The study of solar flares with the extended cellular automaton (X-CA) model

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Abstract

We have developed a new type of cellular automaton (CA) model, the *extended CA* (X-CA), for the study of solar flares. The X-CA model is consistent with the MHD approach, in contrast to the previously proposed CA models (classical CAs), it consistently yields all the relevant physical variables, and it successfully reproduces the observed distributions of total energy, peak flux, and duration of solar flares. We present and discuss the relevant plasma processes and set-up which are implemented by the X-CA model in the framework of a commonly accepted solar flare scenario.

1 Introduction

It is now widely accepted that the solar activity is mainly due to the dissipation of the magnetic energy stored previously in the solar corona. Magnetic energy release manifests itself in a large variety of phenomena, ranging from heating to solar flares. Observations of the radiation signatures of solar energetic particles (especially in the hard X-rays) indicate that the energy release process is fragmented into a large number of sub-events (Vilmer 1993; Trottet 1994; Aschwanden et al. 1995, Vilmer and Trottet 1997). In the last two decades, the availability of several space-born solar instruments, together with the existence of several ground based telescopes, have given the opportunity to perform a number of statistical studies of the flaring activity of the Sun. These types of observations (Dennis 1985; Vilmer 1987; Pick et al. 1990; Crosby et al. 1993; Crosby et al. 1998) revealed that the frequency-distributions of flares as a function of total energy, peak luminosity, and duration are well-defined power laws, extending over several orders of magnitude.

Traditionally, the study of the energy release process was relying on the magnetohydrodynamic (MHD) theory. In this framework, the most common approach was the development of numerical simulation codes, given the complex nature of the energy release problem in the solar corona, which involves strongly nonlinear effects (Einaudi et al. 1996; Georgoulis et al. 1998, and references therein). In MHD numerical simulations, the energy release process (i.e. magnetic reconnection) is simulated in detail, but these simulations are time consuming and can only treat a small number of reconnection events (bursts) and relatively small volumes, leading thus to poor statistics for comparison to solar flare observations.

Alternatively, several cellular automaton (CA) models, based on the approach used in complex dynamical system theory, have been developed in order to explain the solar flare statistics derived from the observations (Lu and Hamilton 1991; Lu et al. 1993; Vlahos et al. 1995; Galsgaard 1996; Georgoulis and Vlahos 1996; 1998). We will term these models *classical* CA models. The classical CA models simulate the energy storage/release process using simple evolution rules, neglecting the details of the processes. These models allow the global modeling of solar active regions, at the expense of understanding in details the underlying physics. The main advantage of them is that they can treat a large number of elementary energy-release events (avalanches) and relatively large volumes, and that they can

