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Particle acceleration by multiple shocks at the hot spots of extragalactic radio sources

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Abstract. We present a model for the acceleration of energetic electrons by an ensemble of oblique shock waves at the hot spots of extended extragalactic radio sources. The energization of the electrons is based on the "shock drift" acceleration mechanism, using the adiabatic treatment, and electrons are subject to synchrotron losses as they travel between the shock fronts. We calculate numerically the energy distribution of electrons and the corresponding intensity of the radiation produced. We compare our results with the analytical solution of the Fokker - Planck equation. Our numerical and analytical results agree well with the existing observations.

Key words: radio galaxies – radio hot spots – particle acceleration – synchrotron radiation – shock waves

1. Introduction

A challenging problem in extragalactic radio astronomy is the need for *in situ* acceleration of energetic electrons in jets. The synchrotron half-life time for the electrons is small compared to the crossing life time of the jet. In this paper we deal with the problem of in situ acceleration of energetic electrons by an ensemble of oblique shock waves and their simultaneous synchrotron losses inside the hot spots of the extended extragalactic radio sources.

Qualitatively a hot spot is a region of a high surface brightness temperature at the outer end of a radio lobe of an extended extragalactic radio source (for review see Perly 1989). The linear dimension of this region is $\approx$ 1-10 kpc. The spectral index of the synchrotron radiation for the hot spots is $0.5 \leq \alpha \leq 1.0$. Hot spots are assumed to be associated with the downstream regions of oblique shock waves (Laing 1982; Lonsdale & Barther 1984), where in situ particle acceleration is taking place. They appear in several morphologies e.g. compact structures or regions with multiple structures inside. One such source was modelled by Rudnik (1988) and the structure inside the hot spot was interpreted as secondary shocks following the bow shock.

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Numerical simulations of non relativistic hydromagnetic flows for the case of jet plasmas indicate that the hot spots are energized by planar terminating mach disk shock (e.g., Norman et al. 1985). The breaking of the assumption of axisymetry in the numerical simulations would lead to a more complex jet termination involving a number of oblique shock waves. We believe that the observations of the hot spots which indicate on one hand, an increase of the surface brightness, and on the other hand, the existence of multiple structures lead to the assumption that *in situ* acceleration of electrons is taking place by multiple oblique shock waves which are present inside the hot spots (Gopal-Krishna & Wiita 1990).

Particles, in general, can be accelerated in shock waves by either *drift* or *diffusive* acceleration mechanism. In the case of the diffusive mechanism, the particle is accelerated as it scatters many times back and forth across the shock front by magnetic irregularities which exist in the upstream and downstream region (Toptyghin 1980; Jokippii 1982; Drury 1983). This mechanism is also called first order Fermi acceleration process. Several applications of the diffusive shock acceleration have been made in order to explain the observed radiation spectrum from extragalactic radio sources (Webb et al. 1984; Heavens & Meisenheimer 1987; Fritz 1989).

In the shock drift acceleration mechanism, the particle gains energy as it moves along the electric field at the shock front (Sarris & Van Allen 1974; Armstrong et al. 1983; Decker 1989). In this case the particle interacts only once with the shock. When MHD turbulence are present upstream and downstream of an oblique shock wave the particles are accelerated from the combined action of the drift and the diffusive mechanism (Decker & Vlahos 1986). Begelman & Kirk (1990) studied the drift acceleration mechanism of electrons by a superluminar oblique shock wave. Their purpose was to present a model for the extragalactic **compact hot spots**. They calculated the resulting electron distribution far downstream from the shock front, estimated the radiation spectrum and investigated the increase on the synchrotron emissivity and the observed cut-offs.

Gopal-Krishna & Wiita (1990) presented a model for multiple shocks at the extragalactic hot spots, in order to explain the observed correlation between the spectral index and the radio luminosity. The shock waves were subject to several requirements (e.g., the compression factors were constants, or were decline monotonically from 4.0 to 2.0, etc). In their model particle acceleration and the synchrotron radiation were treated simultaneously but the electrons were accelerated only by the compression on the shock fronts.

