Signatures of discrete scale invariance in D_{st} time series

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[1] Self-similar systems are characterized by continuous scale invariance and, in response, the existence of power laws. However, a significant number of systems exhibits discrete scale invariance (DSI) which in turn leads to log-periodic corrections to scaling that decorate the pure power law. Here, we present the results of a search of log-periodic corrections to scaling in the squares of D_{st} index increments which are taken as proxies of the energy dissipation rate in the magnetosphere. We show that D_{st} time series exhibit DSI and discuss the consequence of this feature, as well as the possible implications of D_{st} DSI on space weather forecasting efforts. **Citation:** Balasis, G., C. Papadimitriou, I. A. Daglis, A. Anastasiadis, L. Athanasopoulou, and K. Eftaxias (2011), Signatures of discrete scale invariance in D_{st} time series, *Geophys. Res. Lett.*, *38*, L13103, doi:10.1029/2011GL048019.

1. Introduction

[2] Self-similarity is characterized by continuous scale invariance, i.e., invariance with respect to arbitrary magnifying factors [*Anifrani et al.*, 1997]. Its mathematical consequence is the existence of power laws describing the behavior of observables. A system exhibits discrete scale invariance (DSI) if it is invariant under a discrete set of dilatations only [*Anifrani et al.*, 1997]. DSI leads to complex exponents, i.e., log-periodic corrections to scaling.

[3] Fractals have dimensions that are in general real numbers. The generalization from the set of integers to the set of real numbers embodies the transition from the symmetry of translation invariance to symmetry of scale invariance. A fractal distribution points to a power law with a real exponent. If the exponent is taken to be a complex number, log-periodic behavior is obtained. The complex fractal dimension is associated with a DSI, i.e., to the invariance of the system or of its properties only under magnifications that are integer powers of a fundamental ratio. Interestingly, the appearance of DSI signifies a partial symmetry breaking of a continuous scale invariance. A number of studies have proposed spatial and/or temporal log-periodic behavior in ruptures, seismicity, ground-water, bronchial trees, financial systems, and models (for a review see *Malamud et al.* [2005]).

[4] Earth's magnetospheric dynamics in response to solar wind changes resembles the behavior of a complex system which operates out-of-equilibrium and near criticality [*Consolini et al.*, 2005]. In general, theoretical arguments [*Saleur and Sornette*, 1996] indicate that complex exponents are to be expected for out-of-equilibrium and/or quenched disordered systems, such as the Earth's dynamic magnetosphere. *Zhou and Sornette* [2002] performed many tests confirming the remarkable observation of statistically significant log-periodic correction to scaling of three-dimensional fully developed turbulence. Plasma turbulence is a rather ubiquitous feature in the Earth's magnetosphere [*Lui*, 2002].

[5] The D_{st} index has been widely used as a proxy of the ring current strength and consequently of the intensity of geospace magnetic storms the most complex phenomenon of magnetospheric dynamics [*Daglis*, 2006]. Several researchers have developed D_{st} index forecasts using measurements of IMF and/or solar wind plasma parameters. Therefore, a number of empirical models exists, based on differential equations [e.g., *Burton et al.*, 1975; *O'Brien and McPherron*, 2000; *Temerin and Li*, 2002], and on artificial neural networks [e.g., *Lundstedt et al.*, 2002; *Pallocchia et al.*, 2006; *Wei et al.*, 2007]. For instance, *Temerin and Li*'s [2002, 2006] scheme makes successful D_{st} predictions with a time step of 10 minutes whereas the D_{st} index has time step of one hour by using real-time data from ACE as the input to their model (http://lasp.colorado.edu/space_weather/dsttemerin/dstte-merin.html).

[6] Herein, we show that by using the D_{st} index - a typical proxy of magnetospheric dynamics - the derived energy dissipation rate prior to a magnetic storm displays log-periodic oscillations on top of the leading-order power law form. The latter is used for the determination of the time of occurrence of an approaching magnetic storm. The results can be utilized for space weather forecasting purposes.

