such as the solar wind [*Burlaga et al.*, 2007]. Inspired by multifractal concepts, *Tsallis* [1988, 1998] has proposed a generalization of the Boltzmann-Gibbs statistics, which is briefly described here.

[46] The aim of statistical mechanics is to establish a direct link between the mechanical laws and classical thermodynamics. One of the crucial properties of the Boltzmann-Gibbs entropy in the context of classical thermodynamics is extensivity, namely proportionality with the number of elements of the system. The Boltzmann-Gibbs entropy satisfies this prescription if the subsystems are statistically (quasi-) independent, or typically if the correlations within the system are essentially local. In such cases the system is called extensive.

[47] In general, however, the situation is not of this type and correlations may be far from negligible at all scales. In such cases the Boltzmann-Gibbs entropy is nonextensive. *Tsallis* [1988, 1998] introduced an entropic expression

characterized by an index q which leads to a nonextensive statistics,

$$S_q = k\frac{1}{q-1}\left(1 - \sum_{i=1}^{W} p_i^q\right), \qquad (19)$$

where p_i are the probabilities associated with the microscopic configurations, W is their total number, q is a real number, and k is Boltzmann's constant. The value of q is a measure of the nonextensivity of the system: $q \rightarrow 1$ corresponds to the standard extensive Boltzmann-Gibbs statistics.

[48] This is the basis of the so called nonextensive statistical mechanics, which generalizes the Boltzmann-Gibbs theory. The entropic index q characterizes the degree of nonadditivity reflected in the following pseudoadditivity rule:

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B). \qquad (20)$$

[49] The cases $q > 1$ and $q < 1$, correspond to subadditivity, or superadditivity, respectively. For subsystems that have special probability correlations, extensivity is not valid for Boltzmann-Gibbs entropy, but may occur for S_q with a particular value of the index q. Such systems are sometimes referred to as nonextensive [*Boon and Tsallis*, 2005]. The parameter q itself is not a measure of the complexity of the system but measures the degree of nonextensivity of the system. It is the time variations of the Tsallis entropy for a given q (S_q) that quantify the dynamic changes of the complexity of the system. Lower S_q values characterize the portions of the signal with lower complexity.

[50] Herein, we estimate S_q on the basis of the concept of symbolic dynamics and by using the technique of lumping (for details the reader is referred to *Balasis et al.* [2008]). To be more precise, the simplest possible coarse graining of the D_{st} index is given by choosing a threshold C (usually the mean value of the data considered) and assigning the symbols "1" and "0" to the signal, depending on whether it is above or below the threshold (binary partition). Thus, we generate a symbolic time series from a two-letter ($\lambda = 2$) alphabet (0,1), e.g., 0110100110010110... (see also sections 2 and 3.1).

[51] Reading the sequence by lumping of length $L = 2$ one obtains 01/10/10/01/10/01/01/10/.... The number of all possible kinds of blocks is $\lambda^L = 2^2 = 4$, namely 00, 01, 10, 11. Thus, the required probabilities for the estimation of the Tsallis entropy p_{00}, p_{01}, p_{10}, p_{11} are the fractions of the blocks 00, 01, 10, 11 in the symbolic time series.

[52] The S_q for the word length L is

$$S_q(L) = k\frac{1}{q-1}\left(1 - \sum_{(A_1,A_2,\ldots,A_L)} \left[p(L)_{A_1,A_2,\ldots,A_L}\right]^q\right). \qquad (21)$$

[53] Broad symbol-sequence frequency distributions produce high entropy values, indicating a low degree of organization. Conversely, when certain sequences exhibit high frequencies, lower entropy values are produced, indicating a high degree of organization.

5. Results

[54] A way to examine transient phenomena is to divide the measurements into time windows and analyze these windows. If this analysis yields different results for time intervals associated to an intense magnetic storm, for instance, in comparison to time windows associated to the normal state of the magnetosphere, then a transient behavior can be extracted.

[55] In Figure 1 the D_{st} time series is presented. The one year D_{st} data (2001) are divided into five shorter time series (see triangles denoting five distinct time windows in Figure 1). The second and fourth time windows include the D_{st} variations associated to the two intense magnetic storms of 31/3/2001 and 6/11/2001, respectively. Within each of the five time windows, the Shannon, block, *T*, *ApEn* and Tsallis entropy are calculated. Block and Tsallis entropies are computed using the technique of lumping for binary partition (with the mean value as threshold) and block (word) length $n = 2$. The value of the Tsallis q index utilized here for the calculation of nonextensive Tsallis entropy $S_q(q)$ is selected to be 1.8, as indicated by recent analysis [*Balasis et al.*, 2008]. *T* complexity and *ApEn* are calculated using $n = 2$ and $m = 1$, respectively. In the framework of symbolic dynamics theory, various numeric tests have been performed with different candidate lengths of blocks (words) for block and Tsallis entropy as well as with different n and m values for *T* complexity and *ApEn* in order to obtain the optimum parametrization choice for the analysis of the D_{st} time series. In Table 1 the values of all the information measures considered in this paper are given at the five distinct time windows, as well as the time intervals that these windows span.