Achterberg (1990) analysed the diffusive shock acceleration by an ensemble of **identical** shock waves. Anastasiadis & Vlahos (1991) presented a model for the acceleration of particles (ions and electrons) by an ensemble of shock waves in solar flares. In this model the shock waves were not identical and can move randomly inside the acceleration region.

The model presented in this article combines characteristics of several existing models for the acceleration of electrons in the extragalactic radio sources and it is described in Sect. 2. The basic assumptions/characteristics are

1. The existence of an ensemble of oblique shock waves
2. The application of shock drift mechanism for the acceleration of electrons
3. The consideration of synchrotron radiation losses
4. The random choice of the characteristics of the shock waves
5. The random choice of the time interval an electron needed to travel between the shock fronts, which leads to a random number of shock waves.

The results from our numerical model and the analytical calculation of the energy distribution of electrons are presented in Sect. 3. Finally in Sect. 4 we summarise our results and discuss the implications of our model.

2. The model

In this section we present the general aspects of our model. The goal of this study is to estimate the resulting energy distribution of the energetic electrons and the corresponding spectral index of the synchrotron radiation spectrum.

We assume that a number N_S of short lived oblique shock waves are present in an extragalactic hot spot with characteristic dimension L_H = 10 kpc. Each shock wave is considered to be a small scale discontinuity in respect to the overall dimension of the hot spot. Inside this region there is plasma with density $n_o = 10^{-4}$ cm^{-3}, which is moving inside a uniform ambient magnetic field $B_o = 10^{-5}$ G (Ferrari 1983). The flow velocity of the plasma is taken to be non-relativistic and the Alfven velocity is $v_a = 2.18\ 10^8$ cm s^{-1}.

The shock waves are moving with constant velocities V_{Sj} in the direction of the outer end of the hot spot. Their surfaces intersect the ambient magnetic field with different angles (θ_{Bn}). We assume that for each shock wave, the upstream plasma values are: $U_1 = V_{Sj}$, $2.5 \leq M_{A1} \leq 4.0$, $20° \leq \theta_{Bn1} \leq 60°$, $B_1 = 10^{-5}$ G and $\beta_1 = 0.35$, where U_1 is the plasma flow velocity normal to the shock front, $M_{A1} = U_1/v_a$ is the Alfvenic Mach number, θ_{Bn1} is the angle between the shock normal and the direction of the upstream magnetic filed B_1 and β_1 is the plasma parameter beta. The angle θ_{Bn1}, as well as the Mach number M_{A1}, are selected randomly inside the limits given above. It is clear that, intermediate values of θ_{Bn1} (i.e. oblique shocks) are likely to prevail in most astrophysical systems (Decker & Vlahos 1986). One can evaluate the downstream plasma parameters from the upstream ones using the MHD jump conditions, known as *Rankine-Hugoniot* relations, (Tidman & Krall 1971).

A parameter for our model is the number of oblique shock waves which are present. The shock waves are placed inside the region of the hot spot in such a way that, the time (T_S) needed by an electron to travel between them, is a random parameter of our model. Both N_S and T_S are close related to the total time t_H that the ambient plasma spends inside the extragalactic hot spot. Assume that the mean flow velocity of the ambient plasma is $V_p \simeq 7.\ 10^8$ cm s^{-1}, then the total time is $t_H = L_H/V_p \approx 10^6$ yr. It is clear that the acceleration time for the energetic electrons should be of the order of t_H.

The electrons are accelerated by the shock drift acceleration mechanism. As the plasma velocity is non-relativistic we use the *adiabatic treatment* in order to calculate the energy changes of the electrons interacting with each shock wave (Webb et al. 1983; Decker 1989). The adiabatic theory is based on the fact that the magnetic moment of a particle is conserved in a frame of reference where the motional electric field is vanished. We consider only electrons which before the shock encounter, are upstream of the shock and they are transmitted downstream. Since the angle θ_{Bn}, between the shock normal and the ambient magnetic field is random, it is not important to follow the evolution of the pitch angle of the electrons. We choose randomly the pitch angle of the electrons before the electron - shock wave interaction.

