Mitigating Shadowing Effects Through Cluster-Head Cooperation Techniques

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Abstract

In many situations the performance of wireless communication systems decreases especially when they operate over multipath fading channels subject also to shadowing. In this sense, cluster-based networks have been introduced as an efficient solution, offering coverage extension and energy savings. In this paper, we investigate new cluster-head (CH) selection algorithms, where the nodes can select different CHs, according to the corresponding signal strength. Additionally, it is shown that if CHs are equipped with multiple antennas, the negative consequences of the fading/shadowing can be further reduced. The performance of this scheme is theoretically investigated over correlated Nakagami-m fading channels, which are also subject to shadow fading, modelled by the Gamma distribution. The derived statistical metrics are used to obtain numerical evaluated results for the outage and the average bit error probabilities. These results are complemented by computer simulated ones, which validate the accuracy of the proposed analysis.

Index Terms

Cluster-based networks, correlated statistics, decode-and-forward, diversity techniques, multiple-input-single-output (MISO), Nakagami-m fading, shadowing.

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1 Introduction

In recent years, several research activities have been dealing with the assumption of global network infrastructure, proposing novel architectures and technologies towards a dynamic global platform of seamless networks and networked objects [1]. While a new architecture is required to satisfy the demands of these future systems, it is also essential that their development goes through the enhancement of today’s communications networks and standards. In this sense, cellular systems are expected to be a fundamental part of the internet of things (IoT), providing them with crucial benefits, such as ubiquitous coverage and global internetworking [2]. However, most of the connected objects (for example sensors) are subject to several limitations, including power consumption or hardware complexity, which may hinder the direct links to the cellular infrastructure. Towards this problem, cluster-based networks have been introduced, providing coverage extension, which facilitates the connectivity of those objects and conserves their energy [3–8].

1.1 Motivation

The envisaged future applications are expected to operate in wireless communication environments, that are subject to low power constraints, fading and shadowing. While the cluster-based networks offer substantial benefits to such scenarios, the probability of a link failure between the cluster-head (CH) and the nodes cannot be eliminated due to the severe channel conditions. In this regard, the energy efficiency, the reliability and the performance of CH networks can be further enhanced if the cluster nodes are able to switch (or select) between different CHs [9–11]. For example, in [10], the authors proposed a protocol to build multiple independent CHs overlays on top of the physical network, which allows the nodes to switch to another CH in case of a CH failure. The CH switch is based on the node’s residual energy and a cost parameter that is related with the node’s degree of connectivity.

Improving the reliability of wireless links through diversity techniques is an essential concept in wireless communications. Antenna or spatial diversity is an example, highlighting the benefits of combining different replicas of the same information distorted by independent (or near independent) fading paths. Classical diversity reception techniques (at transmit and/or receive sides) include the optimal maximal ratio combining (MRC), the easily implemented selection diversity (SD) and the least complex switch-and-stay combining (SSC) [12]. Additionally, for bridging the performance and implementation gap between MRC and SD, SSC receivers, several hybrid combining techniques have been proposed [12, 13]. Motivated by the fundamental diversity concept, we aim at improving the reliability of cluster-based networks by proposing hybrid link-combining techniques, based on
the assumption that the mobile nodes are in the vicinity of two or more CHs [9], [10]. Moreover, prompted by the absence for a theoretical analysis of such hybrid diversity techniques operating over multipath fading channels in the presence of shadowing, we provide a solid mathematical basis for their performance analysis.

1.2 Contribution

To improve the reliability of cluster-based networks through link diversity, and by extending the research framework of cooperative communications, we investigate two communication protocols, where the nodes can switch between different CHs, according to the corresponding signal strength. More specifically, considering a composite fading environment, where multipath fading coexists with shadowing, and based on the decode-and-forward (DF) strategy, we investigate the case where a message is communicated between a node and an access point (AP), for example base station or gateway, with the assistance of a number of CHs. In this context,

1. we provide a solid mathematical background for describing the stochastic nature of the wireless communication links

2. we propose two novel communication techniques that determine which CH (in the same CH overlay) is the best option to link the nodes with the AP, according to certain QoS criteria.

The composite fading environment, considered in our case, has been employed several times in the past, for example [14–19]. A common observation in all these works is that independent multipath fading conditions have been assumed, in contrast to our work, where exponential-correlated Nakagami-$m$ fading channels, subject also to shadowing, have been considered. Hence, based on the proposed communication techniques, the network reliability significantly increases, especially in cases, where the connection to a CH may be not possible, due to shadowing effects. In contrast to previous works, we relate the CH failure with the quality of the wireless link between the infrastructure (for example base station), and the node, which is affected by multipath fading and shadowing.

1.3 Outline

The remainder of this paper is organized as follows: In Section 2, the system model along with the basic assumptions are presented. In Section 3, a detailed stochastic analysis provides the theoretical framework of various channel models and case studies. Based on this analysis, in Section 4, the different modes of operation are presented, and their corresponding probability density function (PDF), cumulative distribution function (CDF), moments
generating function (MGF) and moments of the output signal-to-noise ratio (SNR) are provided. These results are used in Section 5 to study important performance criteria such as bit error probability (BEP) and the outage probability (OP). In Section 6, using the above mentioned performance metrics several numerical evaluated results are presented, while in Section 7 the concluding remarks of this paper are provided.

2 System and Channel Model

We consider a communication network, where a message is communicated between a node and an AP, for example base station or gateway, with the assistance of a number of CHs [10] (Fig. 1). The APs are considered to support multiple antennas (MA)s, while CH \(i\), with \(i \in (1, N)\), is equipped with a single antenna for the link to the AP and \(L_i\) antennas for the link to the mobile nodes. Employing the DF strategy, the CHs decode the signal and then retransmit the detected version to the AP (in case of the uplink), or the destination nodes (in case of the downlink) [20].