[56] In Figure 2 we depict the block entropy by lumping per letter as a function of the word length ($H(n)/n$ vs n) for the time windows presented in Figure 1. We first focus on the time windows W1, W3 and W5 that represent the quiet time magnetosphere. The associated group of curves of the block entropy per letter lies in the region of high block entropy values (Figure 2). The high block entropy values indicate an underlying strong complexity. We note that a complete absence of structure in the magnetospheric signal, would lead to an horizontal line in the block entropy diagram. This is not the present case. We then focus on the time windows W2 and W4 that include the intense magnetic storms. The estimated entropies drop to significantly lower values for these time windows. This behavior witnesses a significant reduction of complexity of the underlying magnetospheric mechanism: the reduction is more impressive for window W4 that includes the November 2001 magnetic storm.

[57] Figure 3 shows the various entropy measures for the five different windows. We study the temporal evolution of the entropies as the global instability is approaching. The blue time windows are referred to the normal state of magnetosphere, whereas the red windows include the intense magnetic storms of March and November 2001, respectively. The entropies in the red windows (with the exception of Shannon entropy for the fourth time window)

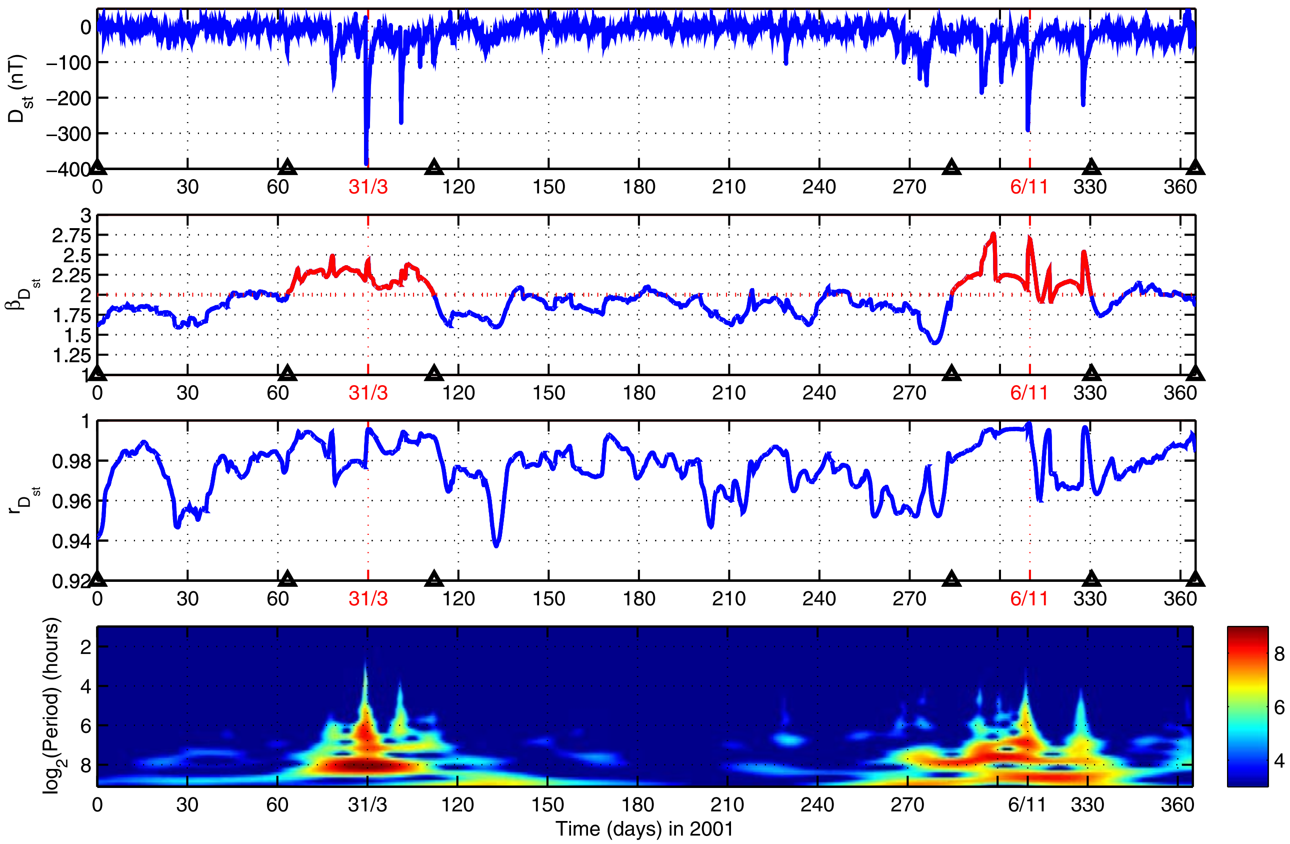

Figure 1. From top to bottom are shown D_{st} time series, spectral exponents $\beta_{D_{st}}$, linear correlation coefficients $r_{D_{st}}$, and the wavelet power spectrum for 2001. The 31 March and 6 November 2001 magnetic storms are marked with red. The red dashed line in the $\beta_{D_{st}}$ plot marks the transition between antipersistent and persistent behavior. The triangles denote the time intervals corresponding to the five time windows discussed in the text.

drop to rather significantly lower values suggesting the appearance of a new distinct state in the magnetosphere, which is characterized by a lower complexity in comparison to that of the blue (normal) epoch of the magnetosphere. We remind that Shannon entropy requires longer time series than the other entropy measures used here, in order to work properly. Furthermore, some entropy measures (e.g., block, Tsallis and in particular *ApEn*) give better (larger) value differences from windows (W1, W3 and W5) to (W2 and W4), thus providing a clearer picture of the transition from prestorm activity to magnetic storms.