We also include synchrotron radiation losses. Assume an electron that has an energy $E_{j,S}$, after the encounter with the *j* shock wave and moves along the ambient magnetic filed B_o, in the upstream region of the (*j+1*) shock wave, losing energy due to the synchrotron radiation, until it reaches that shock front. If the travel time for this electron, between the *j* and the (*j+1*) shock wave is T_{Sj}, the electron will interact with the (*j+1*) shock wave with initial energy E_{j+1}, given by the relation (Pacholczyk 1970)

$$E_{j+1} - E_{j,S} \simeq -2.36 \times 10^{-3} B_o^2 E_{j,S}^2 T_{Sj} \qquad (1)$$

where energies are given in erg, the magnetic field is $B_o = 10^{-5}$ G and the time T_{Sj} is in s. In summary, electrons are accelerated only at the shock fronts, but synchrotron losses are active continuously, playing a very important role on the formation of the final energy distribution.

3. Results

3.1. Numerical calculations

We calculate in this section the spectral index of the synchrotron radiation spectrum and the intensity of the radiated power as a function of frequency.

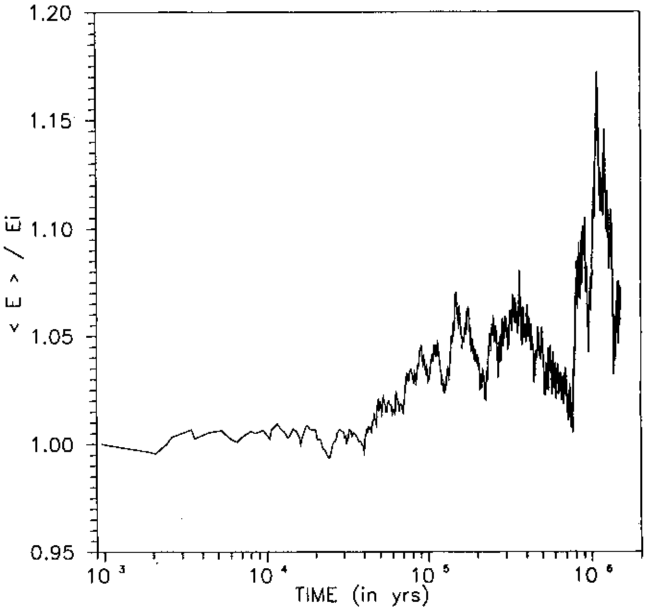

Fig. 1. The time evolution of the mean energy of electrons starting with initial energy E_i = 1 erg, for the model A (N_S = 2691 number of shock waves)

The electrons which are entering the region of the hot spot are injected from the body of the jet of the extragalactic radio source. It is then clear that the injection energy spectrum of the electrons for our model has to be the one suggested from the emitted radiation (assuming that it is synchrotron) of the body of the jet. The spectral index of the radiation for jets is usually around $\alpha_j \simeq -0.6$ (Bridle & Perly, 1984), corresponding to a power law energy distribution with exponent $z = (2|\alpha_j| + 1) \simeq 2.2$. Then the injection spectrum of the electrons used in our study is

$$N(E)dE \propto N_o E^{-z} dE \tag{2}$$

where $N(E)dE$ is the number of electrons having energies in the interval from E to $E + dE$. We can assume that the above energy spectrum holds for energies from 10^{-6} erg up to 1 erg (Gopal-Krishna & Wiita 1990).

We present four different models for the acceleration of the energetic electrons, under the influence of synchrotron losses. The time intervals T_S are selected randomly, as we stated above, taking values from $T_{\min}$ to $T_{\max}$. The corresponding values of the parameters $T_{\min}$ and $T_{\max}$ for each model, named A,B,C and D, are given in yr in Table 1.