Let \(X_{\ell,i}\), with \(1 \leq \ell \leq L_i\), represent the instantaneous SNR per symbol for the link between the \(\ell\)th antenna of the \(i\)th CH and the mobile node (or the link between the \(\ell\)th antenna of the AP and the \(i\)th CH). Assuming Nakagami-\(m\) fading channels, the PDF of \(X_{\ell,i}\)s is given by

\[
f_{X_{\ell,i}}(x) = \frac{m_{\ell,i}^{m_{\ell,i}}x^{m_{\ell,i}-1}}{\Gamma(m_{\ell,i})} \exp\left(-\frac{m_{\ell,i}x}{\bar{x}_{\ell,i}}\right), \quad x \geq 0
\]

where \(0 < m_{\ell,i} < \infty\) is the distribution’s shaping parameter related to fading severity, \(\bar{x}_{\ell,i}\) is the average SNR per symbol and \(\Gamma(\cdot)\) is the Gamma function [21, eq. (8.310/1)]. This model includes the one-sided Gaussian distribution \((m = 1/2)\) and the Rayleigh distribution \((m = 1)\) as special cases. In the limited case where \(m \to \infty\), Nakagami-\(m\) fading channel converges to a non-fading additive white Gaussian noise channel.

In many practical situations, the radio communication links are considered to be affected by shadowing. In these environments, when the multipath components are also subject to shadowing, \(\bar{x}_{\ell,i}\) are randomly varying, becoming thus random variables (RV)s. In this environment the receiver does not average out the envelope fading due to multipath but rather reacts to the instantaneous composite multipath/shadowed signal [12]. In most cases the slow variations in \(\bar{x}_{\ell,i}\) are modeled with the lognormal distribution with PDF

\[
f_{\bar{x}_{\ell,i}}(y) = \frac{\varsigma}{\sqrt{2\pi} \sigma_{\ell,i} y} \exp\left[-\frac{(10 \log_{10}(y) - \mu_{\ell,i})^2}{2\sigma_{\ell,i}^2}\right], \quad y \geq 0
\]

where \(\varsigma = 10/\ln(10)\) and \(\mu_{\ell,i}\) and \(\sigma_{\ell,i}\) are the mean and standard deviation of \(\ln(\bar{x}_{\ell,i})\). An alternative approach, which has been found to be appropriate
for modelling shadowing effects is to employ the gamma distribution [22–24], with PDF given by

\[ f_{\pi_{\ell,i}}(y) = \frac{\alpha_{\ell,i} y^{\alpha_{\ell,i}-1}}{\pi_{\ell,i}^{\alpha_{\ell,i}}} \exp \left( -\frac{\alpha_{\ell,i} y}{\gamma_{\ell,i}} \right), \quad y \geq 0. \]  

(3)

In (3), \( \alpha_{\ell,i} \geq 0 \) is the shaping parameter of the gamma distribution, related to the severity of shadowing (that is, smaller values of \( \alpha_{\ell,i} \) correspond to stronger shadowing), while \( \pi_{\ell,i} \) is the average power of \( \pi_{\ell,i} \).

The composite fading environment that arises when multipath fading is superimposed on shadowing can be obtained by averaging the PDF of the SNR per symbol, in our case (1) is used, over the conditional density of the average SNR per symbol, statistical described by either the lognormal or the gamma distributions. Mathematically speaking, the total probability theorem can be applied for obtaining the PDF of the composite fading SNR as [25, eq. (7.44)]

\[ f_{\text{comp-fad}}(\gamma) = \int_{0}^{\infty} f_{\text{mult-fad}|\pi_{\ell,i}}(\gamma|y) f_{\pi_{\ell,i}}(y) dy \]  

(4)

where \( f_{\text{mult-fad}|\pi_{\ell,i}}(\gamma|y) \) refers to the PDF that describes the multipath fading. It can be easily verified that by using (3), instead of (2), in (4), that is considering gamma distributed shadowing, leads to mathematical more tractable expressions and hence this is the approach that is going to be adopted in our analysis. In the open technical literature and the research framework of cooperative communications over composite fading channels, several works can be found, for example [19, 26–28]. However, to the best of the authors' knowledge, none of them investigates the performance of cooperative communication systems that are subject to exponential-correlated fading and shadowing and this is the main scope of this paper.

### 3 Statistical Framework

In this section, based on a useful expression for the sum of Nakagami-\( m \) RVs, important statistical characteristics of various functions of RVs are investigated. As it will be shown later, these expressions are directly related with the network components signals, which are exchanged in the communication system considered in this paper. Let \( Y_i \) representing the sum of \( X_{\ell,i} \), that is \( Y_i = \sum_{\ell=1}^{L_i} X_{\ell,i} \), and \( \rho_{i,j} \) denoting the exponential correlation among \( X_{\ell,i} \), given as \( \rho_{i,j} = \rho_{i}^{(i-j),0 < \rho_{i} < 1} \), with \( [(i,j) \in (1, \ldots, L_i)] \). Considering also independent and identically distributed (iid) parameters, that is \( \pi_{\ell,i} = \pi_i \) and \( m_{\ell,i} = m_i \), the PDF of \( Y_i \) is closely approximated by [29]

\[ f_{Y_i}(\gamma) = \frac{\gamma^{B_i-1} \exp(-A_{\pi_i} \gamma)}{\Gamma(B_i) A_{\pi_i}^{-B_i}} \]  

(5)
where
\[ A_{\tau_i} = \frac{m_i L_i}{r_i \tau_i}, \quad B_i = \frac{m_i L_i^2}{r_i}, \quad r_i = L_i + \frac{2 \rho_i}{1 - \rho_i} \left( L_i - \frac{1 - \rho_i}{1 - \rho_i} \right). \]

Using [21, eq. (3.351/1)] on the definition of the CDF, that is [25, eq. (4.17)]
\[ F_{Y_i}(\gamma) = \int_0^\gamma f_{Y_i}(\xi) d\xi, \]
the corresponding CDF can be obtained as
\[ F_{Y_i}(\gamma) = \frac{\gamma (B_i, A_{\tau_i} \gamma)}{\Gamma(B_i)} \]
where \( \gamma(\cdot, \cdot) \) is the lower incomplete gamma function [21, eq. (8.350/1)].