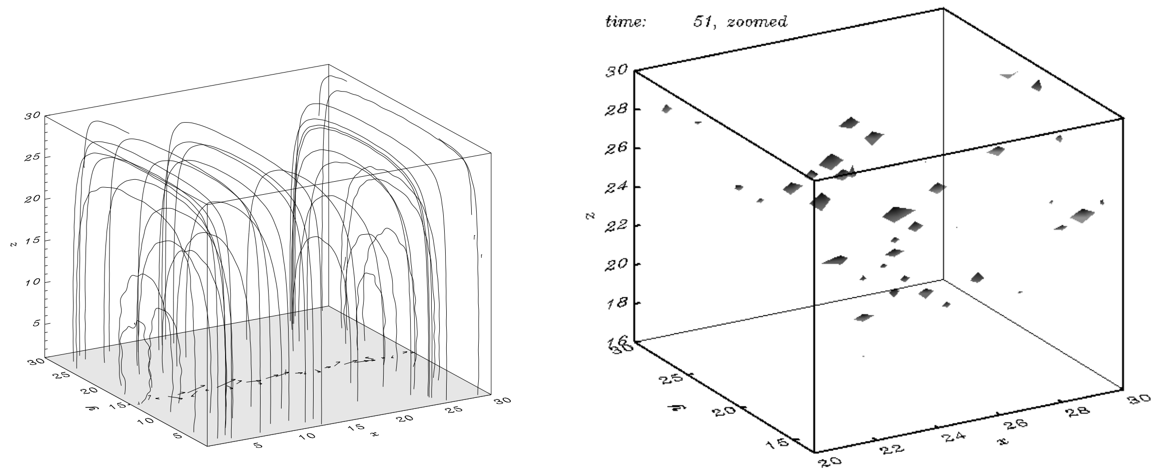

Figure 1. Left: The magnetic topology of the X-CA model, together with a subset of the current vectors.
Right: Current dissipation regions (i.e. volumes enclosing regions of super-critical current) at a temporal snap-shot during a flare.

explain the power-law frequency distributions of the solar flare parameters (see e..g. Georgoulis et al. 2001).

2 Description of the X-CA Model

In our previous work (Isliker et al. 1998), we tried to solve the problem of the vague association of the classical CA model's components with physical variables and processes, which was their most important disadvantage. Our goal was to give a physical interpretation to the classical CAs and to try to connect the two complementary approaches (MHD and CA) for the solar flare problem. Our study revealed the role of several components of the classical CA models, such as the role of the diffusivity, the units used, and the nature of the energy release process. On the other hand, several unsatisfying properties of the classical CA models have been found, such as inconsistencies with the MHD equations (e.g. $\nabla\mathbf{B}$ is uncontrolled), and the non-availability of secondary variables (current density $\mathbf{J}$, electric field $\mathbf{E}$). In order to overcome these problems, we have constructed a new class of CA models for solar flares, termed *extended* CA (X-CA) model.

The X-CA model consists in the combination of a classical CA model with a set-up which is super-imposed onto the classical CA (for details see Isliker et al. 2000, 2001): We use a 3-D cubic grid ($30 \times 30 \times 30$ in the following) with the vector potential $\mathbf{A}$ as the primary grid variable. The magnetic filed $\mathbf{B}$ and the current density $\mathbf{J}$ are secondary variables, as in MHD, and given by Maxwell's equations,

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{1}$$

(which ensures that $\nabla\mathbf{B} = \mathbf{0}$), and

$$\mathbf{J} = \frac{\mathbf{c}}{\mathbf{4}\pi} \nabla \times \mathbf{B}. \tag{2}$$

The electric field is determined by the simple Ohm's law,

$$\mathbf{E} = \eta \mathbf{J}. \tag{3}$$

The central problem is how to calculate the derivatives, since CAs are by their nature discrete models. We differentiate $\mathbf{A}$ and $\mathbf{B}$ by using 3-D cubic spline interpolation, since, evaluating different methods, it turned out that calculating the derivatives in this way has remarkable advantages over other methods.