2. Observation of Fractality in the D_{st} Time Series

[7] Observations suggest that under the influence of the solar wind, the magnetosphere can be channelled into a globally non-equilibrium critical state [*Chang et al.*, 2003; *Consolini et al.*, 2005]. Several studies have highlighted the fractal character of the D_{st} index [*Wanliss*, 2005, *Wanliss et al.*, 2005; *Balasis et al.*, 2006; *Picoli et al.*, 2007; *Yu et al.*, 2007].

[8] A common hallmark of out-of-equilibrium phenomena is their extraordinary complexity. Complex systems self-organize their internal structure and their dynamics, showing novel and surprising macroscopic properties, including coherent large-scale collective behaviors. A universal footprint seen in many complex systems near criticality is the self-affinity for energy release from the system that signals a fractal topology, namely a multi-scale process with no preferred spatial and temporal scales. The fractional

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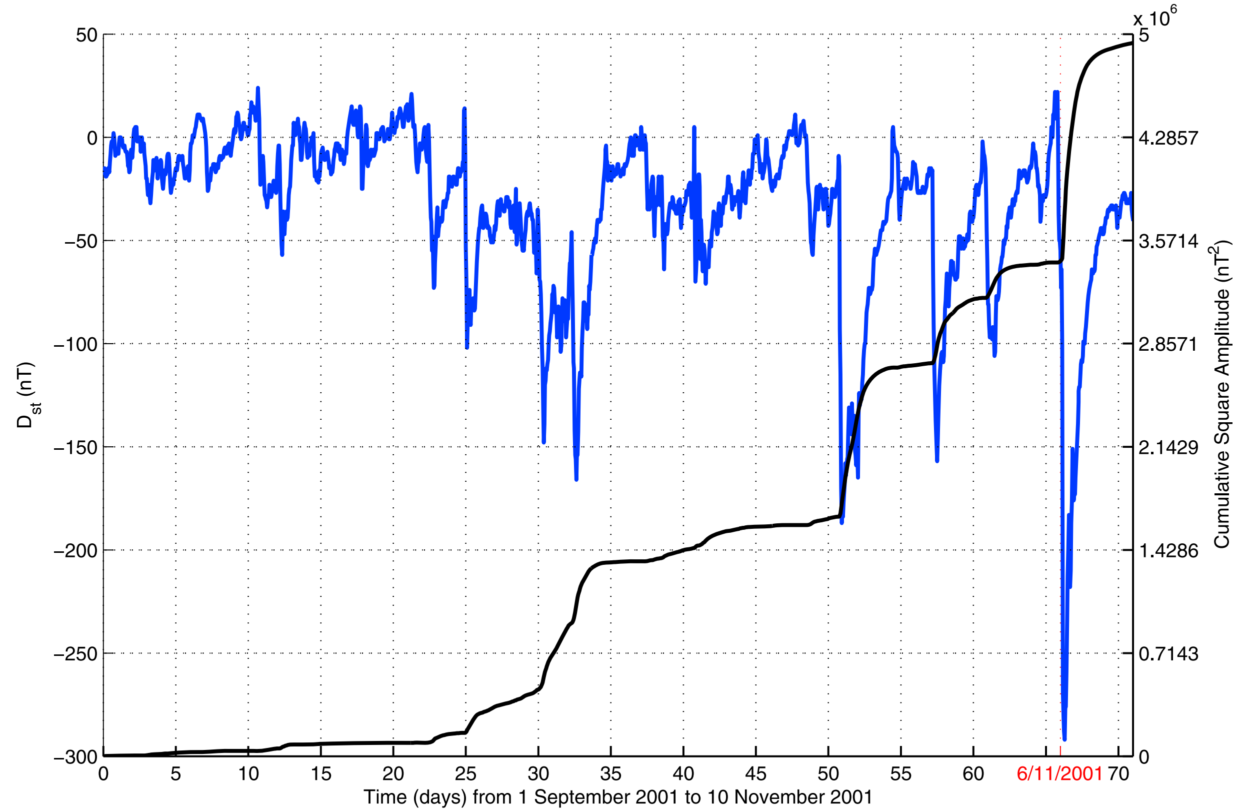

Figure 1. The time series of the D_{st} index from 1/9/2001 to 10/11/2001 along with its cumulative square amplitudes (in black). The magnetic storm of 6 November 2001 is marked with red.

power law relationship is a standard definition of a self-affine structure.

