

# A SOLAR FLARE MODEL IN BETWEEN MHD AND CELLULAR AUTOMATON\*

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## ABSTRACT

The last decade several cellular automata (CA) models have been developed in order to explain the solar flare statistics derived from observations. These models simulate the storage/release process using simple evolution rules, neglecting the details of the processes. The main advantage of this approach is the treatment of a large number of elementary energy release events (avalanches) and the reproduction of the observed solar flare statistics. On the other hand the energy release process has been modeled for years using MHD numerical simulations. In the MHD approach the magnetic reconnection (the energy release process) is simulated in detail, but it fails to treat a large number of reconnection events, which leads to poor statistics for comparison to solar flares. We present here preliminary results of a new type of CA model for solar flares, based on the insights accumulated in MHD simulations and the analysis of the so-far existing CA models. Our goal is to connect the two seemingly totally different, complementary approaches (CA and MHD) for the solar flare problem.

## 1. Introduction

One of the most interesting and still unresolved problems in solar physics is the nature of energy release in the solar atmosphere, which is the driver of the coronal flare activity and possibly of coronal heating. It is believed that flaring activity is mainly due to the dissipation of magnetic energy stored previously in the corona. Observations indicate that this energy release process is fragmented into a large number of sub-events (Kuijpers et al. 1981; Benz 1985; Vilmer 1993; Aschwanden et al. 1995). Concerning the statistical behavior of flaring activity, observations also revealed that the frequency distributions of flares as a function of total energy, peak luminosity, and duration are well-defined power laws, extending over several orders of magnitude (Denis 1985; Vilmer 1987; Ramaty & Murphy 1987; Pick et al. 1990; Crosby et al. 1998).

In the last decade several cellular automaton (CA) models have been developed in order to

explain the solar flare statistics derived from the observations (Lu & Hamilton 1991; Lu et al. 1993; Vlahos et al. 1995; Galsgaard 1996; Georgoulis & Vlahos 1996). These models simulate the storage/release processes using simple evolution rules, neglecting the details of the process. The main advantage of the CA models is that they can treat a large number of elementary energy-release events (in the form of avalanches) and reproduce the observed solar flare statistics. On the other hand, solar flares have been modeled for years using numerical MHD simulations (Einaudi et al. 1996; Georgoulis et al. 1998; and references therein). In this approach the energy release process (magnetic reconnection) is simulated in detail, but these simulations are time consuming and can only treat a small number of reconnection events (bursts), leading to poor statistics for comparison to solar flare observations.

The main question we have posed, the last two years, was how the CA approach can be

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interpreted as a simplified and/or modified MHD approach (Islaker et al. 1998; Vassiliadis et al. 1998). Our goal was to give a physical interpretation of the existing CA models and to connect the two seemingly totally different, complementary approaches (CA and MHD) for the solar flare problem. Our work (Islaker et al. 1998) allowed the extraction of more information from the existing solar flare CA models (e.g. the role of diffusivity, the physical units used, a deeper insight into the assumptions on the energy-release process). On the other hand, several unsatisfying properties of the currently existing CA models have been revealed, (e.g.  $\nabla\mathbf{B}$  is uncontrolled and the interpretation of the basic grid variable is unclear), making thus clear the necessity of an improved CA-model for solar flares.

Our aim is to construct a model for solar flares which is hybrid: It should be a CA model, with all the advantages of CA, namely that it is fast, it models large events, therewith it allows for good statistics. On the other hand, our model should be as much consistent with MHD as possible, with full control over all variables.

This paper is organized as follows: in the next section the basic characteristics (processes) of our model are presented, followed by the overall structure and the necessary definitions (Section 3). In Section 4 we present some preliminary results concerning the solar flare statistics. Finally, in Section 5 we summarize our work and discuss some of the open problems that we plan to address in the near future.