[58] Figure 3 also depicts the Kolmogorov entropy for the five different windows. The formula for Kolmogorov entropy is given from equation (6) by virtue of equation (4): $h = \lim_{n\to\infty}\frac{H(n)}{n}$. Kolmogorov entropies are practically computed by taking the slope of the block entropies $H(n)$ in the diagrams $H(n)$ vs n for each of the five time windows and for n = 1, 2, 3 and 4 (see Figure 2). We note that Kolmogorov entropy follows the behavior of the rest entropy measures (i.e., having lower values in the second and fourth time windows).

6. Fractal Spectral Analysis

[59] During magnetic storms the complex system of the Earth's magnetosphere manifests itself in linkages between space and time, producing characteristic fractal structures [*Consolini and De Michelis*, 2002]. In the work of *Balasis et al.* [2006], the fractal spectral properties of the D_{st} data are examined using wavelet analysis methods. The results

Table 1. Values of the Various Information Measures[a]

Window	Time (Days in 2001)	Shannon Entropy	Block Entropy	T Complexity	Approximate Entropy	Tsallis Entropy	Kolmogorov Entropy	Hurst Exponent
1	0–63.25	0.786817	0.96326	0.21595	1.417287	0.73865	0.2723585	0.4183
2	63.25–112	**0.607611**	**0.822929**	**0.141**	**0.895673**	**0.630496**	**0.1718937**	**0.6148**
3	112–284	0.653712	0.876862	0.1421	1.116397	0.679	0.2137876	0.4219
4	284–330.5	**0.690738**	**0.67392**	**0.07628**	**0.936279**	**0.524507**	**0.0942634**	**0.6122**
5	330.5–365	0.793177	0.978621	0.19772	1.312829	0.748244	0.2668134	0.4857

[a]Values are calculated at the five different time windows indicated in Figure 1. Bold values correspond to the time intervals that include the intense magnetic storms of March and November 2001, respectively. Bold entropy values are in general lower than the entropies of the other windows. Similarly, bold Hurst exponent values are higher than the exponents of the other windows.

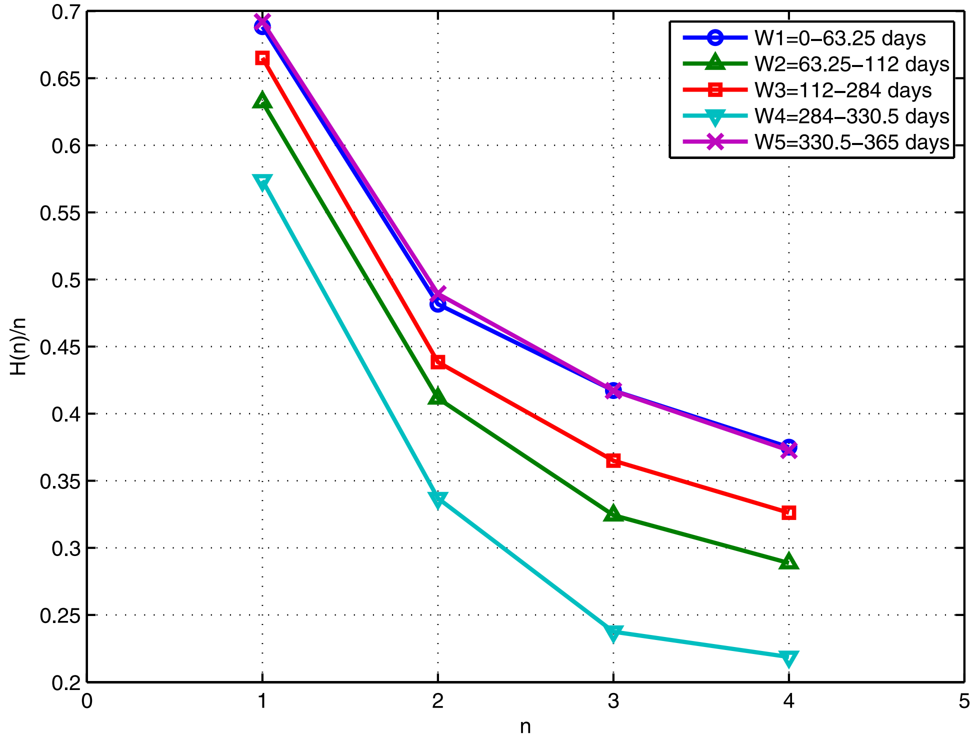

Figure 2. Block entropy per letter (equation (4)), as a function of the word length n (n = 1, 2, 3, and 4) for the five time windows shown in Figure 1. We observe a significant reduction of the block entropy per letter in windows W2 and W4. These windows correspond to the time intervals that include the intense magnetic storms of March and November 2001, respectively.

show that distinct changes in associated scaling parameters emerge as large magnetospheric disturbances approach.

