Diversity Reception over Generalized-K (K_G) Fading Channels

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***Abstract*— A detailed performance analysis for the most important diversity receivers operating over a composite fading channel modeled by the Generalized-K (K_G) distribution is presented. The K_G distribution has been recently considered as a generic and versatile distribution for the accurate modeling of a great variety of short term fading in conjunction with long term fading (shadowing) channel conditions. For this relatively new composite fading model, expressions for important statistical metrics of maximal ratio combining (MRC), equal gain combining (EGC), selection combining (SC) and switch and stay combining (SSC) diversity receivers are derived. Using these expressions and by considering independent but not necessarily identical distributed fading channel conditions, performance criteria, such as average output signal-to-noise ratio, amount of fading and outage probability are obtained in closed form. Moreover, following the moments generating function (MGF) based approach for MRC and SSC receivers, and the Padé approximants method for SC and EGC receivers, the average bit error probability is studied. The proposed mathematical analysis is complemented by various performance evaluation results which demonstrate the accuracy of the theoretical approach.**

***Index Terms*— Generalized-K distribution, multipath/shadow fading, bit error probability (BEP), outage probability, maximal ratio combining (MRC), equal gain combining (EGC), selection combining (SC), switch and stay combining (SSC).**

I. Introduction

In terrestrial and satellite based land-mobile telecommunication systems, radio signals propagate according to the mechanisms of reflection, diffraction and scattering, which in general characterize the radio propagation by three nearly independent phenomena: Path loss variance with distance, shadowing (i.e., long term fading) and multipath fading (i.e., short term fading) [1]. Multipath fading is introduced due to the constructive and destructive combination of randomly delayed, reflected, scattered and diffracted signal components, while shadowing affects the link quality by slow variation of the mean level [2]. In many cases multipath fading and shadowing occur simultaneously. This composite propagation environment consists of multipath fading superimposed by lognormal shadowing, resulting in lognormal based fading models, such as Rayleigh-, Ricean-, or Nakagami-lognormal fading channels [2]. However, such lognormal based fading models are analytically very difficult to handle and therefore rather complicated mathematically expressions have been derived for the performance of digital communication systems operating in such environments, e.g., see [3], [4]. An alternative approach is to employ the Gamma distribution, which is mathematically a more versatile model and also accurately describes fading shadowing phenomena [5]. Based upon the Gamma distribution, several generalized distributions have been proposed serving useful composite fading channel models, e.g., the K [6], [7], the Generalized-K (K_G) [8], [9] and the generalized-Gamma (GG) [10], [11]. Among them, the K_G distribution, which includes the K distribution as a special case, accurately approximates a great variety fading and/or shadowing models [6], [8].

In the past, relatively few contributions concerning digital communications over K_G fading channels have been published, e.g., [8], [9], [12]–[14]. In [8] the K_G distribution was introduced as a generalized fading/shadowing channel model and the performance of coherent binary phase shift keying (BPSK) was evaluated in such a fading environment. In [9] the outage probability (P_{out}) performance, with and without cochannel interference, was evaluated for K_G channels. In the same reference, the derived P_{out} was compared with the P_{out} obtained by using Nakagami-lognormal model, thus verifying the excellent agreement between these two fading models. In [12] an analytical approach for the performance evaluation in a K_G fading channel was presented and the average bit error probability (ABEP) for differential binary phase shift keying (DBPSK) and the P_{out} have been obtained. In [13], [14] a unified performance analysis of various families of modulation schemes and receiver structures operating over K_G fading channel has been presented.

On the subject of diversity, it is well known that it is one of the simplest and yet most efficient techniques to overcome the destructive effects of fading and shadowing in digital communication systems [1], [2]. The most important diversity reception methods employed in digital communication receivers are maximal ratio combining (MRC), equal gain combining (EGC), selection combining (SC) and switch and stay combining (SSC) [2]. Their performance depends on the characteristics of the multipath fading envelopes. Past works concerning diversity over composite fading distributions can be found in [4], [10], [15]–[17]. In [4], the land-mobile satellite channel is modeled as a weighted sum of Ricean and Suzuki distributions and performance evaluation results for M-ary phase shift keying (PSK) and M-ary differential phase shift keying (DPSK) with micro-diversity reception have been presented. In [10], expressions for the P_{out} of a dual-

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branch SC receiver over correlated GG fading channel have been derived. In [16], upper bounds for the bit error outage in Rayleigh fading and Nakagami shadowing were obtained. In a very recent paper [17][1], the performance of SC and MRC was evaluated for the K_G fading channel. However, the results presented in [17] are limited as only the amount of fading (AoF) and the ABEP of BPSK, in terms of integral representation, have been derived. Hence, to the best of our knowledge, a thorough performance analysis for the various diversity receivers over K_G fading channels has not yet been published in the open technical literature. Motivated by this observation, the purpose of our current contribution is to present an analytical study for the performance of MRC, EGC, SC and SSC diversity receivers operating over K_G fading channels.

