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ELECTRON ACCELERATION BY RANDOM DC ELECTRIC FIELDS

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ABSTRACT

We present a global model for the acceleration of electrons in the framework of the *statistical flare* model of Vlahos et al. In this model, solar flares are the result of an internal self-organized critical (SOC) process in a complex, evolving, and highly inhomogeneous active region. The acceleration of electrons is due to localized DC electric fields closely related to the energy-release process in the active region. Our numerical results for the kinetic energy distribution of accelerated electrons show a power-law or an exponential-law behavior, depending on the maximum trapping time of the energetic particles inside the acceleration volume.

Subject headings: acceleration of particles — Sun: flares

1. INTRODUCTION

It has been documented that the energy released during solar flares is separated into three parts: (1) intense localized heating, (2) particle acceleration, and (3) mass flows. It is also clear that a successful model for the radiation emitted during an eruptive energy release in the Sun cannot be constructed without considering the global aspects of the energy-release process (cf. Priest 1992).

A large number of observations on the spatiotemporal evolution of solar flares have been collected (for reviews, see Dennis 1985, 1988; Ramaty & Murphy 1987; Vilmer 1987; Pick, Klein, & Trottet 1990), and interpretations of them have led to a variety of proposals for acceleration processes, such as MHD waves, shock waves, DC electric fields, double layers, coherent acceleration processes, etc. (for reviews, see Heyvaerts 1981; de Jager 1986; Forman, Ramaty, & Zweibel 1986; Scholer 1988; Vlahos 1989; Vlahos et al. 1989; Melrose 1990).

In the existing literature, the transport of energetic particles is decoupled from the acceleration processes in most cases. A few attempts have been made to reconcile the acceleration and the transport of energetic particles (Miller & Ramatty 1989; Ryan & Lee 1991; Hamilton & Petrosian 1992), but none of these approaches make a clear connection between the acceleration mechanism and the energy-release process.

Recent observations of flare radio emission (Kuijpers, van der Post, & Slottje 1981; Benz 1985; Benz & Aschwanden 1992; Vilmer 1993; Trottet 1994; Aschwanden et al. 1995) suggest that the energy released during solar flares is fragmented. Parker (1988, 1989) proposed that a large number of small, unobserved energy releases (10^{24} to 10^{26} ergs), called "nanoflares," may provide essential energy input to the corona. This proposal places great emphasis on magnetic inhomogeneities in the solar corona and changes our view of studying the "explosive" and "quiet" phenomena on the solar surface.

Following the above ideas, a number of qualitative models for energy release in solar flares have been developed (for reviews, see Vlahos 1993, 1996; papers appearing in van den Oord 1994). These models revealed the necessity of studying and understanding the global behavior of the evolution of active regions. Two approaches can be used for this purpose:

1. *MHD simulations* (e.g., Galsgaard & Nordlund 1996; Einaudi et al. 1996).—According to these models, random shearing motions of the magnetic field lines at the photospheric boundary lead to the formation of a number of current sheets inside the active region, where magnetic reconnection occurs. The dynamical evolution of these current sheets drives the creation of supersonic or super-Alfvénic flows and the formation of smaller current sheets (see, for example, Fig. 1 of Galsgaard & Nordlund 1996). The spatial and temporal distribution of the current concentrations is highly intermittent, suggesting the application of statistical theories for the solar activity.

2. *Cellular automata simulations.*—Lu & Hamilton (1991) and Lu et al. (1993) were the first to apply cellular automata simulations to solar flares. They showed that the energy released inside the active region may well be a result of an internal self-organization process. The continuous loading of the active region with new magnetic flux produces several magnetic discontinuities. Simple rules were applied for the redistribution of magnetic fields and the release of magnetic energy in these discontinuities. According to this scenario, the solar flare is not a simple superposition of many smaller energy releases (e.g., microflares), since energy released locally affects the entire structure of the flaring region by redistributing the magnetic field, creating new instabilities in an avalanche-like manner.