We follow the evolution of the mean energy of electrons, which are gaining energy through their interaction with N_S shock waves and lose energy through the synchrotron emission. In Figs. 1 and 2 we present our results for the parameters of the Model A and B respectively for electrons injected with initial energy E_i = 1 erg. For the case of model A, the acceleration is the dominant process as there is a large number of shock waves present. On the other hand for the case of model B, the synchrotron radiation is playing an important role on the formation

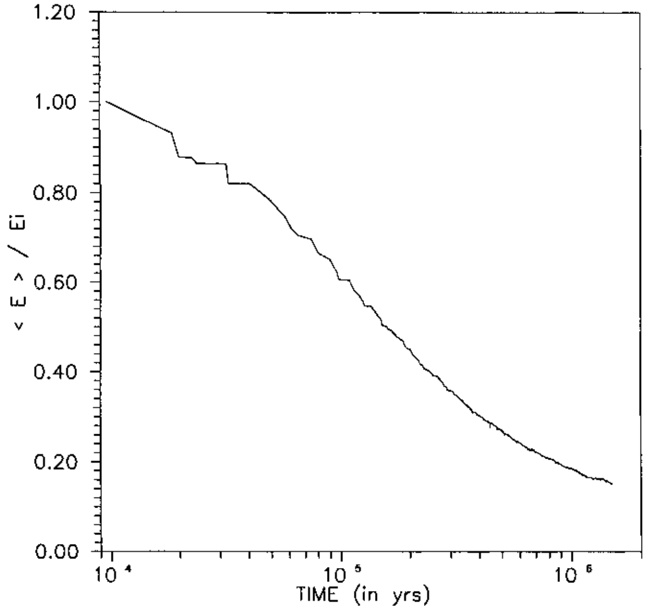

Fig. 2. The same as Fig. 1 but for the model B (N_S = 292 number of shock waves)

Table 1. Results for the Models. $T_{\min}$ and $T_{\max}$ are the boundaries for the random selection of the time interval between the shock fronts and their values are given in yr. N_S is the number of shock waves active. K is the constant of the final power law energy distribution, with an exponent x. The corresponding spectral index is a and x_a is the analytical estimated exponent for the energy distribution

Model	$T_{\min}$	$T_{\max}$	N_S	K	x	α	x_a
A	10^2	10^3	2691	$2.5\ 10^{11}$	9.258	-4.129	2.124
B	10^2	10^4	292	$7.7\ 10^3$	2.786	-0.847	2.322
C	10^2	10^5	27	$4.8\ 10^2$	2.801	-0.900	2.776
D	10^5	$3.\ 10^5$	8	$0.7\ 10^2$	2.917	-0.958	3.028

of the final distribution. Notice that the final energy distribution does not decrease exponentialy, which is characteristic of the synchrotron radiation, since a re-acceleration of the electrons is taking place.

The resulting energy distribution of the energetic electrons, after their interaction with N_S shock waves, for the model C is presented in Fig. 3. The energy distribution for each model can be fitted with a power law distribution, having the general form

$$N_f(E) \propto K E^{-x} \tag{3}$$

where the values of the constant K and the exponent x, for each model, are presented in Table 1. The time needed for the formation of the above energy distributions is $t_f = 1.5\ 10^6$ yr, which is of the order of t_H.