Its organization is as follows. After this introduction, in Section II, the K_G fading model is introduced. In Section III, the most important performance analysis criteria for the different diversity receivers under consideration are studied. In Section IV several numerical performance evaluation results are presented and discussed. Finally, concluding remarks are given in Section V.

II. The K_G Fading Model

Let us consider a diversity receiver operating over a K_G fading channel. The equivalent complex baseband received signal at the ℓth ($\ell = 1, 2, \ldots, L$) branch is $z_\ell = s\, h_\ell + n_\ell$, where s is the transmitted complex symbol with energy $E_s = \mathbb{E}\langle |s|^2 \rangle$, where $\mathbb{E}\langle\cdot\rangle$ denotes expectation and $|\cdot|$ absolute value, h_ℓ is the channel complex gain and n_ℓ is the complex additive white Gaussian noise (AWGN), having single sided power spectral density N_0 assumed to be identical in all branches. Let the fading envelope, $R_\ell = |h_\ell|$, be a K_G random variable (RV) with the following probability density function (PDF) [13]

$$f_{R_\ell}(x) = \frac{4\, m_\ell^{(m_\ell+k)/2}}{\Gamma(m_\ell)\,\Gamma(k)\,\Omega_\ell^{(m_\ell+k)/2}} \times x^{m_\ell+k-1} K_{k-m_\ell}\left[2\left(\frac{m_\ell}{\Omega_\ell}\right)^{1/2} x\right],\ x \geq 0. \tag{1}$$

In the above equation k, m_ℓ are the two shaping parameters, $K_{k-m_\ell}(\cdot)$ is the $(k-m_\ell)$th order modified Bessel function of the second kind [18, eq. (8.407/1)], $\Gamma(\cdot)$ is the gamma function [18, eq. (8.310/1)] and Ω_ℓ is the mean power obtained as $\Omega_\ell = \mathbb{E}\langle R_\ell^2 \rangle / k$. By using different values of m_ℓ and k, (1) can describe a great variety of short-term and long-term fading (shadowing) conditions, respectively. For example, as $k \to \infty$, $f_{R_\ell}(x)$ approximates the well known Nakagami-m fading channel model, while for $m_\ell = 1$ it becomes the K-distribution and approaches Rayleigh-Lognormal (R-L) fading/shadowing channel model [6], [8]. It is also noted that as $m_\ell \to \infty$ and $k \to \infty$, (1) approaches the AWGN (i.e., no fading) channel.

[1]We became aware of this publication after the initial submission of our paper in May 2006.

For the diversity techniques considered it is assumed that the distance among the diversity branches is small, i.e., micro-diversity is studied. Furthermore, it is well known that shadowing occurs in large geographical areas. Thus, it is reasonable to assume that the shadowing effects are not decorrelated so that the shadowing parameter k can be assumed equal for all diversity branches [4].

III. Performance Analysis of the Diversity Receivers

In this section important performance criteria for the diversity receivers under consideration operating over independent but not identical distributed (i.n.d.) fading conditions will be studied. Since the instantaneous signal-to-noise ratio (SNR) per symbol of the ℓth receiving branch is $\gamma_\ell = R_\ell^2\, E_s/N_0$, the corresponding average input SNR will be $\overline{\gamma}_\ell = \Omega_\ell\, k\, E_s/N_0$.

A. Average Bit Error Probability (ABEP)

In order to obtain the ABEP for a large variety of modulation schemes, the moments generating function (MGF) based approach will be used [2], [19]. Therefore, the MGF of the signals at the output of the diversity receivers will be obtained.

1) Maximal Ratio Combining (MRC): Defining γ_{mrc} as the total SNR per symbol at the output of a MRC receiver, the MGF of γ_{mrc} can be easily obtained as $\mathcal{M}_{\gamma_{mrc}}(s) = \prod_{\ell=1}^{L} \mathcal{M}_{\gamma_\ell}(s)$, where $\mathcal{M}_{\gamma_\ell}(\cdot)$ is given in [14, eq. (4)].

2) Equal Gain Combining (EGC): For the EGC receiver is very difficult, if not impossible, to derive a closed-form expression for the MGF output SNR. Instead, it is more convenient to use the Padé approximants method, [20], to accurately approximate the MGF and eventually obtain the ABEP. The Padé approximants can be employed if an expression for the moments output SNR is known. For the EGC such an expression can be obtained as follows.