Several qualitative attempts to study the problem of particle acceleration in the framework of a concrete proposal for a fragmented energy-release process have been made in the past (e.g., Holman 1985; Khan 1989; Haerendel 1994). Anastasiadis & Vlahos (1991, 1994) proposed a model for the acceleration of particles (electrons and ions) by an ensemble of shock waves. In this model, the energy was assumed to be released by means of many localized, small-scale explosive phenomena that drive a number of shock fronts (small-scale, short-lived discontinuities; see simulations by Cargill, Goodrich, & Vlahos 1988; Galsgaard & Nordlund 1996).

In this article, we present a model for electron acceleration based on a random number of localized electric fields closely associated with the energy released during solar flares. We are interested in global characteristics of the energy-release process and the acceleration of high-energy electrons. The approach we use for the energy-release

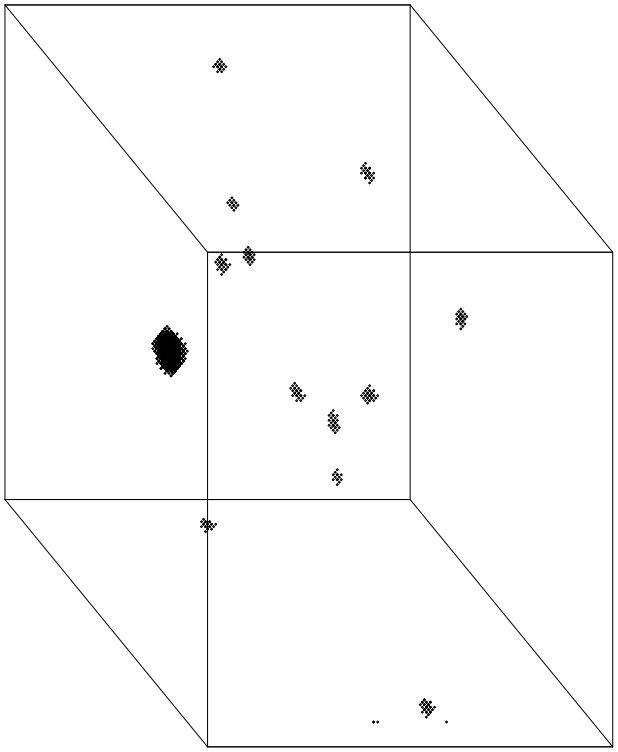

FIG. 1*a*

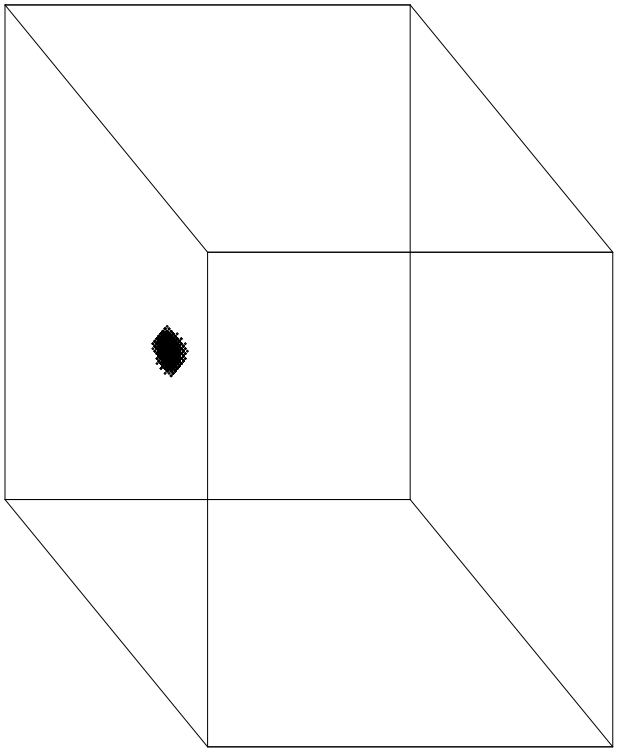

FIG. 1*b*

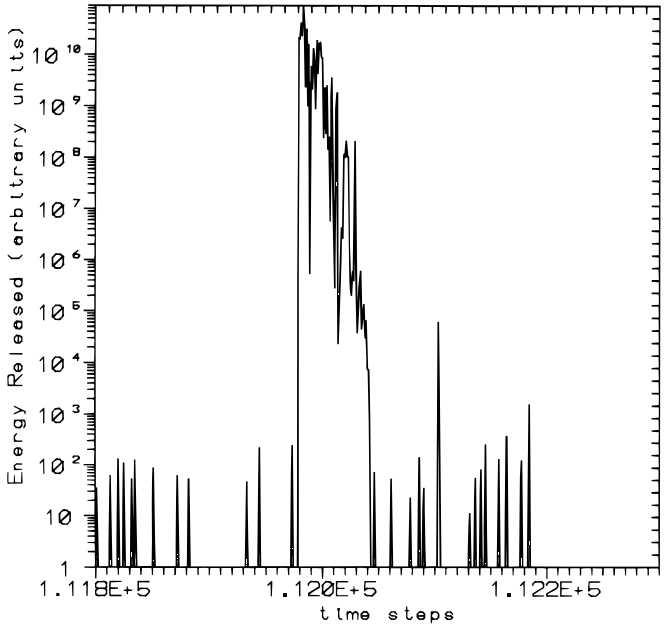

FIG. 1*c*

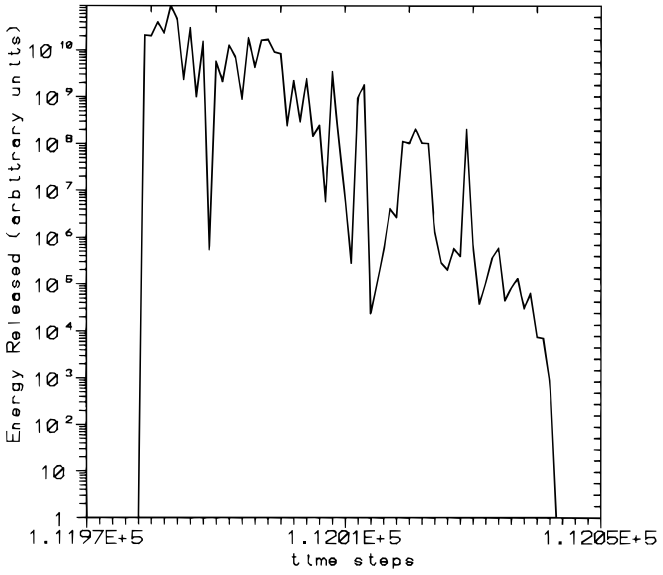

FIG. 1*d*

FIG. 1.—(*a*) Spatial configuration of the thee-dimensional simulation box, illustrating the clustering of unstable grid points. (*b*) The largest cluster formed. (*c*) Energy-release time series corresponding to the configurations in (*a*). (*d*) Profile of the energy-release time series for the cluster presented in (*b*).

process is presented in § 2, while § 3 gives a description of the acceleration model. In § 4 we present our results, and in § 5 we summarize our work and discuss some open problems that require further study.

2. THE ENERGY-RELEASE PROCESS AND THE STATISTICAL FLARE