We can evaluate the spectral index, of the synchrotron radiation spectrum, for each model, using the relation $\alpha = -(x-1)/2$. Further more, we can calculate the intensity of

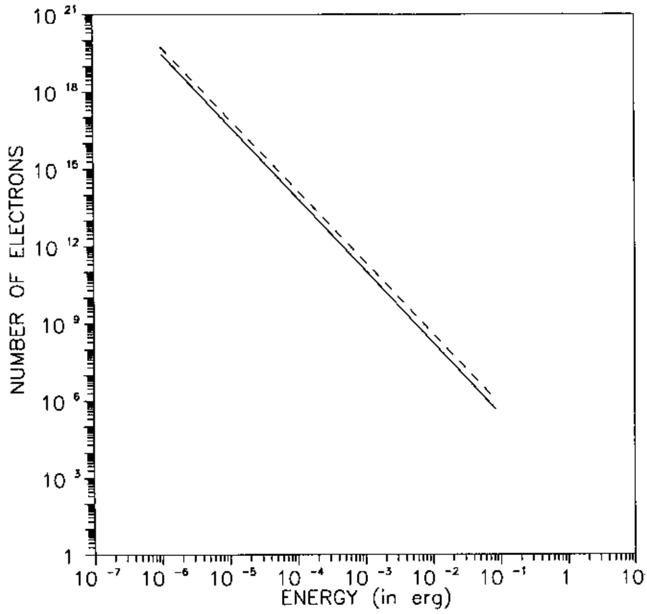

Fig. 3. The resulting energy distribution of electrons for the model parameters C (N_S = 27). The solid line corresponds to the numerical estimation of the distribution and the dashed line to the analytical one. The relative error is 0.8%

the radiated power as a function of frequency, using the formula (De Young 1984)

$$I(\nu) \approx A(x)\frac{e^3}{mc^2}\left(\frac{3e}{2\pi m^3 c^5}\right)^{(x-1)/2} K B_o^{(x+1)/2} \nu^{-(x-1)/2} \quad (4)$$

with

$$A(x) = \frac{\sqrt{3}}{4\pi(x+1)}\Gamma\left(\frac{3x-1}{12}\right)\Gamma\left(\frac{3x+19}{12}\right) \quad (5)$$

where Γ is the Euler gamma function, B_o is the ambient magnetic field, K and x are the given in Eq. (3) and ν is the frequency. In Fig. 4 we plot the intensity, which corresponds to the models B, C and D, as a function of frequency for the range from 10 MHz up to 100 GHz. The above radio band is only a portion of the possible frequency zone, in which the energetic electrons radiate. The model A is not acceptable for hot spots, since the spectral index estimated is out of the observed range ($0.5 \leq \alpha \leq 1$). In this case the acceleration of electrons is the dominant process and the one that is playing the main role on the formation of the final energy distribution.

3.2. Analytical calculation

In this section we estimate analytically the resulting energy distribution of the electrons, gaining energy by interacting with N_S shock waves and losing energy by synchrotron emission. We use a *Fokker-Planck* type transport equation for the electron distribution function (Cocke 1975; Melrose 1980)

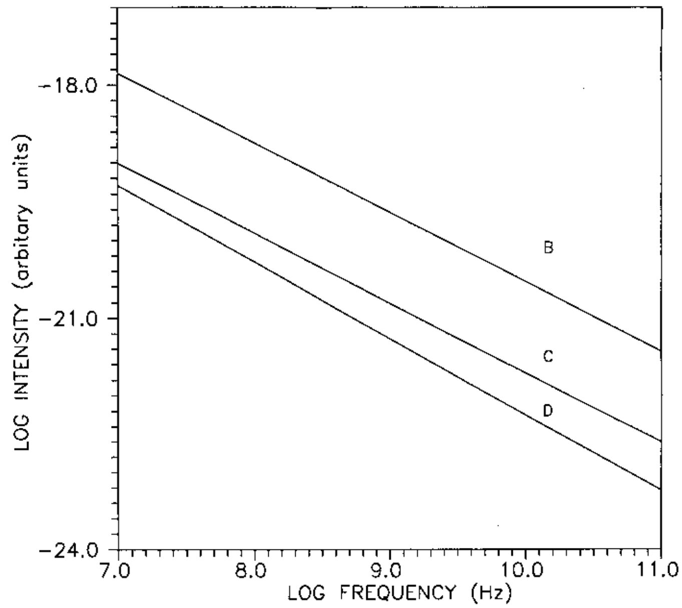

Fig. 4. The intensity of synchrotron radiation as a function of frequency for the models B,C and D