Moments Generating Function Based on the definition of the MGF, that is [25, eq. (5.62)]
\[ M_{Y_i}(s) = \int_0^\gamma \exp(-s \gamma) f_{Y_i}(\gamma) d\gamma, \]
and using [21, eq. (3.351/3)], the MGF of \( Y_i \) is given by
\[ M_{Y_i}(s) = \left( \frac{A_{\tau_i}}{A_{\tau_i} + s} \right)^{B_i}. \]

Moments Similar to the derivation of (7) and based on [25, eq. (5.38)], that is
\[ \mu_{Y_i}(n) = \int_0^\gamma \gamma^n f_{Y_i}(\gamma) d\gamma, \]
the moments can be obtained as
\[ \mu_{Y_i}(n) = \frac{(B_i + n - 1)!}{\Gamma(B_i)} A_{\tau_i}^{-n}. \]

3.1 Sum of Nakagami-\( m \) RVs Averaged on Gamma Variables

The following statistical metrics are related with the combined signals at the multiple antennas of the CHs (or the APs) investigated in the next section. Considering also \( Y_i \) representing the sum of \( X_{\ell,i} \) with PDF provided in (5), while \( \tau_i \) is a RV following the gamma distribution with PDF given in (3). The PDF of \( Y_i \) conditioned on \( \tau_i \) can be obtained by substituting (5) and (3) in the total probability integral, that is (4), and using [21, eq. (3.471/9)], yielding
\[ f_{Y_i}(\gamma) = \frac{2A_{\tau_i}^{\Delta+/2}}{\Gamma(B_i) \Gamma(\alpha_i)} \gamma^\Delta+ -1 K_{\Delta-} \left( 2\sqrt{A_{\tau_i} \gamma} \right) \]
where \( A_{\tau_i} = \alpha_i m_i L_i/(r_i \tau_i) \), \( \Delta+ = \alpha_i + B_i \), \( \Delta- = \alpha_i - B_i \), with identical distributed conditions assumed, that is \( \tau_{\ell,i} = \tau_i \) and \( K_{\nu}(\cdot) \) is the modified Bessel function of the second kind and order \( \nu \) [21, eq. (8.407/1)]. Furthermore, starting from the definition of the CDF [25, eq. (4.17)], expressing \( K_{\nu}(\cdot) \) in terms of the Meijer-G function [30, eq. (03.04.26.0006.01)] and using [31, eq. (26)] a closed-form expression for the CDF of \( Y_i \) can be obtained.
as follows

\[ F_{Y_i}(\gamma) = \left( A_{\gamma_i} \right)^{A_{\gamma_i}} \frac{\Delta_{\gamma_i}}{\Gamma(B_i) \Gamma(\alpha_i)} G^{2,1}_{1,3} \left( A_{\gamma_i} \gamma \left| \begin{array}{c} \frac{1-\Delta_{\gamma_i}}{\Delta_{\gamma_i}}, \frac{1}{\gamma_i} \end{array} \right. \right) \]  

(10)

where \( G^{m,n}_{p,q} \) is the Meijer’s \( G \)-function \([21, \text{eq. (9.301)}]\). Moreover, using \([30, \text{eq. (07.34.03.0727.01)}]\), that is

\[
G_{1,3}^{2,1} \left( \begin{array}{c} a_1 \\ b_1, b_2, b_3 \end{array} | z \right) = \pi \csc \left( \pi \left( b_2 - b_1 \right) \right) \times \left\{ \Gamma \left( 1 - a_1 + b_1 \right) z^{b_1} \tilde{F}_2 \left( 1 - a_1 + b_1, b_1 - b_2 + 1, b_1 - b_3 + 1, z \right) \right. \\
\left. - \Gamma \left( 1 - a_1 + b_2 \right) z^{b_2} \tilde{F}_2 \left( 1 - a_1 + b_2, 1 - b_1 + b_2, 1 - b_1 + b_2, 1 - b_3 + 1, z \right) \right\}
\]

(11)

where \( \tilde{F}_2 (\cdot) \) is the regularized generalized hypergeometric function \([30, \text{eq. (07.32.02.0001.01)}]\) and \( \csc(\cdot) = 1/\sin(\cdot) \), a simplified expression for \( F_{Y_i}(\gamma) \) can be extracted.

In Fig. 2, considering \( m_i = 2, L_i = 3, \rho_i = 0.5, \gamma_i = 5 \text{dB} \) and several values of \( \alpha_i \) for \( i = 1, 2 \), a PDF comparison between a) (9) and b) the PDF that is originated by substituting (5) and (2) in (4), evaluated using simulations, is depicted. In the same figure the corresponding CDFs are also illustrated. It can be easily recognized the close agreement between these two PDFs, especially for higher values of \( \alpha_i \)'s. Moreover, for further validating the fine agreement between the lognormal and the gamma distributed RVs, we employed the Kolmogorov-Smirnov (KS) goodness-of-fit statistical tests. We compared the analytically evaluated lognormal CDF, with the CDF of the gamma distributed RVs, produced via Monte Carlo simulation. Using \( 10^5 \) samples and a significance level of 5%, the KS test was repeated \( 10^3 \) times comparing the test statistics with the critical level as calculated in \([25]\). The average level of acceptance was 97%.