[9] If a time series is a temporal fractal, then a power law of the form $S(f) \propto f^{-\beta}$ is obeyed, with $S(f)$ the power spectral density and f the frequency. The spectral scaling exponent β is a measure of the strength of time correlations. The suitability of the fit of a time series to the power law is represented by the linear correlation coefficient, r, of this representation.

[10] In Figure 1, the D_{st} time series is shown. The data cover 71 days including variations 67 days prior to and 4 days after (1 September 2001 – 10 November 2001) the intense magnetic storm occurred on 6 November 2001 with a minimum D_{st} value of −292 nT. The chosen time interval represents a period of high geomagnetic activity with an abundance of intense magnetospheric events, i.e., 7 magnetic storms in total. Applying the wavelet transform with the Morlet basis as the mother function [*Balasis et al.*, 2006] to the hourly D_{st} values leads to a power spectrum matrix with 65 × (71 × 24) elements, where 65 is the number of frequencies. In Figure 2, the wavelet-estimated power spectral density of the D_{st} time series is shown. When we focus to periods from 2 to 256 hours (see red line in Figure 2) the fit to the power law ($S(f) \propto f^{-\beta}$) is excellent (r is 0.99443). This means that the fractal character of the processes and structures in the magnetosphere underlying a magnetic storm is compact: the activity could be ascribed to a multi-time-scale cooperative activity of numerous activated units, in which an individual unit behavior is dominated by its neighbors, so that all units simultaneously alter their behavior to a common large-scale fractal pattern.

3. Observation of Log-Periodicity in the D_{st} Time Series

[11] DSI manifests itself in data by log-periodic corrections to scaling [*Sornette*, 1998, 2004; *Huang et al.*, 2000; *Gluzman and Sornette*, 2002]. The typical formula for log-periodicity in time is given by

$$E(t) = A + B(t_f - t)^m \{1 + C\cos[\omega \log(t_f - t) + \phi]\}, \quad (1)$$

where $E(t)$ is the dissipative energy released, t_f is the time of the main shock (storm peak), ω is the frequency and ϕ is just an offset.

[12] In Figure 1, the D_{st} cumulative square amplitudes are also shown. We assume that the increase of dissipation during magnetic storms is proportional to the squared fluctuation of D_{st}, i.e, $\overline{D}_{st}^2(t) = (\langle D_{st}\rangle - D_{st}(t))^2$, where $\langle D_{st}\rangle$ is a reasonable threshold selected on the basis of quiet periods. Various thresholds (i.e., −50, −40, …, 0, …, 40 and 50 nT) were used to conduct the analysis presented next. It was overall found that for $\langle D_{st}\rangle = 0$ and when ignoring positive D_{st} values optimum results are yielded in terms of fitting D_{st} data to the theoretical curve given by equation (1) and finding from there the time of occurrence of the intense magnetic storm. A significant increase in the rate of energy release as the intense magnetic storm of 6 November 2001