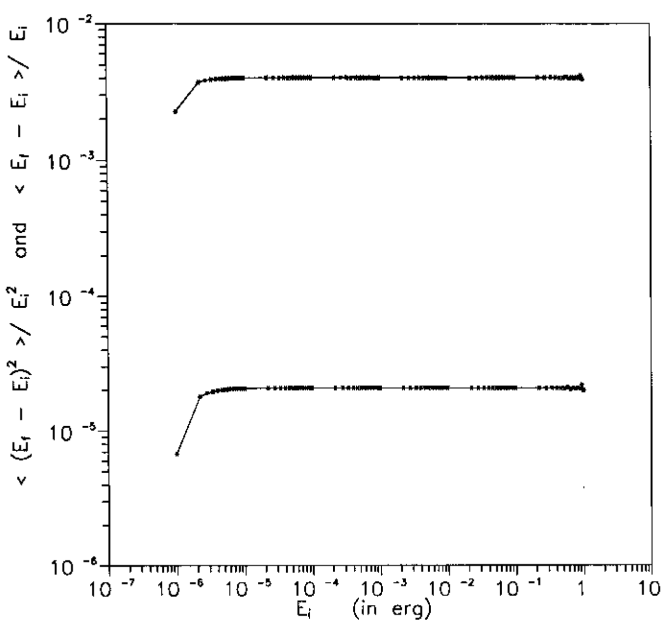

Fig. 5. The mean energy change and the mean of the square of the energy change for 10^3 electrons interacting with a single shock, as a function of their initial energy

$$\frac{\partial}{\partial t}N(E,t) = -\frac{\partial}{\partial E}\left(<\frac{dE}{dt}> N\right) + \frac{1}{2}\frac{\partial^2}{\partial E^2}\left(<\frac{dE^2}{dt}> N\right) + \beta\frac{\partial}{\partial E}(E^2 N) + Q(E,t) \quad (6)$$

where the additional terms on the right hand side of the equation allow, respectively, synchrotron losses and gains due to an external source. From Eq. (1), we found that the parameter at the synchrotron term is $\beta = 2.36\ 10^{-3}\ B_o^2 = 2.36\ 10^{-13}\ \mathrm{s^{-1}\ erg^{-1}}$.

We can assume that the coefficient of the systematic acceleration can be written as

$$< \frac{dE}{dt} >=< \nu_s(\Delta E) >= \frac{N_S}{t_f} < \Delta E > \quad (7a)$$

where ΔE is the energy change per electron per shock wave interaction and ν_s is the frequency of the electron - shock wave interactions, which depends on the number of shock waves present and the total time t_f = 1.5 10^6 yr = 4.74 10^{13} s. The ensemble average represent the mean value of the energy change for a population of electrons having the same initial energy. Similarly the diffusion coefficient in energy is

$$< \frac{dE^2}{dt} >=< \nu_s(\Delta E)^2 >= \frac{N_S}{t_f} < (\Delta E)^2 > \quad (7b)$$

We can now calculate numerically, the mean energy change and the mean of the square of the energy change of 10^3 electrons, interacting with a single shock, as a function of their initial energy (see Fig. 5). We found that $< (\Delta E) >= \alpha_1 E$ and $< (\Delta E)^2 >= \alpha_2 E^2$, with $\alpha_1 \approx 0.4\ 10^{-2}$ and $\alpha_2 \approx 0.205\ 10^{-4}$. If we replace these values in Eq. (7) we found that

$$< \frac{dE}{dt} > \approx 8.4\ 10^{-17}\ N_S\ E \approx A_1 E \quad (8a)$$

and

$$< \frac{dE^2}{dt} > \approx 4.3\ 10^{-19}\ N_S\ E^2 \approx A_2 E^2 \quad (8b)$$

where $A_1 = 8.4\ 10^{-17}\ N_S\ \mathrm{s^{-1}}$ and $A_2 = 4.3\ 10^{-19}\ N_S\ \mathrm{s^{-1}}$.