Solar flares are associated with active regions, where the magnetic fields are intense and highly concentrated. It is believed that active regions are highly inhomogeneous and complex dynamical systems, because the magnetic field emerging from the convection zone is subject to random shuffling from the photosphere. Owing to these motions, the magnetic field lines interact randomly and create a large number of current sheets in different sites. The sizes of these current sheets vary over a wide range.

In order to simulate the above physical picture, Vlahos et al. (1995), Georgoulis, Kluiving, & Vlahos (1995), and

Georgoulis & Vlahos (1996) have in a series of recent articles developed a cellular automaton model based on the concept of self-organized criticality (SOC) (Bak, Tang, & Wiesenfeld 1987, 1988; Tang & Bak 1988). Their main goals were to explore the origin of the frequency distribution of solar flares, and to gain some insight into the problems of coronal heating by nanoflares and the variability of occurrence rates of solar flares. In these articles, "flares" are considered not as isolated localized explosions but rather as randomly appearing clusters of reconnection sites in a highly inhomogeneous topology. The active regions are continuously in a flaring state, but the spatiotemporal evolution of the explosions changes randomly inside each active region. Vlahos et al. (1995) labeled this type of activity "statistical flare" and used a cellular automaton model to describe the energy-release process.

The basic rules of the statistical flare model are:

1. A three-dimensional, cubic lattice represents an active region. A random value of the magnetic field is assigned to each grid point (initial loading).