[60] If a time series is a temporal fractal then a power law of the form $S(f) \propto f^{-\beta}$ is obeyed, with $S(f)$ the power spectral density and f the frequency. The spectral scaling exponent β is a measure of the strength of time correlations. The goodness of the fit of a time series to the power law is represented by the linear correlation coefficient, r, of this representation. The wavelet transform with the Morlet wavelet as the basis function [*Balasis et al.*, 2005; *Mandea and Balasis*, 2006; *Balasis and Mandea*, 2007] (see report at http://www.sciencemag.org/content/vol314/issue5798/twil.dtl) was applied to 1-year-long D_{st} time series from 2001.

[61] The nonstationary character of the D_{st} index requires methods that can appropriately treat such nonstationarities. In practice, the condition of stationarity for nonstationary signals can be satisfied by dividing the signal into blocks of short, pseudostationary segments [*Akay*, 1997]. On the other hand, recent studies show that the wavelet transform can remove effects due to nonstationarities present in the time series [*Amaral et al.*, 1998].

[62] In Figure 1 the D_{st} time series and its wavelet power spectrum are plotted. Power spectral densities (PSDs) were estimated in the frequency range from 2 to 128 h using a 256-h moving window and an overlap of 255 samples. For each PSD parameters r and β were derived. In Figure 1 the D_{st} spectral parameters $r_{D_{st}}$ and $\beta_{D_{st}}$ are shown: $r_{D_{st}}$ is always above 0.9 and $\beta_{D_{st}}$ takes values between 1 and 3. (Regarding the error estimates for the fractal spectral analysis, these are either low or negligible for the spectral parameters of the D_{st} data from 2001, as shown in Figure 3 of *Balasis et al.* [2006].)

[63] The temporal evolution of $r_{D_{st}}$ indicates that the fit to the power law is excellent. This means that the fractal character of the underlying processes and structures is compact: the activity could be ascribed to a multi-time-scale cooperative activity of numerous activated units, in which an individual unit behavior is dominated by its neighbors, so that, all units simultaneously alter their behavior to a common large-scale fractal pattern. In the case of the two intense magnetic storms, we observe a further increase of $r_{D_{st}}$ as the main phase approaches: a region with $r_{D_{st}} > 0.99$ is observed during the last stage of precursory activity. The gradual increase of $r_{D_{st}}$ indicates that the clustering of activated events in more compact fractal structures is strengthened with time. Such elementary activated events could, in the case of magnetic storms, be successive stages of acceleration and earthward transport of ions, for example, due to substorm-induced impulsive electric fields [*Daglis et al.*, 2004]. Substorms as well as regular convection result in multiple ring currents with a distribution of growth/decay times [*Liemohn and Kozyra*, 2003].

[64] The temporal evolution of $\beta_{D_{st}}$ means that the time profile of the D_{st} time series is qualitatively analogous to fractional Brownian motion (fBm) [*Heneghan and McDarby*, 2000], possessing long-range temporal correlations. More precisely, the observed fractal law ($S(f) \propto f^{-\beta}$) indicates the existence of long-term memory. This means that the current value of the geomagnetic signal is correlated not only with its most recent values but also with its long-term history in a scale-invariant, fractal manner. The distribution of the $\beta_{D_{st}}$ exponent is also shifted to higher values as the intense magnetic storms approach. This shift reveals several features