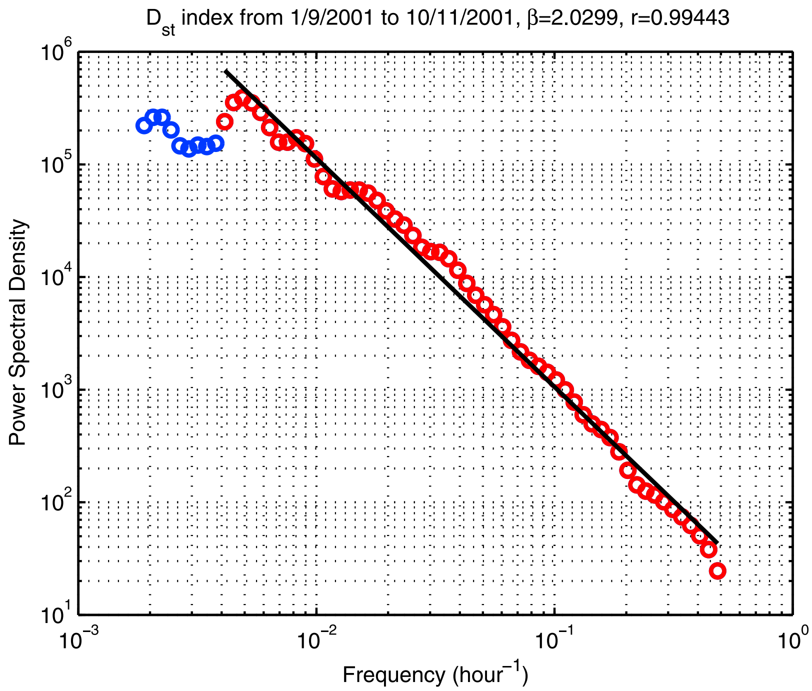

Figure 2. The power spectral density function of the D_{st} data, shown in Figure 1, from periods of 2 hours to 512 hours. The black line indicates a fitted power law of the form $S(f) \propto f^{-\beta}$ estimated to periods 2–256 hours (red points).