3.1.1 Moments Generating Function

By substituting (9) in the definition of the MGF \([25, \text{eq. (5.62)}]\), and using \([21, \text{eq. (6.643/3)}]\), the MGF of \( Y_i \) can be expressed as

\[
M_{Y_i}(s) = \mathcal{W} \left( \frac{A_{\gamma_i}}{s}, \alpha_i, B_i \right)
\]

(12)

where

\[
\mathcal{W}(x, y, z) = x^{y \frac{x-1}{z}} \exp \left( \frac{x}{2} \right) W_{\frac{z}{2}, \frac{z}{2}} \left( x \right)
\]

with \( W_{\lambda, \mu} (\cdot) \) denoting the Whittaker function \([21, \text{eq. (9.220)}]\).
3.1.2 Moments

The moments of \( Y_i \) can be easily obtained by substituting (9) in the definition of the moments [25, eq. (5.38)] as

\[
\mu_{Y_i}^{(n)} = \mathcal{R} \left( A_{\gamma_i}, \alpha_i, B_i \right)
\]

where

\[
\mathcal{R} (x, y, z) = x^{-n} \frac{\Gamma(y+n)\Gamma(z+n)}{\Gamma(z)\Gamma(y)}.
\]

3.2 Maximum of Two Sums of Nakagami-\( m \) Variables

The following statistical metrics are related with the CH selection process investigated in the next section. Let us define a new RV \( Z = \max(Y_1, Y_2) \), with \( Y_i \) representing the sum of \( X_{\ell,i} \). In that case, and if \( Y_1 \) and \( Y_2 \) are independent, the CDF of \( Z \) can be expressed as [25, eq. (6.55)],

\[
F_Z(\gamma) = F_{Y_1}(\gamma)F_{Y_2}(\gamma)
\]

while the corresponding PDF can be obtained by differentiate \( F_Z(\gamma) \) with respect to \( \gamma \) as

\[
f_Z(\gamma) = f_{Y_1}(\gamma)f_{Y_2}(\gamma) + F_{Y_1}(\gamma)f_{Y_2}(\gamma).
\]

In the above equation, \( F_{Y_i}(\gamma) \) is given by (10), while \( f_{Y_i}(\gamma) \) is given by (9).

3.2.1 Moments Generating Function

By substituting (9) and (10) in (15) and using the definition of the MGF, a closed-form expression for the MGF of \( Z \) cannot be obtained. To overcome this, the Meijer-G function appearing in (10) is expressed in terms of the \( p \tilde{F}_q(\cdot) \), with the aid of (11), and then by employing the infinite series representation of \( p \tilde{F}_q(\cdot) \), [30, eq. (07.32.02.0001.01)], integrals of the following form appear

\[
I = \int_0^\infty x^{c_1} \exp (-C_1 x) K_{c_2} \left( C_2 x^{1/2} \right) dx
\]

where \( c_1, c_2, C_1, C_2 \in \mathbb{R} \) with \( c_1 > |c_2/2| \). These integrals can be solved in closed form, using [21, eq. (6.631/3)], and hence after some mathematical manipulations, the following exact expression for the MGF of \( Z \) can be extracted

\[
M_Z(s) = \sum_{j=1}^{2} \mathcal{D}_j \mathcal{D}_{\xi-\frac{s}{A_{\gamma,j}+\Delta_{\gamma,j}}} \exp \left( \frac{A_{\gamma,j}}{2s} \right)
\times \left[H_1 \left( \alpha_\xi, \alpha_j, A_{\gamma_j}, B_j, A_{\gamma_\xi}, B_\xi, s \right) - H_1 \left( B_\xi, \alpha_j, A_{\gamma_j}, B_j, A_{\gamma_\xi}, \alpha_\xi, s \right) \right]
\]

\[17\]
where
\[
\mathcal{H}_1(x, y, z, q, w, r, s) = \sum_{t=0}^{\infty} \frac{\Gamma(x + t + y) \Gamma(x + t + q)}{\Gamma(x - r + 1) x (x - t + 1) t!} s^{x+t} t! \\
\times W_{1-x-q-1-s-t} \left( \frac{y z}{s} \right)
\]
while \( \mathcal{D}_i = \frac{A_{\xi + j}}{1 - A_{\xi + j} (\alpha)} \), \( (\cdot)_p \) is the Pochhammer’s symbol [21, p. xliii], with \( p \in \mathbb{N} \), and \( \xi = 3 - j \). In Section 6, the convergence rate of the series appearing in (17) is also examined, verifying the fast convergence of it.

3.2.2 Moments

Substituting (9) in the definition of the moments, expressing \( K_{\nu}(\cdot) \) in terms of the Meijer-G function [30, eq. (03.04.26.0006.01)] and using [31, eq. (26)] the \( n \)th order moment of \( Z \) can be obtained in closed form as

\[
\mu_Z(n) = \sum_{j=1}^{2} \mathcal{D}_j \mathcal{D}_\xi A_{\eta_j} \psi_{2,3}^2 \left( \frac{A_{\eta_j}}{A_{\eta_j}} \right) 1 - \frac{\Delta_{\xi + j} + 1 - \alpha_j - \beta_j}{2}, -n, 1 - \beta_j - \Delta_{\xi + j} - n \right)
\]

3.3 Sequences of Two Sums of Nakagami-\( m \) Variables

The following statistical metrics are related with the CH switching process investigated in the next section. Let us consider two independent sequences of RVs \( Y_{1,j} \) and \( Y_{2,j} \), with \( j \in \mathbb{N} \), where for each \( j \) the PDF of \( Y_{i,j} \) is described by (9). Furthermore, let us define a new sequence \( Z_n \) given as

\[
Z_j = Y_{1,j} \text{iff } \begin{cases} Z_{j-1} = Y_{1,j-1} \text{ and } Y_{1,j} \geq \delta \\
\text{or } Z_{j-1} = Y_{2,j-1} \text{ and } Y_{2,j} < \delta \end{cases}
\]

where \( \delta \) is a predefined threshold. It can be proved that by following the analysis provided in the appendix of [32], the CDF of \( Z_j \) has the following form

\[
\mathcal{F}_{Z_j}(\gamma) = \begin{cases} C \left[ \mathcal{F}_{Y_{1,j}}(\gamma) + \mathcal{F}_{Y_{2,j}}(\gamma) \right], \gamma \leq \delta \\
C \left[ \mathcal{F}_{Y_{1,j}}(\gamma) + \mathcal{F}_{Y_{2,j}}(\gamma) - 2 \right] + \mathcal{D}, \gamma > \delta \end{cases}
\]