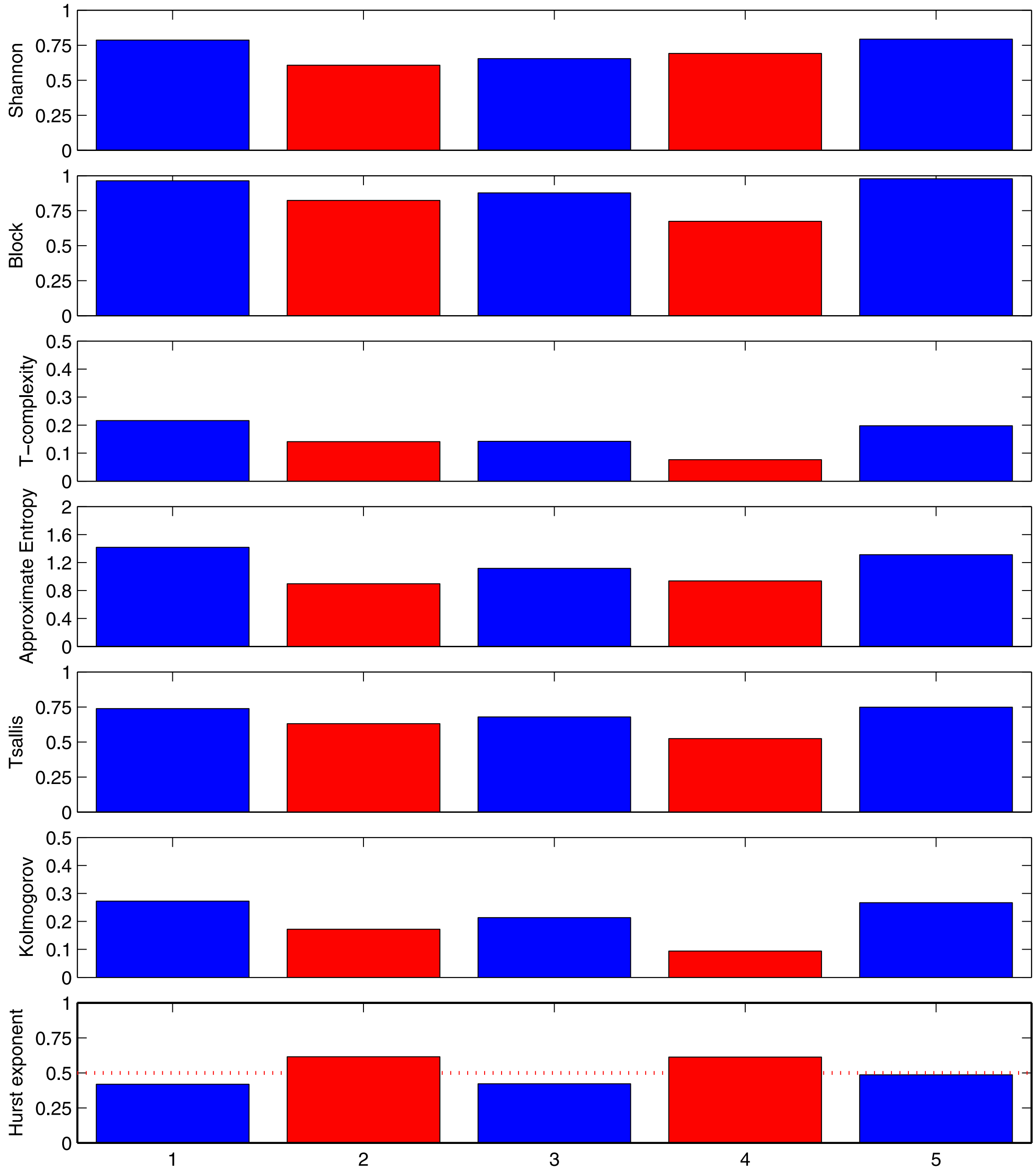

Figure 3. A comparison of practical information measures for the D_{st} time series. From top to bottom are shown Shannon entropy, block entropy, T complexity, approximate entropy, Tsallis entropy, Kolmogorov entropy, and Hurst exponent. The values of all the measures were calculated at the five time windows that derived after the initial D_{st} time series was divided into five shorter time intervals as shown in Figure 1. The red dashed line in the Hurst plot marks the transition between antipersistent and persistent behavior.

of the underlying mechanism. As $\beta_{D_{st}}$ increases, the spatial correlation in the time series also increases. This behavior indicates a gradual increase of the memory, and thus a gradual reduction of complexity in the underlying dynamics. This suggests that the onset of an intense magnetic storm may represent a gradual transition from a less orderly state to a more orderly state [see also *Sitnov et al.*, 2001].

[65] The β exponent is related to the Hurst exponent, H, by the formula [*Turcotte*, 1997]

$$\beta = 2H + 1 \tag{22}$$

with $0 < H < 1$ ($1 < \beta < 3$) for the fBm random field model [*Heneghan and McDarby*, 2000]. The exponent H characterizes the persistent/antipersistent properties of the signal [*Balasis et al.*, 2006].

[66] The range $0 < H < 0.5$ ($1 < \beta < 2$) indicates antipersistency, which means that if the fluctuations increase in a period, it is likely to decrease in the interval immediately following and vice versa. Physically, this implies that fluctuations tend to induce stability within the system (negative feedback mechanism). If $0.5 < H < 1$ ($2 < \beta < 3$) then the signal exhibits persistent properties, which means