The total conditional SNR per symbol at the output of the EGC combiner is $\gamma_{egc} = [E_s/(LN_0)]\, \mathbb{E}\left\langle \left(\sum_{\ell=1}^{L} R_\ell\right)^2 \right\rangle$ [1], while its nth order moment $\mu_{\gamma_{egc}}(n) = \mathbb{E}\langle \gamma_{egc}^n \rangle$, is $\mu_{\gamma_{egc}}(n) = \mathbb{E}\left\langle [E_s/(LN_0)]^n \left(\sum_{\ell=1}^{L} R_\ell\right)^{2n} \right\rangle$. Hence, by substituting $\gamma_\ell = R_\ell^2 E_s/N_0$, using the multinomial identity [21, eq. (24.1.2)], and after some straight-forward mathematical manipulations yields

$$\mu_{\gamma_{egc}}(n) = \frac{(2n)!}{L^n} \sum_{\substack{n_1, n_2, \ldots, n_L \\ n_1+n_2+\cdots+n_L=2n}}^{2n} \frac{\prod_{\ell=1}^{L} \mu_{\gamma_\ell}(n_\ell/2)}{n_1! n_2! \cdots n_L!}, \tag{2}$$

where $\mu_{\gamma_\ell}(\cdot)$ is given in [14, eq. (5)].

3) Selection Combining (SC): Similar to the EGC receiver, the Padé approximants will be also employed to obtain the ABEP of the SC. Let γ_{sc} denote the SNR per symbol at the output of the SC receiver. The cumulative distribution function (CDF) of γ_{sc}, $F_{\gamma_{sc}}(\gamma)$, is mathematically expressed as $F_{\gamma_{sc}}(\gamma) = \prod_{\ell=1}^{L} F_{\gamma_\ell}(\gamma)$, where $F_{\gamma_\ell}(\cdot)$ is given in [14, eq. (3)].

In principal, for any L, the PDF of γ_{sc} can be obtained by differentiating $F_{\gamma_{sc}}(\gamma)$ with respect to γ. However, even for small values of L (e.g., $L \geq 3$) such differentiation is very complicated and does not lead to closed-form expressions for the moments. Thus, the case of $L = 2$, which is of most practical interest, will be considered here yielding

$$f_{\gamma_{sc}}(\gamma) = \frac{\pi^2 \csc[\pi(k-m_1)] \csc[\pi(k-m_2)]}{\Gamma(m_1)\Gamma(m_2)\Gamma(k)^2} \times \mathcal{H}_2(2)\mathcal{H}_1(1)\mathcal{H}_2(1)\mathcal{H}_1(2), \quad (3)$$

where

$$\mathcal{H}_1(\ell) = \Xi_\ell^{m_\ell} \frac{\gamma^{m_\ell}\,\Gamma(m_\ell)\,{}_1F_2(m_\ell; 1+m_\ell, 1-k+m_\ell; \Xi_\ell\,\gamma)}{\Gamma(1+m_\ell)\Gamma(1-k+m_\ell)} - \Xi_\ell^{k} \frac{\gamma^{k}\,\Gamma(k)\,{}_1F_2(k; 1+k, 1+k-m_\ell; \Xi_\ell\,\gamma)}{\Gamma(1+k)\Gamma(1+k-m_\ell)},$$

$$\mathcal{H}_2(\ell) = \Xi_\ell^{m_\ell} \frac{\gamma^{m_\ell-1}\,{}_0F_1(1-k+m_\ell; \Xi_\ell\,\gamma)}{\Gamma(1-k+m_\ell)} - \Xi_\ell^{k} \frac{\gamma^{k-1}\,{}_0F_1(1+k-m_\ell; \Xi_\ell\,\gamma)}{\Gamma(1+k-m_\ell)},$$

with $\Xi_\ell = k m_\ell/\overline{\gamma}_\ell$ and ${}_pF_q(\cdot)$ being the generalized hypergeometric function [18, eq. (9.14/1)], where p, q are integers.

By substituting (3) in $\mathbb{E}\langle\gamma_{sc}^n\rangle$, using [21, eq. (07.22.26.0004.01)], [21, eq. (07.17.26.0007.01)] and [21, eq. (07.34.04.0011.01)], the nth order moment of γ_{sc}, $\mu_{\gamma_{sc}}(n)$, can be obtained in closed form as

$$\mu_{\gamma_{sc}}(n) = \frac{\pi^2 \csc[\pi(k-m_1)] \csc[\pi(k-m_2)]}{\Gamma(m_1)\Gamma(m_2)\Gamma(k)^2} \times \mathcal{G}(m_1,k,m_2,k,1)\,\mathcal{G}(k,m_1,m_2,k,1) \times \mathcal{G}(m_2,k,m_1,k,-1)\,\mathcal{G}(k,m_2,m_1,k,-1), \quad (4)$$

with

$$\mathcal{G}(x,y,z,w,\ell) = \frac{\Xi_1^x\,\Xi_2^{-x-n}}{x(x-y)!} \Bigg\{ \Gamma(x+z+n) \times \frac{{}_pF_q\left[x+z+n, x, w+x+n; 1+x-y, 1+x; \left(\frac{\Xi_1}{\Xi_2}\right)^\ell\right]}{(-1)^{z+x+n}\Gamma(1-w-x-n)} - \Gamma(x+w+n) \times \frac{{}_pF_q\left[x+w+n, x, z+x+n; 1+x-y, 1+x; \left(\frac{\Xi_1}{\Xi_2}\right)^\ell\right]}{(-1)^{w+x+n}\Gamma(1-z-x-n)} \Bigg\}.$$