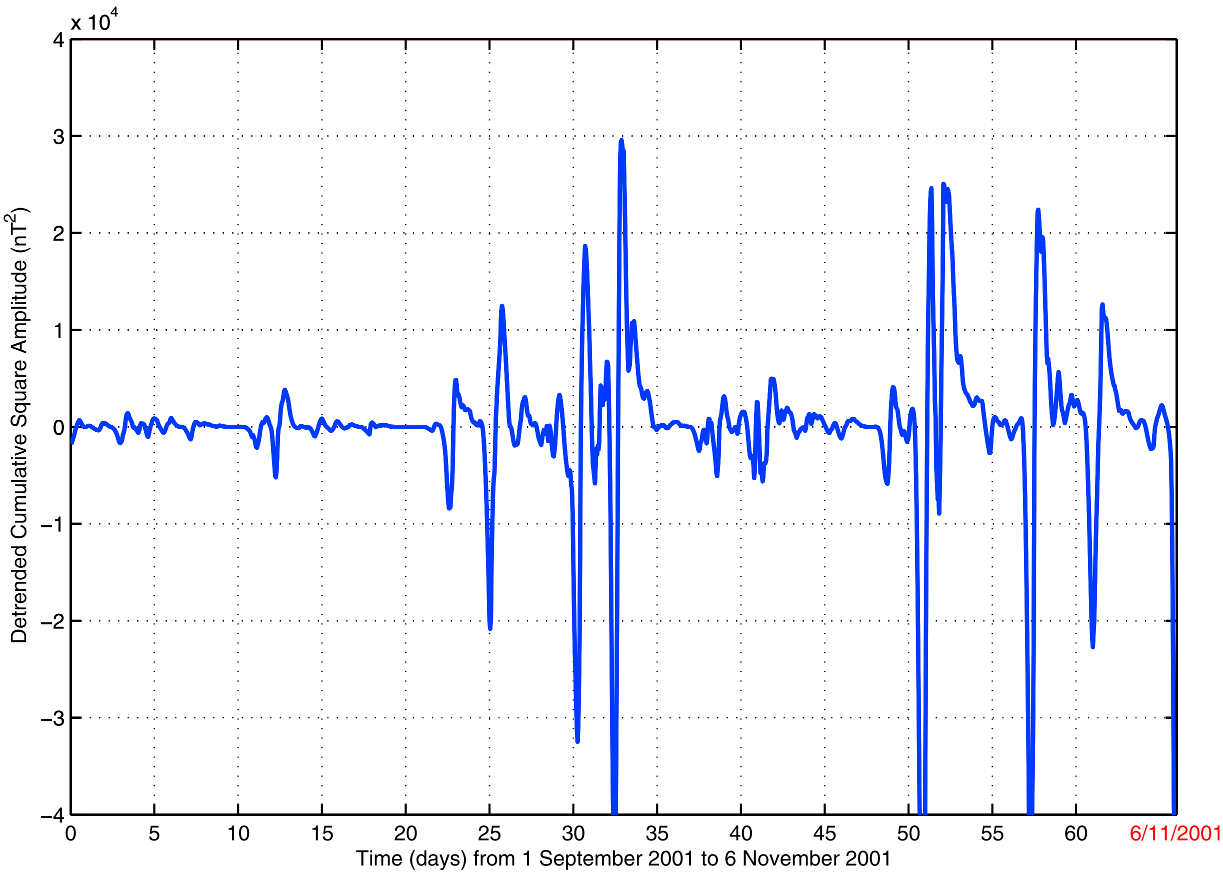

Figure 3. The detrended time series of the cumulative square amplitudes of D_{st} shown in Figure 1 from 1/9/2001 to 6/11/2001.