## 2. Description of the Model

Our model is constructed for a slab geometry, with 2-D vector-fields in a 2-D space  $(x,y)$ , assuming that in the  $z$ -direction changes are negligible. The 2-D grid is quadratic, with the basic grid-variables the  $z$ -component of the vector potential  $\mathbf{A}$  and a 2-D velocity field  $\mathbf{u}$ .

*1. Initial Setup.* We start with a random vector potential in the plane  $(x,y)$  with  $A_z(x,y) \in [-0.3,0.3]$ , an initial random, spatially uncorrelated disturbance. According to MHD, the magnetic field  $\mathbf{B}$  and the current  $\mathbf{J}$  are secondary grid-variables, namely derivatives of  $\mathbf{A}$ :  $\mathbf{B}=\nabla\times\mathbf{A}$  and  $4\pi/c \mathbf{J}=\nabla\times\mathbf{B}$ . The above derivatives are calculated by performing a 2-D

cubic spline interpolation through the entire grid. This is done to have MHD consistent definitions of  $\mathbf{B}$  and  $\mathbf{J}$  and in order to secure the equation  $\nabla\mathbf{B}=0$  which is not present in the so far existing CA models. The velocity field (the second primary grid variable) is a random function, uncorrelated in time and space, with a probability distribution of power law form with index  $-1.8$  in the range  $[u_1,u_2]$ , corresponding to Alfvén velocity values of magnetic field from 1G to 300 G.

*2. Temporal Evolution.* The evolution of the vector potential is according to the induction equation, expressed in terms of the vector potential:

$$\frac{\partial\mathbf{A}}{\partial t} = (\mathbf{u} \times \mathbf{B}) + n\nabla^2\mathbf{A} - n\nabla(\nabla\mathbf{A}) \quad (1)$$

where  $n$  is the diffusivity. The first term on the right hand side of the above equation (convective term) dominates the loading phase of our CA model ( $n$  is generally small in the corona). The second and the third terms (diffusive terms) on the right hand side dominate in the bursting phase, where  $n$  due to some instability has drastically increased.

*2.1 Loading.* In the loading phase,  $A_z$  evolves only according to the convective term of eq. (1), which is implemented in the form of an iteration with

$$\delta A_z(t, \mathbf{x}) = (\mathbf{u}(t, \mathbf{x}) \times \mathbf{B}(t, \mathbf{x}))_z \delta t \quad (2)$$

so that

$$A_z(t+1, \mathbf{x}) = A_z(t, \mathbf{x}) + \delta A_z(t, \mathbf{x}) \quad (3)$$

with  $\delta t$  a free parameter. We note that using the above approach no field is added to our grid, but the existing field is shuffled, deformed through the fluid and that the loading depends on the pre-existing magnetic field  $\mathbf{B}$ .

*2.2. Instability Criterion.* The loading phase is iterated until a local, current dependent threshold is exceeded, which causes the diffusivity  $n$  to increase (assuming that  $n$  has become anomalous) and the diffusion process

to dominate over the convection. We use the following instability criterion

$$\frac{\sigma}{|\sigma|} |\mathbf{J}(\mathbf{x})| > J_{cr} \quad (4)$$

where  $\sigma$  is a topological factor which is positive if  $A_z$  is convex in one direction and anti-convex in the other direction, else it is negative. This factor identifies with its sign current sheet like topologies with scale length the grid size.

**2.3 Bursting.** If the threshold is exceeded ( eq. (4) is fulfilled) at a grid site, then  $A_z$  evolves solely according to the diffusive part of the induction equation (eq.(1)) in the local neighborhood of this site (the site plus its 4 nearest neighbors). The redistribution rules are derived as the asymptotic solution of the diffusive equation with fixed boundaries and can be summarized as follows:

$$\begin{aligned} A_z(t, x_{n,n}) &\rightarrow A_z(t+1, x_{n,n}) = \\ &= -\langle B_y^{before} \rangle_{n,n} (x_{n,n} - x_{ij}) + \\ &+ \langle B_x^{before} \rangle_{n,n} (y_{n,n} - y_{ij}) + f(z) \end{aligned} \quad (5)$$

where  $x_{n,n}$  runs over the four nearest neighbors and the central point  $x_{ij}$ , and  $\langle B^{before} \rangle_{n,n}$  is the average value of the magnetic field before the burst in the local neighborhood. We note that this redistribution rule is similar to Lu & Hamilton (1991) but not equal, as in their case the magnetic field is conserved.