4) Switch and Stay Combining (SSC): In the case of SSC the MGF based approach can be employed as follows. Let γ_{ssc} denote the SNR per symbol at the output of a SSC receiver and γ_T the predetermined switching threshold at both diversity branches. The CDF of γ_{ssc}, $F_{\gamma_{ssc}}(\gamma)$, is given by [2]

$$F_{\gamma_{ssc}}(\gamma) = \begin{cases} [P_1P_2/(P_1+P_2)] \sum\limits_{\ell=1}^{2} F_{\gamma_\ell}(\gamma), & \gamma \le \gamma_T \\ [P_1P_2/(P_1+P_2)] \times \sum\limits_{\ell=1}^{2} \left[F_{\gamma_\ell}(\gamma) + F_{\gamma_\ell}(\gamma)/P_\ell - 1\right], & \gamma > \gamma_T \end{cases} \quad (5)$$

where $P_\ell = F_{\gamma_\ell}(\gamma_T)$. By differentiating (5) with respect to γ, the PDF of SSC can be obtained as

$$f_{\gamma_{ssc}}(\gamma) = \begin{cases} \frac{P_1P_2}{P_1+P_2} \sum\limits_{\ell=1}^{2} f_{\gamma_\ell}(\gamma), & \gamma \le \gamma_T \\ \frac{P_1P_2}{P_1+P_2} \sum\limits_{\ell=1}^{2} f_{\gamma_\ell}(\gamma)\,(1+1/P_\ell), & \gamma > \gamma_T \end{cases} \quad (6)$$

where $f_{\gamma_\ell}(\cdot)$ is given in [14, eq. (2)].

Substituting (6) in $\mathbb{E}\langle\exp(-s\gamma_{ssc})\rangle$, the MGF of γ_{ssc}, $\mathcal{M}_{\gamma_{ssc}}(s)$, can be expressed as

$$\mathcal{M}_{\gamma_{ssc}}(s) = \frac{P_1P_2}{P_1+P_2} \sum_{\ell=1}^{2} \left[\mathcal{M}_{\gamma_\ell}(s) + \frac{\mathcal{M}_{\gamma_\ell}(s)}{P_\ell} - \frac{1}{P_\ell} \int_0^{\gamma_T} \exp(-s\gamma) f_{\gamma_\ell}(\gamma) d\gamma \right]. \quad (7)$$

The above definite integral can be evaluated, in a straight forward way, via numerical integration. Moreover, the optimum value of γ_T, γ_T^*, minimizing the ABEP can be obtained by numerically solving $[\theta P_b(E)/\theta\gamma_T]|_{\gamma_T^*=\gamma_T} = 0$ [2, pp. 428].

B. Average Output SNR (ASNR) and AoF

Using the previously derived expressions for the EGC (see (2)) and SC (see (4)) moments, the ASNR and the AoF ($\text{AoF} = \mu_2/\mu_1^2 - 1$) can be obtained. For MRC and SSC the moments will be derived next.

1) MRC: It is well known that γ_{mrc} equals to the sum of the individual diversity branches SNRs [1], i.e., $\gamma_{mrc} = \sum_{\ell=1}^{L} \gamma_\ell$. Following a similar procedure used in deriving (2), the nth order moment of γ_{mrc} can be obtained as

$$\mu_{\gamma_{mrc}}(n) = \sum_{n_1=0}^{n} \sum_{n_2=0}^{n} \cdots \sum_{\substack{n_L=0 \\ n_1+n_2+\cdots+n_L=n}}^{n} \frac{n!}{n_1!n_2!\cdots n_L!} \prod_{\ell=1}^{L} \mu_{\gamma_\ell}(n_\ell). \quad (8)$$

2) SSC: Using (6) in $\mathbb{E}\langle\gamma_{ssc}^n\rangle$ and [21, eq. (03.04.21.0007.01.30)] and after some mathematical manipulations the nth order moment of γ_{ssc}, $\mu_{\gamma_{ssc}}(n)$, is expressed as

$$\mu_{\gamma_{ssc}}(n) = \frac{P_1P_2}{P_1+P_2} \sum_{\ell=1}^{2} \Bigg\{ \mu_{\gamma_\ell}(n) \left(1 + \frac{1}{P_\ell}\right) - \frac{\pi \csc[\pi(k-m_\ell)]\,\gamma_T^n}{P_\ell\,\Gamma(k)\Gamma(m_\ell)} \Big[(\Xi_\ell\,\gamma_T)^{m_\ell} \Gamma(m_\ell+n) \times {}_p\widetilde{F}_q(m_\ell+n; 1-k+m_\ell, 1+m_\ell+n; \Xi_\ell\,\gamma_T) - (\Xi_\ell\,\gamma_T)^k \times \Gamma(k+n)\,{}_p\widetilde{F}_q(k+n; 1+k-m_\ell, 1+k+n; \Xi_\ell\,\gamma_T) \Big] \Bigg\}, \quad (9)$$

where ${}_p\widetilde{F}_q(\cdot)$ is the regularized generalized hypergeometric function [21, eq. (07.32.02.0001.01)].