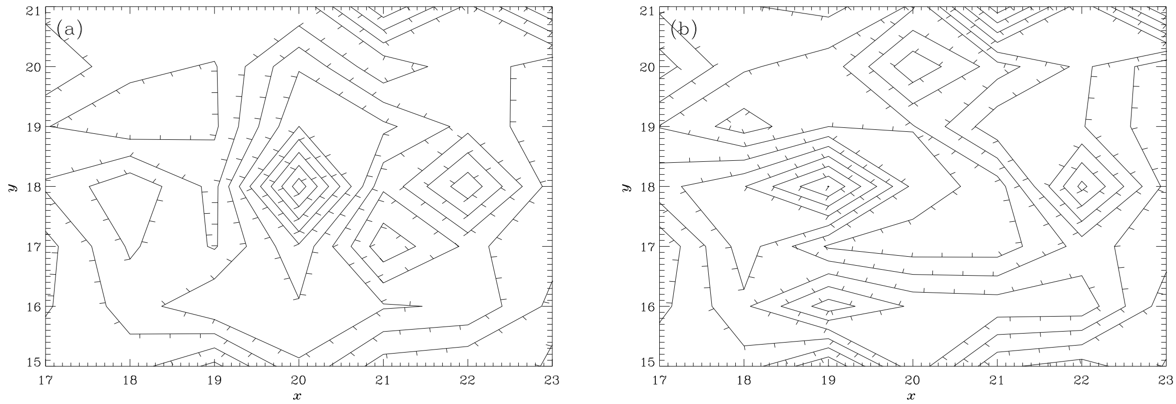

Figure 2. Contour-plot of the current-density, before (left), and after (right) a burst, which occurs in the middle of the plot (the z-coordinate is fixed).

The dynamic evolution of the X-CA has two distinct phases. In the loading phase, random field increments $\delta\mathbf{A}$ are dropped at random sites. If a local instability criterion is fulfilled, local bursts (relaxing redistributions of $\mathbf{A}$) are triggered, during which energy is released, whose amount is determined as Ohmic dissipation ($\sim \eta\mathbf{J}^{\mathbf{2}}$). As a critical quantity either $d\mathbf{A}$, the stress in $\mathbf{A}$, as defined in the classical CAs (e.g. Lu & Hamilton 1991), or the current density $\mathbf{J}$ is used.

3 Results and Conclusions

The X-CA model yields power-law frequency distributions for the diverse flare parameters which are compatible with the observations, and, unlike the classical CAs, the X-CA yields insight into the set-up and the physical processes during flares:
1. The vector-potential, magnetic field, and current exhibit large-scale organization and quasi-symmetries. Depending on the directionality of loading and the boundary conditions, the global topology of the magnetic field has two varieties: Either it forms an arcade of magnetic field lines (Fig. 1, left), or it forms closed magnetic field lines around and along a neutral line.
2. The temporal evolution of the model follows the MHD induction equation, which, expressed in terms of the vector potential, writes

$$\frac{\partial\mathbf{A}}{\partial t} == \mathbf{v}\wedge\mathbf{B} + \eta\frac{\mathbf{c}^{\mathbf{2}}}{\mathbf{4}\pi}\nabla^{\mathbf{2}}\mathbf{A} - \eta\frac{\mathbf{c}^{\mathbf{2}}}{\mathbf{4}\pi}\nabla(\nabla\mathbf{A}) + \nabla\chi, \tag{4}$$

where η is the diffusivity and χ an arbitrary function.
2.a) The loading increments are of the form $\delta\mathbf{A} \sim \mathbf{v}\wedge\mathbf{B}$, with $\mathbf{v}$ the implicitly assumed velocity of a plasma which flows upwards and expands, so that loading implements the action of the convective term in Eq. (3).
2.b) Bursts are localized diffusion processes, they implement the solution to the diffusive part of Eq. (3) in one time-step, with some characteristic simplifications, though.
3. Bursts occur there where the currents are large, and after a burst the local current is relaxed (Fig. 2). An important innovation of the X-CA model is the direct use of the current in the instability criterion. With this modification, the X-CA directly implements Parker's (1993) flare scenario that an instability is triggered if the current $\mathbf{J}$ exceeds some threshold, since this implies that, through a current driven instability, the resistivity increases to anomalous values, and diffusion dominates over convection in the time evolution.
4. Important for future applications to particle acceleration is that the X-CA makes the electric fields explicitly available as a function of time and space.

5. The current-dissipation is spatially and temporally fragmented into a large number of practically independent, dispersed, and disconnected dissipation regions with the shape of current-surfaces, which vary in size and are spread over a considerable volume (Fig. 1, right).

In conclusion, the X-CA model is a model for energy release through current dissipation, and it represents a realization of Parker's (1993) flare scenario. It allows interesting future developments, one of which is the introduction of particles into the system, which will allow to model the radiation signatures of the flaring plasma and the accelerated particles. This will put the comparison of CA models to observations on new grounds.

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