**3. Triggering.** We assume that in case a burst occurs a disturbance may travel through the grid. This is possible if we assume that waves or beams of accelerated particles can travel away from the activated site, reaching some random grid sites. The number of sites to be triggered is a free parameter that controls the activity level of our model. We model the triggering the same way as the loading (using eq. (2)-(3)) but with the upper boundary for the velocity  $u_2=c/3$ , in order to mimic loosely the propagation of fast electrons.

### 3. Structure of the CA - Definitions

The overall structure of our CA model, in numbering steps, is

1) initializing

- 2) loading
- 3) scanning: searching for unstable sites. If there are none, return to loading (2)
- 4) scanning and bursting: redistribute the unstable sites, identified in the scanning (3) or (6).
- 5) triggering: only if bursts occurred in (4)
- 6) scanning: search for unstable sites. If there are any go to bursting (4), else return to loading (2).

Following, we define the time step, the released energy per time step, a burst, a flare, the duration of a flare, the total flare energy and the peak-flux of a flare. These definitions are important for the solar flare statistics.

As one *time step* in our model is considered the steps (4) and (5) together. The *released energy per time step* is the sum of all the energy released by bursts in this time step. A *burst* is considered a single redistribution event in step 4. We term a *flare* the loop (4,5,6) from the occurring of the first burst in (4) until the activity has died out and the model has returned via the scanning (6) to loading (2). The *duration* of the flare is the number of time steps it lasted. The *total flare energy* is the sum of all energies released in the duration of the flare. The *peak-flux* is the maximum of the energies of all the time steps of the flare.

## 4. Results

In this Section we present some preliminary results from our CA model. All results presented are for a 20x20 spatial grid.

For visualization of the grid and the redistribution rules, in Figures 1 and 2 the interpolated magnetic field lines are presented for the half grid size. The two figures have one time step difference. In Figure 1 the site (7,8) is unstable. Note the field line structures of that site after one time step in Figure 2. As the sight was unstable, it relaxed by redistributing the magnetic field, affecting its neighborhood.

In Figure 3 the frequency distribution of the total flare energy  $E_{tot}$  (solid line) and the peak-flux  $E_{peak}$  (dashed line) of 1000 flares are presented. The x-axis is energy in ergs. Both frequency distributions have an extended power law shape with index  $-1.87$  for  $E_{tot}$  and  $-2.12$  for  $E_{peak}$ .

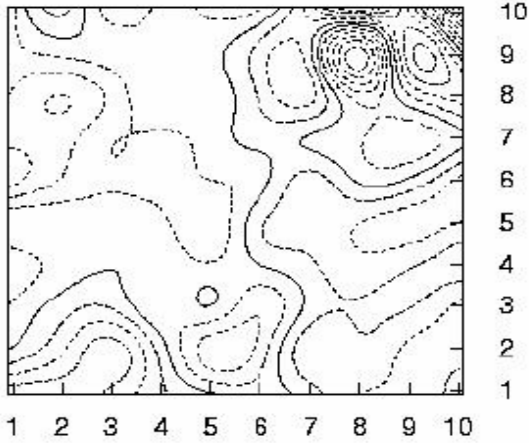


Figure 1: Magnetic Field line plot of a region in our CA model at a given time  $t$ . The sight (7,8) is unstable, as the threshold is exceeded.