C. Outage Probability (P_{out})

Since P_{out} is defined as the probability that the output SNR falls below a given threshold, γ_{th}, it can be easily obtained for SC (using $F_{\gamma_{sc}}(\gamma)$) and SSC (see (5)) as $P_{out}(\gamma_{th}) = F_\gamma(\gamma_{th})$. For MRC and EGC, P_{out} can be mathematically expressed as $P_{out}(\gamma_{th}) = F_\gamma(\gamma_{th}) = \mathcal{L}^{-1}\left[\frac{\mathcal{M}_\gamma(s)}{s}; \gamma\right]\Big|_{\gamma=\gamma_{th}}$, where $\mathcal{L}^{-1}(\cdot)$ denotes inverse Laplace transformation. However, since there is no closed-form expression available for the MGF of EGC, as well as, the MGF of MRC does not lead to closed-form expression for the CDF, the MGFs are approximated with Padé approximants. In this way, P_{out} can be easily evaluated by numerical techniques [20], [22].

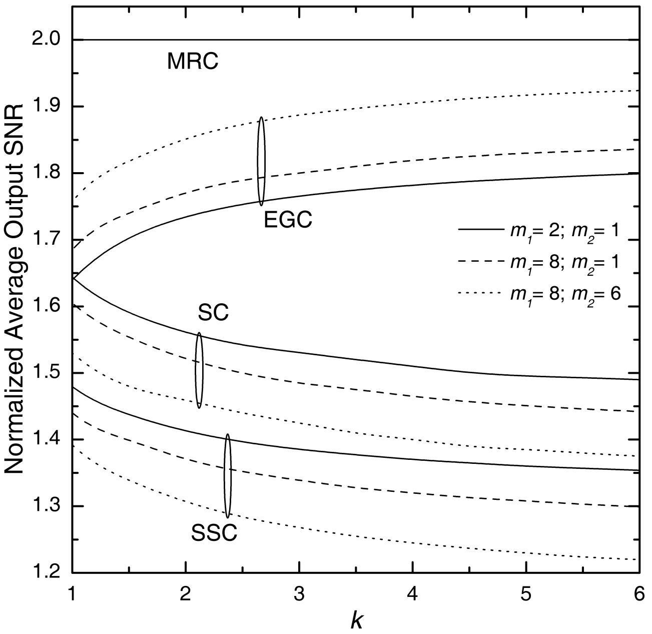

Fig. 1. Normalized average output SNR (ASNR) of MRC, EGC, SC and SSC diversity receivers, for $L = 2$, versus k for different values of m_1 and m_2.

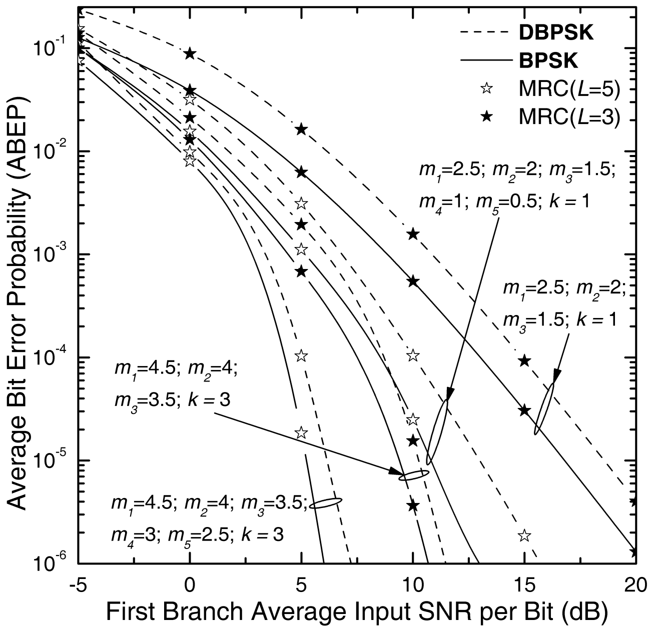

Fig. 2. MRC diversity employing DBPSK and BPSK signaling formats: Average bit error probability (ABEP) versus the first branch average input SNR per bit for different values of k and m_ℓ.