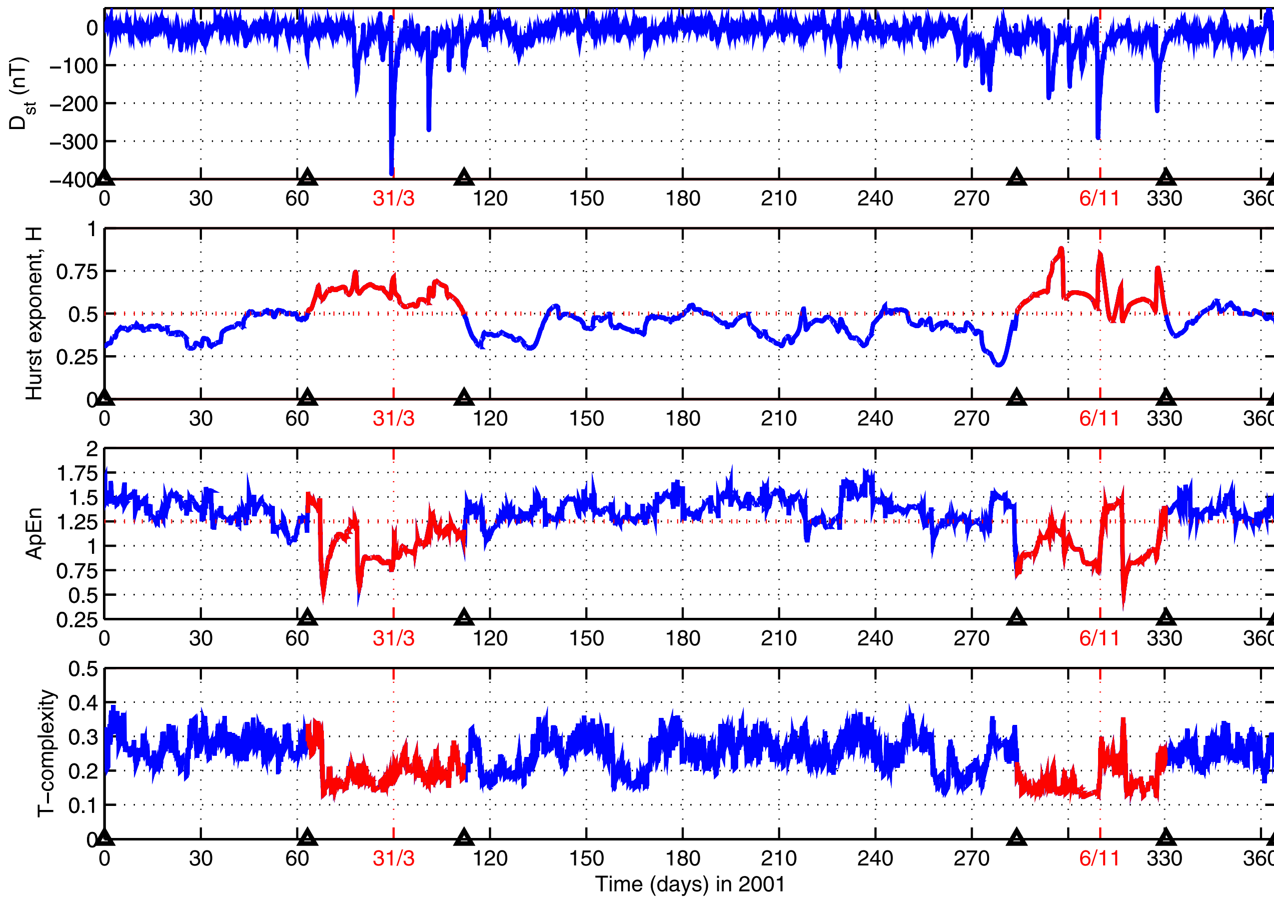

Figure 4. From top to bottom are shown D_{st} time series along with time variations of Hurst exponents H and approximate entropies *ApEn* and T complexities. The 31 March and 6 November 2001 magnetic storms are marked with red. The red dashed line in the H plot marks the transition between antipersistent and persistent behavior. The red dashed line in *ApEn* plot marks the boundary value suggested in this paper for the transition to the lower complexity characterizing the different state of the magnetosphere. The triangles denote the time intervals corresponding to the five time windows discussed in the text.

that if the amplitude of fluctuations increases in a time interval it is likely to continue increasing in the immediately next interval. In other words, the underlying dynamics is governed by a positive feedback mechanism. The value $H = 0.5$ ($\beta = 2$) suggests no correlation between the repeated increments. Consequently, this particular value takes on a special physical meaning: it marks the transition between antipersistent and persistent behavior in the time series.

[67] The $\beta_{D_{st}}$ values during the quiet period (i.e., well before and after March 31 and November 6 2001 magnetic storms) are between 1 and 2 indicating antipersistency [*Balasis et al.*, 2006], while the observed systematic increase of the spectral exponent during this stage (after day 30 and 270, respectively) indicates that the fluctuations become more correlated with time. We draw attention to the fact that $\beta_{D_{st}}$ exceeds 2 and therefore D_{st} exhibits persistent properties [*Balasis et al.*, 2006] around March 31 and November 6 2001 magnetic storms (see the parts of the $\beta_{D_{st}}$ plot marked with red in Figure 1), coupled with a significant acceleration of the energy release (see its wavelet plot in Figure 1).

[68] In Figure 3 we also show the average values of the Hurst exponents H calculated at the same five time windows as the entropy measures. We stress that the antipersistent epochs ($0 < H < 0.5$) correspond to the epochs of higher entropy values (first, third and fifth time windows given in blue in Figure 3), while, the persistent epochs ($0.5 < H < 1$) corresponds to the epochs of lower entropies (second and fourth time windows given in red in Figure 3). This finding further supports the existence of two different epochs referring to two distinct states of the magnetic storm evolution. Antipersistent behavior and higher entropy measures correspond to a regular undisturbed magnetosphere while persistent behavior and lower entropy measures correspond to a disturbed storm time magnetosphere.

7. Conclusions and Discussion

[69] Magnetic storms are undoubtedly among the most important phenomena in space physics and also a central subject of space weather. They have severe impacts on both spaceborne and ground-based technological systems, as well as, possibly, on weather and climate [*Daglis et al.*, 2001].