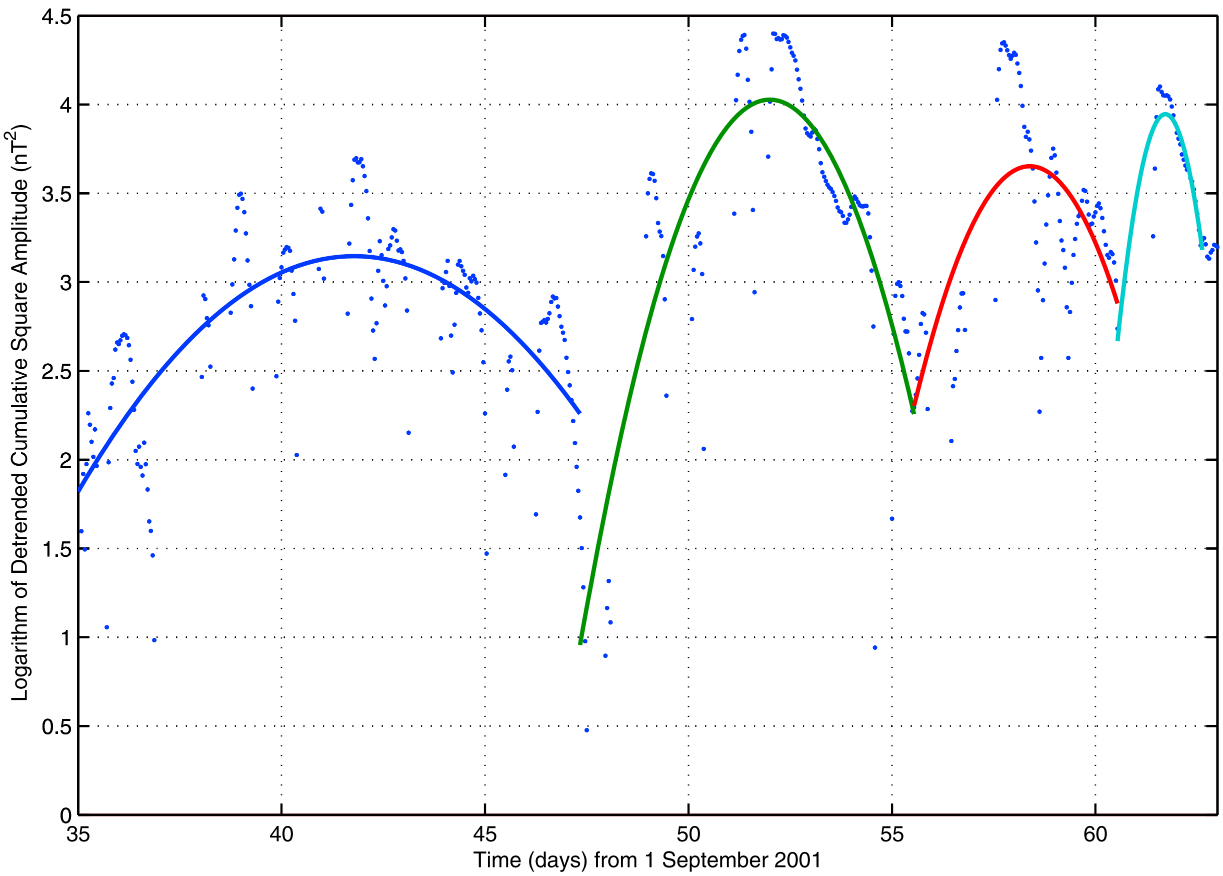

Figure 4. The logarithm of the detrended time series of Figure 3 from day 35 onwards. First, we note four distinct time intervals with their corresponding maxima. We then fit a parabolic curve to the local maximum of each distinct time interval.

approaches is observed. This implies that the system of the Earth's magnetosphere is not only near the peak of the magnetic storm in the sense of having power law correlations, but also in terms of exhibiting high susceptibility. We focus on Figure 1 and in the energy oscillations observed prior to the main magnetic event (0–67 days). One can observe a trace of oscillations modulating the pure power law behavior in the energy density.

[13] Discrete scale invariance (when present) dictates that t_f would have been preceded by a series of local maxima t_1, t_2, t_3, … the successive intervals of which will follow a geometric series defined by $\frac{t_{n+1}-t_n}{t_{n+2}-t_{n+1}} = \lambda$. To accurately detect these extrema (the "knees" in the energy dissipation rate graph) and capture the dynamics of the small fluctuations we construct the cumulative square amplitude time series (ignoring positive D_{st} values) and detrend the new time series by using a moving average with a 25-hour long window according to $y_n^{detr} = \frac{1}{25}\sum_{i=n-12}^{n+12} y_i$ (Figure 3).

[14] Keeping only the positive values, since we are interested in local maxima and taking the logarithm of the detrended time series (as a visualization aid, i.e., to downscale the y-axis which covers 4 orders of magnitudes) we can see that from day 35 onwards there are four distinct time intervals which are the most likely precursor candidates (Figure 4). The local maximum of each time interval can be easily estimated using a parabolic fit of the form $a_n + b_n(t_n - t)^2$ (Figure 4).

[15] From these maxima we get four values for t_n {41.78, 52.00, 58.51, 61.72} which yield an average λ value of 1.75889. The recursive formula $t_{n+1} = t_n + (t_n - t_{n-1})/\lambda$ for these parameters converges rapidly and predicts a final $t_f = t_\infty = 66.10$ days.

[16] Using this t_f as a given value, we can now proceed with the fitting of the log-periodic power law to the dissipative energy of Figure 1. First, we use a pure power law (by setting $C = 0$ in equation (1)) in order to reduce the free parameters to just three: A, B, m. In Figure 5 this pure power law is shown along with the dissipative energy derived by D_{st}^2. Then keeping these values we add the log-periodic component and estimate the other three parameters (C, ω, ϕ) of equation (1). To get the final values, we use the estimated parameters as starting points and perform a last fit for all of them, keeping only t_f constant. (The fitting was achieved using least squares and it is $R^2 = 98.56$.)

[17] Eventually, in Figure 5 a power law with log-periodic oscillations, as given by equation (1), fitted to the cumulative D_{st}^2 amplitudes is shown (see Table 1 in Text S1 of the auxiliary material for the fitted values of the parameters of Equation (1)).[1] It is clear that a law of this form can adequately describe the observations. Fitting now equation (1) only for t_f yields $t_f = 66.30 \pm 0.07$ days, which corresponds

[1]Auxiliary materials are available in the HTML. doi:10.1029/2011GL048019.