1) MRC: In the Appendix it is shown that, under the assumption of independent and identical distributed (i.i.d.) fading conditions, $\mathcal{M}_{\gamma_{mrc}}(s)$ can be represented in a more compact form as

$$\mathcal{M}_{\gamma_{mrc}}(s) = \mathcal{F}(m,k,L)\left(\frac{\Xi}{s}\right)^{\tau}\exp\left(\frac{\Xi L}{s}\right), \tag{10}$$

where

$$\begin{aligned}\mathcal{F}(m,k,L) = &\sum_{N=0}^{L}\underset{n_0+n_1+\cdots+n_{m-1}=N}{\sum_{n_0=0}^{N}\sum_{n_1=0}^{N}\cdots\sum_{n_{m-1}=0}^{N}}\;\underset{p_0+p_1+\cdots+p_{k-1}=L-N}{\sum_{p_0=0}^{L-N}\sum_{p_1=0}^{L-N}\cdots\sum_{p_{k-1}=0}^{L-N}}\\
&\times\binom{L}{N}\frac{N!}{n_0!n_1!\cdots n_{m-1}!}\frac{(L-N)!}{p_0!p_1!\cdots p_{k-1}!}\\
&\times\left[\frac{\Gamma(m-k)}{\Gamma(m)}\right]^{N}\left[\frac{\Gamma(k-m)}{\Gamma(k)}\right]^{L-N}\\
&\times\prod_{i=0}^{m-1}\left[\frac{(1-m)_i\,(-1)^i}{i!(k-m+1)_i}\right]^{n_i}\prod_{j=0}^{k-1}\left[\frac{(1-k)_j\,(-1)^j}{j!(1-k+m)_j}\right]^{p_j},\end{aligned}$$

and $\tau = \sum_{i=0}^{m-1}(in_i) + \sum_{j=0}^{k-1}(jp_j) + Lm + (k-m)N$ with $(\cdot)_n$ denoting the Pochhammer's symbol [18, eq. (9.749)].

By applying the inverse Laplace transform in (10) the PDF of γ_{mrc} can be obtained in closed form as

$$\begin{aligned}f_{\gamma_{mrc}}(\gamma) =&\mathcal{F}(m,k,L)\,\Xi^{(1+\tau)/2}\,L^{(1-\tau)/2}\\
&\times\gamma^{(\tau-1)/2}\,I_{\tau-1}\left(2\sqrt{\Xi L}\,\gamma^{1/2}\right),\end{aligned} \tag{11}$$

where $I_\nu(\cdot)$ is the νth order modified Bessel function of the first kind [18, eq. (8.40)].

Furthermore, $F_{\gamma_{mrc}}(\gamma)$, can be obtained using [21, eq. (03.02.21.0011.01)] as

$$F_{\gamma_{mrc}}(\gamma) = \mathcal{F}(m,k,L)\left(\frac{\Xi\gamma}{L}\right)^{\tau/2} I_\tau\left(2\sqrt{\Xi L}\gamma^{1/2}\right). \tag{12}$$

Clearly, using (12), the P_{out} of MRC for i.i.d. fading conditions can be obtained in a straight-forward manner as $P_{out}(\gamma_{th}) = F_{\gamma_{mrc}}(\gamma_{th})$.

IV. Performance Evaluation Results and Discussion

Using the analysis presented in Section III, various numerical performance evaluation results will be presented here. For the multipath channel, the exponentially decaying power delay profile (PDP) has been considered [23], i.e., $\overline{\gamma}_\ell = \overline{\gamma}_1\exp\left[-\delta(\ell-1)\right]$, where δ is the power decaying factor. In order to obtain a fair comparison between the i.i.d. and i.n.d. fading conditions, the total average input SNRs of the branches are assumed to be identical for both cases. For the i.n.d. conditions, we consider the most general case where different shaping parameters, m_ℓ, have been assumed among the different branches, e.g., m_1, m_2, $\ldots$, m_L, which for the i.i.d. fading condition become $m = (m_1 + m_2 + \cdots + m_\ell)/L$.

In Fig. 1, the ASNR for MRC$(2,2)$, EGC$(2,2)$, SC (for $L = 2$) and SSC has been plotted as function of the k shaping parameter and for several values of m_ℓ. It is depicted that as k and/or m_ℓ increase, i.e., the channel fading conditions improve, the ASNR for SC and SSC is decreasing while for EGC it is increasing. Moreover the ASNR performance of the MRC, which is constant and independent of k and m_ℓ, is always better as compared to the other diversity reception techniques. Comparing SC and SSC, the later always has worst performance.