[70] The results of the present work establish an interesting link between dynamics and information. They show that D_{st} fluctuations are the natural carriers of information of the impending magnetic storm. More precisely, we have seen that a combination of nonlinear with linear statistical

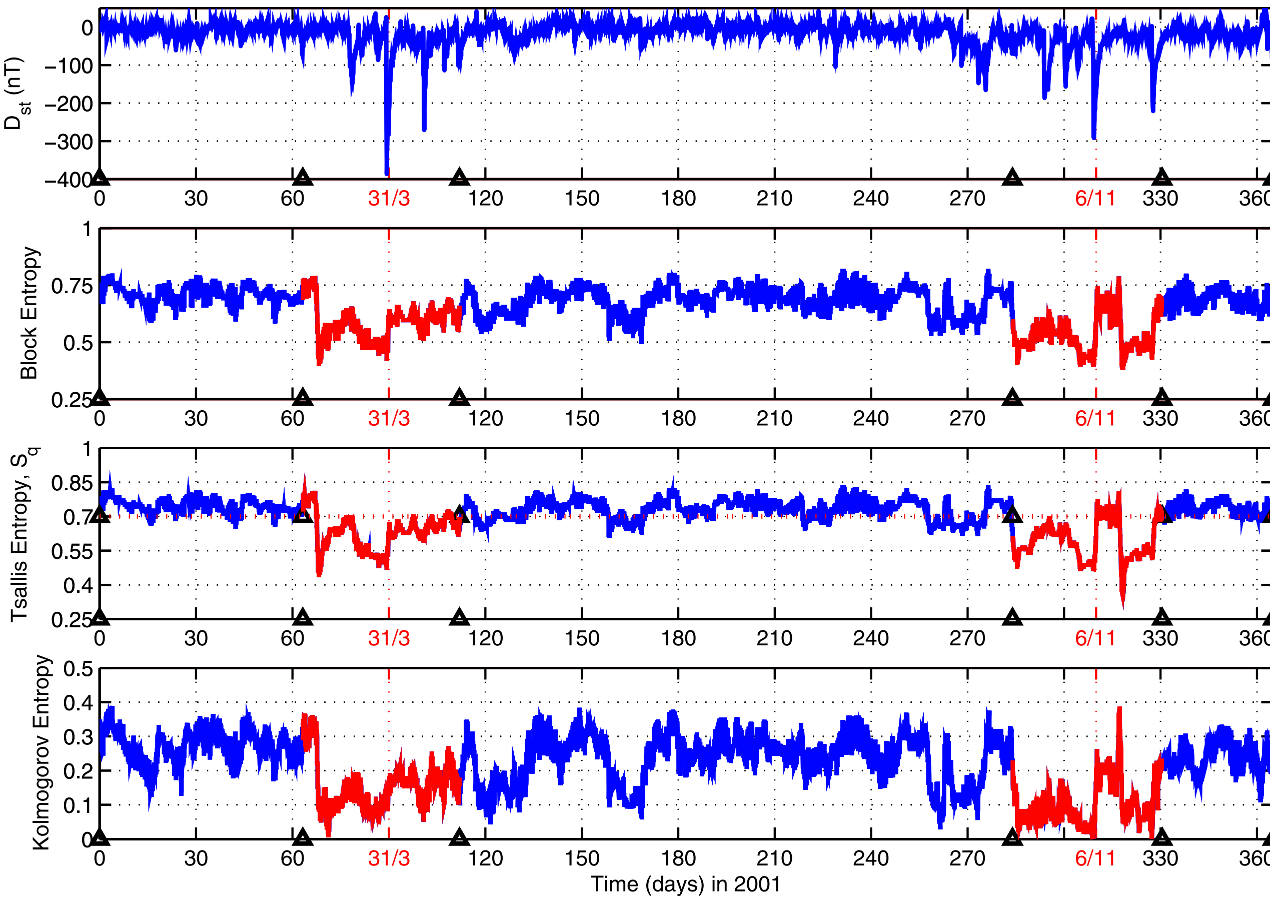

Figure 5. From top to bottom are shown D_{st} time series along with time variations of block entropies, Tsallis entropies S_q, and Kolmogorov entropies. The 31 March and 6 November 2001 magnetic storms are marked with red. The red dashed line in the S_q plot marks the boundary value suggested in this paper for the transition to the lower complexity characterizing the different state of the magnetosphere. The triangles denote the time intervals corresponding to the five time windows discussed in the text.

approaches allows one to extract rich information hidden in the D_{st} time series.

[71] In this paper, we analyze D_{st} time series by introducing a fairly large variety of information measures in the search of appropriate and effective entropic quantities to study the complex character of magnetospheric dynamics. This is a challenging task and requires a great amount of computational efforts and numerical trials in order to achieve it. We would like to point out that it is the first time, at least to our knowledge, that a significant number of modern entropy measures are applied to the problem of dynamical complexity in the Earth's magnetosphere.

[72] Block entropy, T complexity, approximate entropy, nonextensive Tsallis entropy and Kolmogorov entropy sensitively show the complexity dissimilarity among different "physiological" (quiet time) and "pathological" states (intense magnetic storms). They imply the emergence of two distinct patterns: (1) a pattern associated with the intense magnetic storms, which is characterized by a higher degree of organization, and (2) a pattern associated with normal periods, which is characterized by a lower degree of organization.

[73] The present study confirms the conclusions of a previous work based on an independent linear fractal spectral analysis (Hurst exponent) using wavelet transforms. The Hurst exponent analysis also shows the existence of two different patterns: (1) a pattern associated with the intense magnetic storms, which is characterized by a fractional Brownian persistent behavior, and (2) a pattern associated with normal periods, which is characterized by a fractional Brownian antipersistent behavior.