where

\[
C = \frac{\mathcal{F}_{Y_{1,j}}(\delta) \mathcal{F}_{Y_{2,j}}(\delta)}{\mathcal{F}_{Y_{1,j}}(\delta) + \mathcal{F}_{Y_{2,j}}(\delta)}, \quad D = \frac{\mathcal{F}_{Y_{1,j}}(\gamma) \mathcal{F}_{Y_{2,j}}(\gamma) + \mathcal{F}_{Y_{1,j}}(\delta) \mathcal{F}_{Y_{2,j}}(\gamma)}{\mathcal{F}_{Y_{1,j}}(\delta) + \mathcal{F}_{Y_{2,j}}(\delta)}.
\]
Furthermore, in (20) \( F_{Y_{i,j}}(\cdot) \) is given by (10). The corresponding PDF of \( Z_j \) can be easily obtained as
\[
f_{Z_j}(\gamma) = \left\{ \begin{array}{ll}
C \left[ f_{Y_{1,j}}(\gamma) + f_{Y_{2,j}}(\gamma) \right] , & \gamma \leq \delta \\
C \left[ f_{Y_{1,j}}(\gamma) + f_{Y_{2,j}}(\gamma) \right] + \mathcal{P}, & \gamma > \delta 
\end{array} \right. \tag{21}
\]
where
\[
\mathcal{P} = \frac{f_{Y_{1,j}}(\gamma)F_{Y_{2,j}}(\delta) + F_{Y_{1,j}}(\delta)f_{Y_{2,j}}(\gamma)}{F_{Y_{1,j}}(\delta) + F_{Y_{2,j}}(\delta)}
\]
and \( f_{Y_{i,j}}(\cdot) \) is given by (9).

3.3.1 Moments Generating Function
Substituting (21) in the definition of the MGF [25, eq. (5.62)] and using (9) and [21, eq. (6.643/3)] the MGF of \( Z_j \), can be obtained as
\[
\mathcal{M}_{Z_j}(s) = C \sum_{i=1}^{2} \left[ W \left( \frac{A_{\gamma i}}{s}, \alpha_i, B_i \right) + \frac{1}{F_{Y_{i,j}}(\delta)} \int_{\delta}^{\infty} f_{Y_{i,j}}(\gamma) \exp(-s\gamma) d\gamma \right].
\tag{22}
\]
It is noted that in (22), the definite integral cannot be obtained in closed form and hence numerical evaluation methods will be employed, using any of the well-known mathematical software packages, for example MATHEMATICA or MAPLE.

3.3.2 Moments
Substituting (21) in the definition of the moments, using (9), using [21, eq. (6.561/16)] and following a similar procedure to that used for deriving (10), the \( n \)th order moment of \( Z_j \) can be obtained in closed form as
\[
\mu_{Z_j}(n) = C \sum_{i=1}^{2} \left[ \left( 1 + \frac{1}{F_{Y_{i,j}}(\delta)} \right) R \left( A_{\gamma i}, \alpha_i, B_i \right) \right. \\
\left. - \frac{A_{\gamma i}^{\frac{1}{2}+n}}{F_{Y_{i,j}}(\delta)} A_{\gamma i}^{\frac{1}{2}} G_{1,3}^{2,1} \left. \left( \frac{A_{\gamma i}^{\frac{1}{2}+n}}{A_{\gamma i}^{\frac{1}{2}}, -\frac{1}{2}, -\frac{1}{2}, -n} \right) \right) \right]. \tag{23}
\]

4 Mode of Operation and Received Signal Statistics
In this section, important statistical metrics of the received signal for both links in each side of the CH, namely the link between the AP and the CH (link 1) and the link between the CH and the nodes (link 2), will be studied,
assuming shadowing and correlated effects are present. More specifically, the important situation where fully correlated shadowing is present, will be investigated. This is the case where the different paths simultaneously exhibit identical shadowing effects and has been studied many times in the past, for example [16, 17]. This type of shadowing is of great practical interest, since it arises in situations where the distance among the antennas is considerable smaller than the shadowing coherence distance [33, 34].

Considering link 1 and the downlink case the AP transmits a message to the CHs, employing the maximal ratio transmission (MRT) technique and assuming that perfect channel state information is available at the AP. For the uplink case, the CHs retransmit the decoded message, received from the nodes, to the AP, which combines the received signal replicas at each antenna by terms of MRC. Let $X_{\ell,i}$ representing the instantaneous SNR for the link between the $\ell$th antenna of the AP and the $i$th CH. Considering correlation among the links for the multipath effects, the total instantaneous SNR between the AP and the CH can be expressed as the sum of the individual SNRs $\gamma_i = \sum_{\ell=1}^{L_i} X_{\ell,i}$. In case that fully correlated shadowing effects are also present, the analysis presented in Section 3.1 can be applied and the PDF of the output SNR, $\gamma_i$, is given by (9), the CDF by (10), the MGF by (12) and the moments by (13).

Considering link 2 and assuming CH $i$ is equipped with $L_i$ antennas, the instantaneous SNR between the mobile nodes and each of the $i$th CH is $\gamma_i = \sum_{\ell=1}^{L_i} X_{\ell,i}$. Next, two strategies for selecting the CH to be connected to are presented, namely a) the CH selection algorithm (indicated as SE), and b) the CH switching algorithm (indicated as SW). In Fig. 3, the mode of operation (in terms of a flowchart) for both these strategies is depicted.

### 4.1 CH Selection

In CH selection strategy, the CH providing the link with the highest instantaneous SNR is selected for the connection between the nodes and the AP. In this case the instantaneous SNR at the output of the nodes for the downlink (or at the output of the CHs for the uplink) is $\gamma_{\text{se}} = \max(\gamma_1, \gamma_2, \ldots, \gamma_N)$, see Fig 3.b. Considering the practically important case where the nodes may select and connect between 2 CHs, and based on the analysis presented in Section 3.2, the corresponding statistics of the output SNR, $\gamma_{\text{se}}$, are given as: the PDF by (15), the CDF by (14), the MGF by (17), while the moments by (18).