In Figs. 2 - 4, the ABEP is plotted as a function of the first branch average input SNR per bit, $\overline{\gamma}_{b,1} = \overline{\gamma}_1/\log_2 M$, for several values of the shaping parameters m_ℓ, k and different receiver structures. Fig. 2 illustrates the ABEP of BPSK and DBPSK for different values of k and m_ℓ, employing MRC$(5,5)$ and MRC$(3,3)$, and for $\delta = 0.5$. As expected,

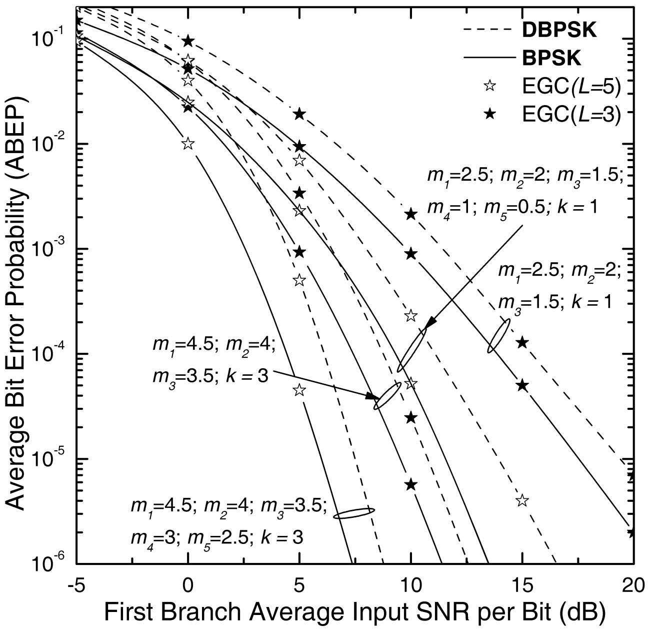

Fig. 3. EGC diversity employing DBPSK and BPSK signaling formats: average bit error probability (ABEP) versus the first branch average input SNR per bit for different values of k and m_ℓ.

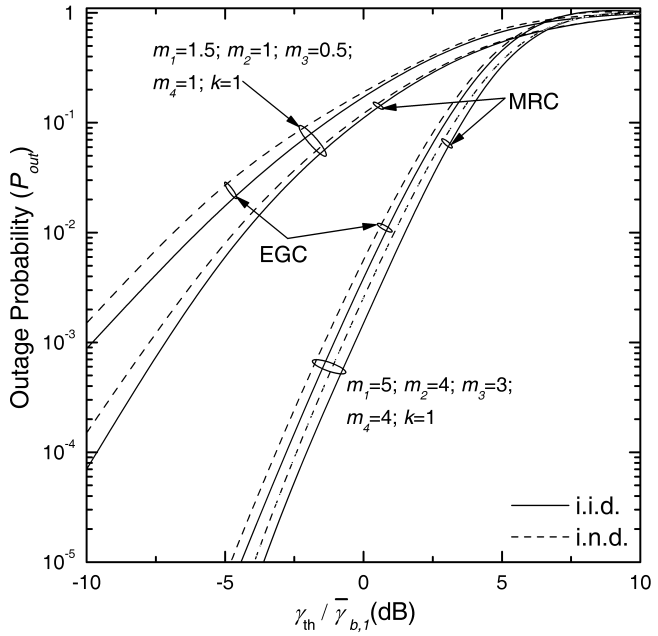

Fig. 5. MRC and EGC diversity employing 4 branches: outage Probability (P_{out}) versus the normalized outage threshold per bit for several values of k and m_ℓ.

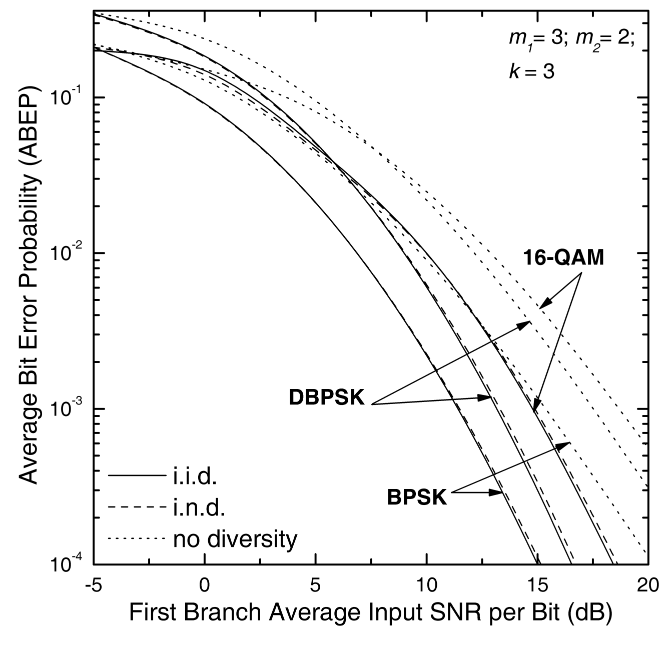

Fig. 4. SSC diversity employing DBPSK, BPSK and 16-QAM signaling formats: average bit error probability (ABEP) versus the first branch average input SNR per bit.