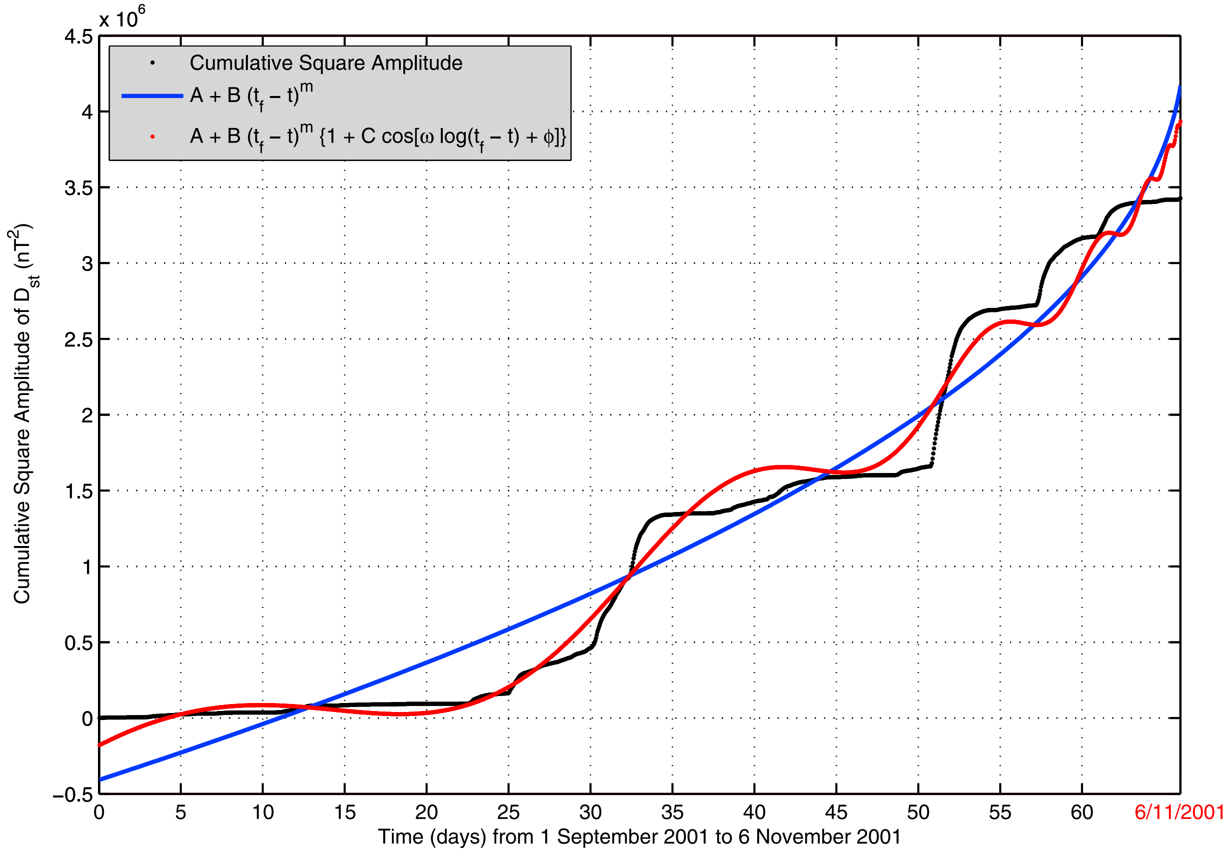

Figure 5. A fit of the cumulative square amplitudes of D_{st}, shown in Figure 1, to a pure power law (blue line) and to a power law with log-periodic oscillations given by equation (1) (red line). The magnetic storm of 6 November 2001 is marked with red.

within a 95% confidence limit to the interval [66.23, 66.37] for t_f. We note that the storm time, t_f, predicted by the DSI theory is 1.92 hours ahead of the actual time of the event (the real storm peak is at 66.38 days).

[18] As expected, the log-periodic oscillations are modulated in frequency with a geometric increase of the frequency on the approach to the time t_f: the intermittent accelerations and quiescences of ring current enhancements around the power law acceleration become more closely spaced as the main event is approaching. The aforementioned behavior reflects a preparation stage for major geomagnetic storms, in which prior activity enhancements occur at particular discrete times (as represented by λ) and not in a continuous fashion: these discontinuities in turn mirror the localized and threshold nature of the underlying mechanism. It is this "punctuated" physics which gives rise to the scaling precursors modeled mathematically by the log-periodic correction to scaling.