### 4.1.1 High SNR Analysis

For high values of the average input SNR, that is $\bar{\gamma}_i \geq 30$dB, simplified expressions for the MGF are evaluated. Starting from (17), using [30, eqs.
yields the following closed-form approximated expression for $M_{\gamma_{se}}(s)$

$$M_{\gamma_{se}}(s) \cong \sum_{j=1}^{2} \frac{D_j D_{\xi} \pi \csc(\pi \Delta_{\xi})}{s} \exp\left(\frac{A_{\tau_j}}{2s}\right) \times \left[ H_2\left(\alpha_{\xi}, \alpha_j, A_{\tau_j}, B_j, A_{\tau_{\xi}}, B_{\xi}, s\right) - H_2\left(B_{\xi}, \alpha_j, A_{\tau_j}, B_j, A_{\tau_{\xi}}, \alpha_{\xi}, s\right) \right]$$

where

$$H_2(x, y, z, q, w, r, s) = \frac{\Gamma(x+y) w^{-x} \Gamma(x+q)}{\Gamma(x+r+1)} W_{\frac{x+y}{2}, \frac{r-x}{2}}\left(\frac{y^2}{s}\right).$$

### 4.2 CH Switching

For the CH switching strategy and in order to maintain a predefined QoS constraint the mobile nodes are able to switch between different CHs, according to the corresponding signal strength. More specifically, a CH is used for the communication link as long as the provided instantaneous SNR is above a predefined threshold; otherwise, the other CH within the same CH overlay is selected [35, eq. (62)], see Fig 3.a. In this way, unnecessary switchings can be avoided, compared to the CH SE mode. According to the protocol’s mode of operation, the final instantaneous output SNR, $\gamma_{sw}$, will have the CDF given in (20), by substituting $\gamma_i$ with $\gamma_i$ and representing the predefined QoS threshold $\delta$ as $\gamma_{\tau}$. Hence, based on the analysis presented in Section 3.3, the corresponding PDF of $\gamma_{sw}$ is given in (21), the CDF by (20), while the MGF by (22) and the moments by (23).

#### 4.2.1 High SNR Analysis

Considering high values of the average input SNR, that is $\gamma_i \geq 30$dB, and starting from (22), representing the Bessel function as a generalized hypergeometric function, namely $0F_1(\cdot)$ [30, eq. (03.04.27.0005.01)], and by employing [30, eq. (07.17.06.0003.01)] a closed-form approximated expression can be obtained as

$$M_{\gamma_{se}}(s) \cong C \sum_{i=1}^{2} \left\{ W\left(\frac{A_{\tau_i}}{s}, \alpha_i, B_i\right) + \frac{1}{\mathcal{F}_{\gamma_i}(\gamma_{\tau})} \frac{1}{\Gamma(\alpha_i)} \Gamma(B_i) \right\} \times \left[ \Gamma(\Delta_{\tau}) \left(\frac{A_{\tau_i}}{s}\right)^{B_i} \Gamma(B_i, s\gamma_{\tau}) + \Gamma(-\Delta_{\tau}) \left(\frac{A_{\tau_i}}{s}\right)^{\alpha_i} \Gamma(\alpha_i, s\gamma_{\tau}) \right]$$

where $\Gamma(\cdot, \cdot)$ is the upper incomplete Gamma function [21, eq. (8.350/2)].
5 Performance Analysis

In this section, using the previously derived expressions for the instantaneous SNR of the links of both CH sides, important performance quality indicators are studied. More specifically, the performance is studied using the OP and the BEP.

5.1 Outage Probability

The OP is defined as the probability that the output SNR falls below a predetermined outage threshold $\gamma_{th}$ and hence in our scenario an outage could occur if either one of the two links is in outage [36]. More specifically, for the communication link 1 and $i$th CH, the OP can be obtained as $P_{out1} = F_{\gamma_i}(\gamma_{th})$ using (10), while for link 2 can be obtained as $P_{out2} = F_{\gamma_{out}}(\gamma_{th})$ using (14), for the CH selection algorithm, and (20), for the CH switching algorithm. Hence, the end-to-end (E2E) OP can be easily obtained as the complement event of having both links operating above a $\gamma_{th}$, that is $P_{out_{tot}} = P_{out1} + P_{out2} - P_{out1}P_{out2}$. It is noted that for the CH switching case the optimal switching threshold $\gamma^*_{\tau}$ for minimum OP, can be obtained for $\gamma^*_{\tau} = \gamma_{th}$ [35].

5.2 Average Bit Error Probability

In the DF strategy the CHs decode and retransmit the received signal to the destination (the AP or the nodes), thus resulting an overall probability of error equal to [36]

$$P_{b_{tot}} = P_{b_1} + P_{b_2} - 2P_{b_1}P_{b_2}$$

(26)

where $P_{b_{1}}$ denoted the average BEP obtained in link 1 and $P_{b_{2}}$ is the average BEP obtained in link 2. In case of $P_{b_1}$, a directly evaluation of the BEP can be performed by averaging the conditional symbol error probability, $P_e(\gamma)^2$, over the PDF of $\gamma_i$, that is

$$P_{s_1} = \int_0^\infty P_e(\gamma)f_{\gamma_i}(\gamma)d\gamma.$$  

(27)

More specifically, for binary phase shift keying (BPSK) and square $M$-quadrature amplitude modulation (QAM), $P_e(\gamma) = \text{Aerfc}(\sqrt{\gamma})$, where \text{erfc} denotes the complementary error function [21, eq. (8.250/1)] and $A,B$ constants depending on the specific modulation scheme. Hence, substituting (9) in (27) and using [30, eq. (07.34.21.0011.01)] as well as [30, eq. (07.34.03.0890.01)] the SEP can be obtained in closed form as

$$P_{s_1} = \frac{\sqrt{\pi} \csc(-\pi \alpha_i)}{\Gamma(\alpha_i)\Gamma(B_i)} [\mathcal{H}_3(\alpha_i, B_i) - \mathcal{H}_3(B_i, \alpha_i)]$$