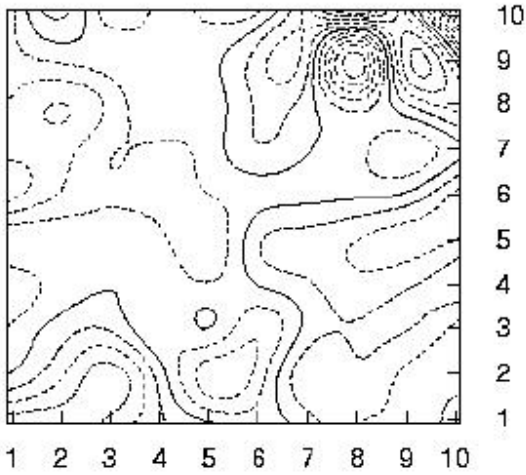


Figure 2: The same as in Figure 1, but for time  $t+1$ . Note the different structure of the magnetic field lines at the neighborhood of the sight (7,8).

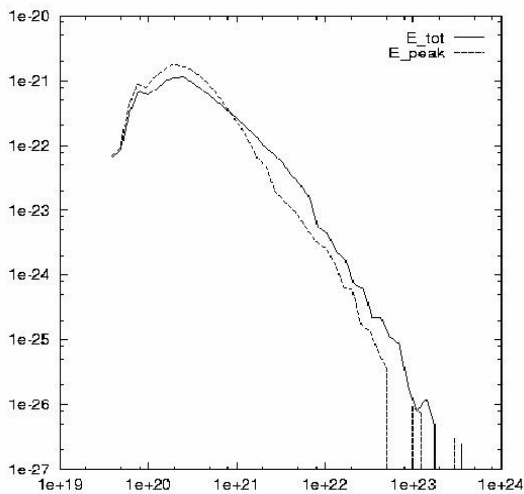


Figure 3: The frequency distributions of the total flare energy  $E_{tot}$  and the peak-flux  $E_{peak}$  for 1000 flares. The x-axis is energy in ergs.

Finally, in Figure 4 the duration distribution of the 1000 flares is presented. Note that the x-axis is in time steps.

## 5. Summary and Discussion

We have presented preliminary results from a new type of CA model that we have constructed at the interplay of CA and MHD. Our aim was to remove many unsatisfying properties of the currently existing CA models that we have revealed in our previous work (Islaker et al. 1998) and to connect the MHD approach with the CA one. For instance the interpretation of the grid variables is clear and consistent, and  $\nabla \mathbf{B} = 0$  is fulfilled. We believe that this kind of connection can allow us in the near future to perform studies, not only on the solar flare statistics problem but also on the problems of radiation and/or acceleration of particles in solar flares (see for example Anastasiadis, Vlahos and Georgoulis 1997).

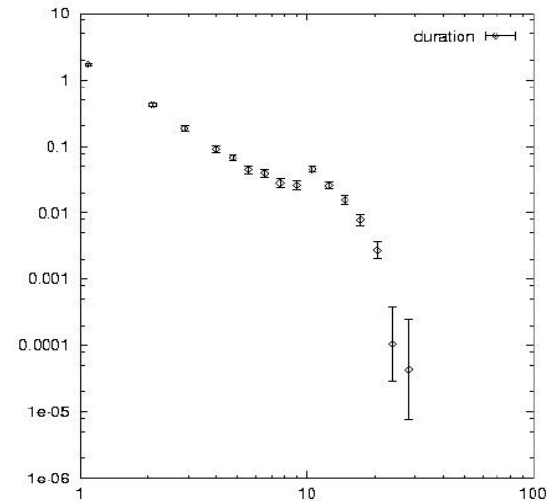


Figure 4: The duration distribution of 1000 flares. The x-axis is in time steps (arbitrary units).

Our model is able to produce extended power laws for the frequency distributions of total released energy and peak-flux for solar flares. In order these results to be consistent with the recent observations of solar flare statistics an extensive parametric study, in respect to the free parameters of the model, is needed. The main free parameters in our model are: the intensity of the loading (the  $\delta t$ ), the distribution of the turbulent velocity field (index and range), the number of triggered burst sights, the threshold for bursting  $J_{cr}$  and the percentage of the grid which is loaded in each loading step.

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