[74] We stress that the antipersistent time windows correspond to the time windows of higher entropies, while the persistent time windows correspond to the time windows of lower entropies. Importantly, a recent analysis presented by *Carbone and Stanley* [2007] shows that anticorrelated time series, with Hurst exponent $0.5 < H < 1$, are characterized by entropies greater than correlated time series having $0.5 < H < 1$. This suggestion is in agreement with our results.

[75] An important remark is the agreement of the results between the linear analysis in terms of the Hurst exponent and nonlinear entropy analyses. A combination of linear and nonlinear analysis techniques can offer a firm warning that the onset of an intense magnetic storm is imminent.

[76] Figure 4 gives the temporal evolution of D_{st} along with corresponding time variations of the Hurst exponent, the *ApEn* and, the T complexity for the whole year of 2001. Figure 5 presents the same temporal evolution of the magnetospheric signal but with corresponding time variations of the block, Tsallis and Kolmogorov entropy. All the

relative entropy measures were calculated using a moving time window of 256 h. We see how nicely the entropy measures identify the different complexity regimes in the D_{st} time series (see the red part of the corresponding plots). Figures 4 and 5 further demonstrate that the *ApEn* and Tsallis entropy along with Hurst exponent yield superior results in comparison to the other entropy measures regarding the detection of dynamical complexity in the Earth's magnetosphere (i.e., offer a clearer picture of the transition). A possible explanation for this is that Tsallis is an entropy obeying a nonextensive statistical theory, which is different from the usual Boltzmann-Gibbs statistical mechanics. Therefore, it is expected to better describe the dynamics of the magnetosphere, which is a nonequilibrium physical system with large variability. On the other hand, *ApEn* is more stable when dealing with nonstationary signals of dynamical systems (such the magnetospheric signal) than the other entropy measures presented here.

[77] Figures 4 and 5 could also serve for placing boundary values or thresholds for *ApEn* and Tsallis entropy in order to distinguish the different magnetospheric states. Along these lines we are driven to potentially suggest the limits of 1.25 and 0.7 for the *ApEn* and Tsallis entropy, respectively.

[78] *Johnson and Wing* [2005] explored the nonlinear behavior of the magnetosphere as characterized by the planetary 3-h-range index, K_p, which is designed to measure solar particle radiation by its magnetic effects (http://www-app3.gfz-potsdam.de/kp_index/index.html). They have demonstrated that strong nonlinear magnetospheric dependencies are statistically significant up to 1 week, in accordance with the frequency range (2–128 h) used in the fractal spectral analysis to estimate significant long-range temporal correlations, but also to the frequency range indicated by the 256 h time interval utilized to derive all the entropy measures.

[79] Recently, *Consolini et al.* [2008] attempted a verification of the magnetospheric nonequilibrium dynamics by investigating the long-term evolution of the Earth's magnetosphere, as monitored by D_{st}. They were able to provide a proof of the existence of a steady state far from equilibrium for the Earth's magnetosphere.

[80] Other studies also indicate the existence of two different regimes in the dynamics of magnetosphere. *Sitnov et al.* [2001] suggest that the substorm dynamics resembles second-order phase transitions, while magnetic storms, are shown to reveal the features of first-order nonequilibrium transitions. The antipersistency/persistency well meet the second-order/first-order phase transition correspondingly. Metastability and topological complexity of magnetic field, emerging from *Chang*'s [1999] model also justify the evidence for transition from prestorm activity to magnetic storms found in our study. Furthermore, *Chang et al.* [2003, 2004] and *Vörös et al.* [2005] described intermittent turbulence in space plasmas which is consistent with the ideas derived here. Recently *Vörös et al.* [2008] examined the statistical properties of magnetic fluctuations in the Venusian magnetosheath and wake regions. They found multiscale turbulence at the magnetosheath boundary layer and near the quasi-parallel bow shock.

[81] Additionally, similar behavior to our observations (i.e., reduction of multiscale complexity) was observed in high-latitude geomagnetic activity prior to strong substorms using a different methodology. *Uritsky and Pudovkin* [1998] and *Uritsky et al.* [2001] presented cellular automata models which allowed interpretation of the observed effects in terms of transitions between critical, supercritical and subcritical states. *Uritsky et al.* [2006] provided evidence for similar behavior in the spatial scaling of the auroral brightness. *Wanliss et al.* [2005] applied symbolic dynamics analysis to D_{st} time series for modeling magnetic storms. They presented evidence for intermittency and non-Gaussianity, which are reflective of large magnetic storms. It was also suggested that the ring current is always out of equilibrium and may undergo state changes via multiplicative cascades.

[82] Finally, it is known that the semiannual variation in the D_{st} index is excessively large compared to all other indices of geomagnetic activity [*Mursula and Karinen*, 2005]. This has been interpreted in terms of a separate nonstorm component which is not related to storms or the ring current. Therefore it would be useful at some point in the future to perform similar information dynamics analysis with a corrected D_{st} index for seasonal effects. Ultimately, the methodology applied in this paper for the analysis of the D_{st} index with respect to intense magnetic storms can serve as a starting point for future space weather applications regarding forecasting of major geospace events (e.g., magnetic storms).

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