(28)

$^2$Since $E_s = E_b \log_2 M$, $\overline{P}_{b_1} \approx \overline{P}_{s_1} / \log_2 M$. 

where
$$\mathcal{H}_3(x, y) = \Gamma(x) \Gamma \left( x + \frac{1}{2} \right) \left( \frac{A_\gamma}{B} \right)^x p \hat{F}_q \left( x, x + \frac{1}{2} ; x - y + 1, x + 1 ; \frac{A_\gamma}{B} \right).$$

For non-coherent binary frequency shift keying (BFSK) and differential binary PSK (DBPSK), $P_e(\gamma) = A \exp(-B \gamma)$. Following a similar procedure as for deriving (12), $P_{s1}$ can be expressed in closed form as
$$P_{s1} = AW \left( \frac{A_\gamma}{B}, \alpha_i, B_i \right). \quad (29)$$

Furthermore, $P_{b2}$ can be easily obtained using the MGF expressions derived in the previous section, that is (17) for the CH selection algorithm and (22) for the CH switching algorithm, and following the MGF-based approach [12]. Specifically, the BEP can be calculated: i) directly for non-coherent DBPSK, that is $P_{b2} = 0.5 \mathcal{M}_{\gamma_{\text{out}}}(1)$; and ii) via numerical integration for Gray encoded $M$-PSK, that is $P_{b2} = \frac{1}{\pi \log_2 M} \int_0^{\pi/M} \mathcal{M}_{\gamma_{\text{out}}} \left[ \log_2 M \frac{\sin^2(\pi M) \sin^2 \phi}{\sin^2 \phi} \right] d\phi$. Moreover, for the CH switching algorithm, the optimum value of $\gamma_T$, that is $\gamma_T^*$, which minimizes the BEP can be obtained by numerically solving $[\theta P_{b1}(E)/\theta \gamma_T]_{\gamma_T^*} = 0$ [12, pp. 428].

6 Numerical Results and Discussion

In this section, selected numerical performance evaluation results are presented and discussed. These results include performance comparisons of several communication scenarios, employing various performance criteria and different fading and shadowing channel conditions. Firstly, E2E system performance results are presented, in terms of OP and BEP. Then, special attention is given to the CH-nodes communication link (link 2), where its performance is thoroughly investigated in order to better understand the performance improvement of CH-selection algorithms in various communication scenarios. The parameters considered in each communication scenario, can be found in Table 1. It should be noted that when small values for $m$ (or $\alpha$) are considered, strong multipath fading (or shadowing) effects are present and vice versa.

The rate of convergence of the infinite series expressions in (17) has been investigated in Table 2. In this table, the minimum number of terms, $N_{\text{min}}$, for convergence of (17) in the range of $\pm 10^{-4}$% are provided, considering iid fading conditions, different values of $m, \alpha, \gamma, \rho$ and $L = 2$. It is clear that only a few terms are needed in order to achieve the target accuracy, while the required terms increase by increasing $m, \alpha$ and/or decreasing $\gamma, \rho$. Our research has also shown that similar rates of convergence were also obtained for different values of $L$.

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For the iid fading conditions $m_i = m, \alpha_i = \alpha, \rho_i = \rho, L_i = L, \gamma_i = \gamma$ for $i = 1, 2$. 

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6.1 E2E Performance Results

In this subsection, iid fading and shadowing conditions have been also assumed. In Fig. 4, considering CHs with $L_2 = 2$ antennas (with $\rho = 0.2$), various values of the number of antennas $L_1$ (with $\rho = 0.2$) that the AP supports and DBPSK modulation scheme, the average BEP is plotted as a function of the average SNR per hop. Furthermore, the following fading conditions have been considered $m = 2$, $\alpha = 1.5$ for both links, while the investigation concerns a) single CH communication, b) CH with selection algorithm, and c) CH with switching. In this figure, it is clearly depicted that the worst performance is obtained in the single CH scenario, while the best is obtained when CH selection algorithm is employed. It is also depicted that the performance also improves when the number of the AP branches increases. In Fig. 5, considering the CH selection algorithm, the OP is plotted as a function of the normalized SNR per hop for different fading conditions and number of antennas employed in the AP and the CHs. More specifically, we have considered severe fading/shadowing conditions, that is $m = 1, \alpha = 1$, light fading/shadowing conditions $m = 3, \alpha = 3$. In this figure, it is illustrated that in all cases (for $L_1 = 4, L_2 = 2$ and for $L_1 = 2, L_2 = 4$) the best performance is obtained when light fading conditions are considered in link 1 and heavy fading conditions are considered in link 2. This is mainly due to the fact that the negative consequences of fading/shadowing have been effectively countermeasured using the proposed CH selection algorithm. It should be noted that similar performance result observations have been also reported in [37,38]. Specifically, in both these works it was verified that the system performance, in both terms of BEP and OP, increases in case that the number of receiver antennas employed also increases.

6.2 CH-nodes Performance Results

In Fig. 6, in order to investigate the energy efficiency that the CH selection algorithms induce to the nodes, the normalized power consumption is plotted as a function to the BEP of DBPSK. We have assumed $m = 2$, $L_2 = 2$, $\alpha = 2$ for all CH communication link, several values of $\rho$ and optimum values for the $\gamma_r$ for the case of switching algorithm. In this figure it is shown that for the same QoS target, the lowest normalized power consumption performance is obtained when the CH selection algorithm is used, with, however, the switching algorithm having quite close performance. Another interesting observation in this figure is that the power consumption also increases when the correlation coefficient of the multipath channel increases, since the diversity gain that is induced by the MAs reduces.