4. Discussion and Conclusions

[19] Magnetospheric research has gradually advanced from the early days of considering primarily macroscopic processes treatable by the magnetohydrodynamics (MHD) equations to the present-day realization of the real complexity of the magnetosphere with its rich array of physical processes involving multiple spatio-temporal scales as well as cross-scale coupling [*Lui*, 2002].

[20] The emergence of discrete scale invariance in a system is translated to the fact that observations related to the system obey a power law with an additional log-periodic component. *Johansen and Sornette* [1998] suggested that DSI is quite general, albeit subtle phenomenon. They pointed out that the practical problem in uncovering an underlying DSI is that standard ensemble averaging procedures destroy it as if it was pure noise.

[21] Herein, the squares of the negative values of the D_{st} index increments have been taken as proxies of the energy dissipation rate in the Earth's magnetosphere. We have shown that a power law with log-periodic oscillations fits well the cumulative square amplitudes of D_{st} time series which include an intense magnetic storm. Based on the theory of log-periodic corrections to scaling we have inferred a theoretical value for the time of the occurrence of the extreme magnetospheric event which is 1.92 hours ahead of the real time that the magnetic storm peak D_{st} value took place. The theoretical curve we have used is able to fit D_{st} data starting 66.25 days prior to the intense magnetospheric event. The last data used to achieve this theoretical occurrence time came from day 62.63, almost four days before the actual event (real occurrence time is at 66.38 days).

[22] In this study we are interested in the temporal evolution of intense magnetic storms. Magnetic storms are defined as the global ground geomagnetic disturbance, which is correctly represented by the D_{st} index. We do not study the evolution of the ring current and therefore there is no need to correct the D_{st} index to account for the effect of other magnetospheric current systems in an effort to deduce information on the ring current from the D_{st} index. There-

fore, a corrected D_{st} index, such as the magnetopause-corrected D_{st}^{*} index [e.g., *Burton et al.*, 1975; *Asikainen et al.*, 2010], is neither required, nor adds anything to our study. Nevertheless, an analysis that was performed for corresponding D_{st}^{*} data inferred an occurrence time derived from theoretical arguments based on DSI methodology almost 10 hours prior to the actual occurrence time of the intense magnetic storm, i.e., worse than the occurrence time found from D_{st} data (for the DSI analysis of D_{st} data see Figures 1 and 2 and Table 2 in Text S1 of the auxiliary material).

[23] The results reported in Figure 5 show a discrepancy between the cumulative trend and the fitted one both in timing and values especially from day 50 on. A possible explanation for the observed discrepancy may be the fact that the parametric approach used in this work has the drawback of becoming unstable in presence of noise as reported by *Zhou and Sornette* [2002].

[24] The evidence of DSI in the temporal evolution of D_{st} offers new insight towards better understanding of the magnetospheric dynamics and can contribute towards a more precise theoretical treatment of the Earth's magnetosphere.

[25] Our approach achieved a successful prediction of a major magnetic storm. It is useful to compare our results with the monitoring of the degree of organization of the system by using an appropriate organization criterion. A previous investigation of D_{st} time series using nonextensive Tsallis entropy [*Tsallis*, 1988, 2009] as a complexity measure has shown that Tsallis entropy attains its lower values as the intense magnetic storm of 6 November 2001 approaches [*Balasis et al.*, 2008, 2009]. This in turn means that the system of the Earth's magnetosphere exhibits a higher degree of organization (lower complexity) around this extreme space weather event.

[26] We believe that the convergence of the results presented in this paper with other well-established methods of D_{st} forecast (discussed in the Introduction) can potentially increase the reliability of forecasting techniques and can therefore improve space weather forecasting and modeling.

[27] **Acknowledgments.** The D_{st} data are provided by the World Data Center for Geomagnetism, Kyoto (http://swdcwww.kugi.kyoto-u.ac.jp/).

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