BPSK outperforms DBPSK, and their performance improves with improving fading channel conditions. Similar behavior is observed for EGC diversity reception employing these two signaling formats (see Fig. 3). These performance results confirm that, under the same channel conditions, the performance of a MRC receiver is always better as compared to an equivalent EGC by at least 3 dB. In Fig. 4, the ABEP of DBPSK, BPSK and 16-quadrature amplitude modulation (QAM) of a SSC receiver for $\delta = 0$ and 0.5 is illustrated. Again BPSK has the lowest ABEP and for the i.i.d. fading conditions the ABEP is better as compared to the equivalent i.n.d.

In Fig. 5, P_{out} is plotted as a function of the normalized outage threshold per bit of the first branch $\gamma_{th}/\overline{\gamma}_{b,1}$, for several values of m_ℓ and k for MRC and EGC. It can be observed that as m_ℓ and/or k increase, P_{out} decreases. Additionally MRC gives always better performance as compared to EGC and by considering i.i.d. fading conditions, P_{out} improves.

V. Conclusions

The performance of MRC, EGC, SC and SSC diversity receivers operating over independent but non identical distributed K_G fading channels has been analyzed. Novel expressions for the statistics of these diversity receivers, including the PDF, CDF, MGF, and the moments output SNR have been derived. Capitalizing on these statistical metrics important performance analysis criteria for several channel conditions and diversity receivers structures have been studied. Various performance evaluation results have been also presented verifying the usefulness of the proposed analysis.

Appendix
Derivation of Equation (10)

The MGF of MRC for the i.i.d. K_G fading channel can be expressed as

$$\begin{aligned}\mathcal{M}_{\gamma_{mrc}}(s) &= \left[\left(\frac{\Xi}{s}\right)^{\beta/2} \exp\left(\frac{\Xi}{2s}\right) W_{-\beta/2,\alpha/2}\left(\frac{\Xi}{s}\right)\right]^L \\ &= \left(\frac{\Xi}{s}\right)^{(\beta L)/2} \exp\left(\frac{\Xi L}{2s}\right) W_{-\beta/2,\alpha/2}\left(\frac{\Xi}{s}\right)^L,\end{aligned} \tag{A-1}$$

where $\alpha = k - m$ and $\beta = k + m - 1$. Using [18, eq. (9.220)], (A-1) can be rewritten as

$$\begin{aligned}\mathcal{M}_{\gamma_{mrc}}(s) &= \left(\frac{\Xi}{s}\right)^{(\beta\, L)/2} \exp\left(\frac{\Xi\, L}{2s}\right)\\ &\times \left[\frac{\Gamma(-\alpha)}{\Gamma(m)}\left(\frac{\Xi}{s}\right)^{(\alpha+1)/2} \exp\left(-\frac{\Xi}{2\,s}\right) {}_1F_1\left(k;\alpha+1;\frac{\Xi}{s}\right)\right.\\ &\left. + \frac{\Gamma(\alpha)}{\Gamma(k)}\left(\frac{\Xi}{s}\right)^{(-\alpha+1)/2} \exp\left(-\frac{\Xi}{2s}\right) {}_1F_1\left(m;\alpha+1;\frac{\Xi}{s}\right)\right]^L.\end{aligned} \tag{A-2}$$

With the aid of the binomial identity (A-2) simplifies to

$$\begin{aligned}\mathcal{M}_{\gamma_{mrc}}(s) &= \sum_{N=0}^{L} \binom{L}{N} \left[\frac{\Gamma(-\alpha)}{\Gamma(m)}\right]^N \left[\frac{\Gamma(\alpha)}{\Gamma(k)}\right]^{L-N}\\ &\times \left(\frac{\Xi}{s}\right)^{Lm+\alpha N} {}_1F_1\left(k;\alpha+1;\frac{\Xi}{s}\right)^N {}_1F_1\left(m;1-\alpha;\frac{\Xi}{s}\right)^{L-N}.\end{aligned} \tag{A-3}$$

Using [18, eq. (07.20.03.0025.01)], and for positive integer values of k and m, (A-3) can be expressed in terms of finite sums as

$$\begin{aligned}\mathcal{M}_{\gamma_{mrc}}(s) &= \sum_{N=0}^{L} \binom{L}{N} \left[\frac{\Gamma(-\alpha)}{\Gamma(m)}\right]^N \left[\frac{\Gamma(\alpha)}{\Gamma(k)}\right]^{L-N}\\ &\times \left(\frac{\Xi}{s}\right)^{Lm+\alpha N} \left[\exp\left(\frac{\Xi}{s}\right) \sum_{i=0}^{m-1} \frac{(1-m)_i\,(-\Xi/s)^i}{i!(\alpha+1)_i}\right]^N\\ &\times \left[\exp\left(\frac{\Xi}{s}\right) \sum_{j=0}^{k-1} \frac{(1-k)_j\,(-\Xi/s)^j}{j!(-\alpha+1)_j}\right]^{L-N}.\end{aligned} \tag{A-4}$$

Making use in (A-4) of the multinomial identity [21, eq. (24.1.2)] and after some straight-forward mathematical manipulations, (10) can be obtained.

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