The normalization is defined by dividing the consumed power by the highest power consumed for obtaining BEP equal to $10^{-9}$.
In Fig. 7, the OP of the CH selection algorithm is plotted as a function of $\alpha$, for several values of the average input SNR and for iid and independent but non identical distributed (ind) fading parameters. More specifically, for the ind case we have considered $m_1 = 1$, $\rho_1 = 0.7$ and $m_2 = 4$, $\rho_2 = 0.1$, for the iid case we have assumed for both CHs $m = (m_1+m_2)/2$, $\rho = (\rho_1+\rho_2)/2$, while in all cases $L = 2$ and $\alpha_i = \alpha$.

It can be easily observed that the performance improves by increasing $\alpha$, that is the shadowing effects lessen, with however decreasing rate of improvement. Another interesting observation is that the best performance is obtained when iid fading conditions are considered.

Finally, in Fig. 8, the OP of the CH selection algorithm is plotted as a function of $\rho$, considering ind and iid fading conditions, and several values of $L$. More specifically, for the ind case we have considered $a_1 = 1, m_1 = 1, a_2 = 3, m_2 = 4$, for the iid case we have $m = (m_1+m_2)/2$, $\alpha = (\alpha_1+\alpha_2)/2$, while for both cases $\rho_i = \rho$ and $L_i = L$. For comparison purposes the corresponding performance of single CH with MRC (or MRT) case is also illustrated. In this figure, it is depicted that the performance decreases as $\rho$ increases, while the best performance is also obtained in case of iid fading. Furthermore, for both cases of selection algorithms, it is important to note the performance improvement when CH selection is employed, as compared to single CH reception. For comparison purposes, computer simulation performance results are also included in the figures, verifying the validity of the proposed theoretical approach.

7 Conclusions

In this paper considering a composite fading environment (modelled by the exponential-correlated Nakagami-$m$ fading and gamma distributed shadowing), a communication scenario where an AP is connected to the nodes through CHs, employing DF strategy, is studied. In this context, CH selection algorithms were proposed, where the nodes can switch between different CHs, according to the corresponding signal strength, in order to maintain a predefined QoS constraint. Additionally, it was shown that when CHs are equipped with MA capabilities the negative consequences of multipath fading/shadowing are further degrading. Employing the proposed scheme, the reliability of CH-based networks significantly increases, while the power consumption decreases. These improvements are more apparent in cases where the connection to a CH may be not possible, due to multipath and/or shadowing effects, for example in a mobile scenario. The theoretical results were validated via computer simulations. Our future research activities will include the scenario where a node may select a CH to connect to among $L$ available as well as simpler diversity reception techniques.
References


**Tables’ Captions**

Table 1: Parameter Values for the Communication Scenario Considered.

Table 2: Minimum number of terms, $N_{\text{min}}$, for convergence of (17) in Range of $\pm10^{-4\%}$.

**Figures’ Captions**

Fig. 1: System model.

Fig. 2: Comparison between proposed PDF and CDF and the corresponding ones obtained when lognormal shadowing is present.

Fig. 3: The mode of operation.

Fig. 4: The E2E average BEP versus the SNR per hop for several values of $L_1$.

Fig. 5: The E2E OP versus the normalized outage threshold per hop, $\gamma_{\text{th}}/\overline{\gamma}$, for various fading conditions.

Fig. 6: The node normalized power consumption as a function of the average BEP, for several values of the correlation coefficient.

Fig. 7: The OP of CH selection algorithm as a function of the shadowing shaping parameter $\alpha$.

Fig. 8: The node normalized power consumption as a function of the average BEP, for several values of the correlation coefficient.
Table 1: Parameter Values for the Communication Scenario Considered.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values (scenario dependent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>APs antennas</td>
<td>$L_1 \in (2, 4)$</td>
</tr>
<tr>
<td>CHs antennas (for the AP link)</td>
<td>1</td>
</tr>
<tr>
<td>CHs antennas (for the nodes link)</td>
<td>$L_2 \in (2, 4)$</td>
</tr>
<tr>
<td>nodes antennas</td>
<td>1</td>
</tr>
<tr>
<td>correlation among AP antennas</td>
<td>$\rho \in (0, 1)$</td>
</tr>
<tr>
<td>correlation among CHs antennas</td>
<td>$\rho \in (0, 1)$</td>
</tr>
<tr>
<td>topology</td>
<td>randomly deployed APs, CHs and nodes</td>
</tr>
<tr>
<td>modulation scheme</td>
<td>DBPSK</td>
</tr>
<tr>
<td>fading parameter</td>
<td>$m \in (1, 4)$</td>
</tr>
<tr>
<td>shadowing parameter</td>
<td>$\alpha \in (0.5, 8)$</td>
</tr>
</tbody>
</table>
Table 2: $N_{\text{min}}$ for convergence of (17) in the range of ±10⁻⁴.

<table>
<thead>
<tr>
<th>$\gamma$ (dB)</th>
<th>$\rho = 0.3$</th>
<th>$\rho = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m = 1, \alpha = 1$</td>
<td>$m = 1, \alpha = 1$</td>
</tr>
<tr>
<td>5</td>
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<td>10</td>
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<td>7</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Note: $\gamma$ values correspond to the dB values listed.
Figure 1: System model.
Figure 2: Comparison between the proposed PDF and CDF and the corresponding ones obtained when lognormal shadowing is present.
Start, $t=0$

One of the two CHs is selected

Estimate the instantaneous SNR of the selected CH

$\gamma_{out,t} = \gamma_{select}$

$t = t + 1$

Select the other CH

Is it above threshold $\gamma_{select} > \gamma_i$?

Yes

Remain to the selected CH

No

$t = t + 1$

Estimate the instantaneous SNR of all CHs $\gamma_i, i \in \{1,2,\ldots,N\}$

Select the CH to be connected to according to $\gamma_{out} = \max(\gamma_1, \gamma_2, \ldots, \gamma_N)$

$t = t + 1$

Figure 3: The mode of operation
Figure 4: The E2E average BEP versus the SNR per hop for several values of $L_1$. 
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Figure 6: The node normalized power consumption as a function of the average BEP, for several values of the correlation coefficient.
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