We notice that $A_1 >> A_2$ and $\beta >> A_2$ for our models, so we ignore the diffusion term and seek a steady state solution for the energy distribution of the electrons, e.g.

$$E\frac{dN}{dE}(\beta E - A_1) + (2\beta E - A_1)\,N + Q(E) = 0 \quad (9)$$

with the source term $Q(E)$ given by Eq. (2). The solution of the above equation is

$$N_a = \frac{N_o}{(z-1)}\frac{E^{-z}}{(\beta E - A_1)} \quad (10)$$

where $z = 2.2$ is the exponent of the injection energy distribution of electrons. Using Eq. (10) and the corresponding values of the constants β and A_1 for our models we calculate the energy distribution function N_a (see Fig. 3 for the model C, dashed line)

$$N_a(E) \propto K_a E^{-x_a} \quad (11)$$

where the exponent x_a for each model is given in Table 1.

In model A the environment produces a large number of shock waves (N_S = 2691), the electrons are accelerated in high energies, and the synchrotron radiation play no role on the formation of the final energy distribution. The medium is very efficient accelerator and the process is coherent, so we can not approach the evolution of the distribution with a transport equation.

Models B, C and D are producing distribution functions which are in good agreement with the observation. The corresponding errors between the analytical and the numerical exponent are 16%, 0.8% and 3%, for the models B, C and D, respectively.

4. Summary and discussion

We presented a model for the acceleration of energetic electrons at the hot spots of extended extragalactic radio sources. We calculated the resulting energy distribution of electrons and the corresponding spectral index of the synchrotron radiation emitted from such a distribution. We used a numerical and analytical approach.

In our numerical estimates, we simulated the environment of a hot spot, using a scenario based upon the following assumptions:

1. We use the shock drift mechanism for the acceleration of electrons by an ensemble of oblique shock waves, with random characteristics
2. We consider synchrotron radiation losses for the electrons.

A basic parameter on our model is the time needed by an electron to travel between the shock fronts (T_S). This number is related to the number of shock waves driven by the flow in the hot spot. We analysed four different models based on the value of the time interval T_S and we conclude that

1. For a broad set of values of the parameter T_S, the environment produces distribution functions with the corresponding spectral index of the synchrotron radiation, been in a good agreement with the observations (models B, C and D). This means that our approach *is not sensitive* to the number of the shock waves (N_S), unless this number becomes unrealistically high and the synchrotron losses do not play important role on the formation of the final distribution function (model A).
2. It is clear that as the parameter T_S increases or the dimensions of the hot spot become smaller, fewer shock waves are necessary in order to form the observed distribution. We believe that for a compact hot spot or for $T_S \approx 10^6$ yr, no more than one shock wave is necessary to produce the observed radiation spectrum (Begelman & Kirk 1990).

We used a Fokker-Planck type transport equation for the energization of electrons. We included the acceleration by the shocks and the synchrotron losses and search for a steady state solution. Comparing our analytical and numerical results, we found that the errors are 16%, 0.8% and 3% for models B,C and D respectively, but there is big error for model A (77%).

Our conjecture is that the acceleration of electrons in the body of the jet of extragalactic radio sources, can also be modulated with the procedure described above, using different values for $L_{\rm H}$ and $T_{\rm S}$. If this is proven to be the case, the energetic electrons are the tracers of non-linear discontinuities (shocks) appearing randomly inside the flow. These are short lived coherent formations inside a random set of waves (turbulence). In such a case the main difference between the body of the jet and the hot spots of extragalactic radio sources, are the characteristics of the turbulent flow (e.g. the distance between the shock waves, their Mach number, etc.). This final point is currently under study.

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