2. Randomly selected grid points gain increments, δB, also selected randomly out of the power-law probability distribution function (loading)

$$P(\delta B) \sim (\delta B)^{-5/3} , \tag{1}$$

where $P(\delta B)$ is the probability of occurrence of a certain increment δB. Notice that this distribution is a power law in which the exponent is a multiple of $\frac{1}{3}$. Such exponents occur frequently in phenomenological theories of turbulence due to Kolmogorov's second refined hypothesis (Kluiving 1995). Since the increments δB in equation (1) are supposed to be products of a highly turbulent process in the convection zone, the particular form of the exponent (i.e., that it is a multiple of $\frac{1}{3}$) is loosely explained. It is clear, however, that the connection between equation (1) and the turbulent convection zone should be analyzed further (see Galsgaard 1996; Georgoulis & Vlahos 1997).

3. The continuous loading generates local discontinuities, which excite currents ($\nabla \times \boldsymbol{B} \propto \boldsymbol{J}$). If a certain critical threshold, $B_{\rm cr}$, is exceeded, the topology is considered unstable and is relaxed by redistributing the magnetic field to the neighborhood and releasing energy (the relaxation process).

Thus the ambient magnetic gradients, developed due to loading activity, give rise to local instabilities that are relaxed by releasing excess magnetic energy through a large number of reconnection events. The amount of energy released during the restructuring of the magnetic field at a single grid point i is given by the relation (Georgoulis et al. 1995; Georgoulis & Vlahos 1996)

$$\epsilon_{ri} \sim (B_i - \tfrac{6}{7} B_{\rm cr})^2 , \tag{2}$$

where B_i is the local magnetic field at the i grid point. This energy is treated as the magnetic energy release during magnetic reconnection.

By performing a number of iterations, we are able to obtain spatiotemporal information on the energy released (in arbitrary units). In Figure 1*a*, we demonstrate the clustering of current sheets of all sizes for a small time interval. The largest cluster is presented in Figure 1*b*. The corresponding energy-release time series are given in Figures 1*c* and 1*d*, respectively. Notice that the temporal evolution of the energy released for the cluster depicted in Figure 1*b* mimics very closely the well-known behavior of an isolated flare. The cluster undergoes many dramatic changes during a few time steps. It is obvious that a large cluster is surrounded by many smaller clusters that play an important role for the energy released from the active region during a "flare" and the subsequent acceleration of energetic particles.

In the next section, we use the evaluated energy-release time series (corresponding to the entire active region, not the one related to a single cluster) for the acceleration of test particles. We study the interaction of particles with a number of different clusters surrounding the main event. The energy-release time series obeys a well-defined double power-law frequency distribution (see Fig. 1 of Georgoulis & Vlahos 1996), exhibits a scale-invariant behavior, and encloses a self-similar nature (Vlahos et al. 1995; Georgoulis et al. 1995). Because of the self-similar structure of the time series, the acceleration will follow a similar behavior even if we used the internal structure of an isolated cluster. It turns out that owing to self-similarity, our results are not affected by the size of the simulation box, provided that the box is not too small (not smaller than $50 \times 50 \times 50$).

3. MODEL FOR ELECTRON ACCELERATION

We propose here a model for the acceleration of electrons in a dynamically evolving active region. Our acceleration model is based on the interaction of electrons with a number of reconnecting current sheets (RCS). Two basic ingredients for the estimation of the energy gained by these electrons are the strength of the electric field inside the RCS and the effective acceleration length along this field.

We assume that the magnitude of the electric field inside each RCS is comparable to the convective electric field, because of the ambient plasma flow and the local magnetic field. If we assume that the flow velocity of the plasma inside the flaring region is of the order of the Alfvén speed, $v_{\rm A}$, then the convective electric field is given by the relation

$$E = \left| -\frac{\boldsymbol{v}_{\rm A} \times \boldsymbol{B}}{c} \right| \approx \frac{B^2}{c(4\pi n m_i)^{1/2}} , \tag{3}$$

where B is the magnetic field, c is the speed of light, n is the ambient plasma density, and m_i is the proton mass. From the above relation, we find that the convective electric field in volts cm^{-1} is

$$E(t) \approx 2.184 \times 10^3 B^2(t) n^{-1/2} . \tag{4}$$

Litvinenko (1996) performed a similar calculation in order to find a relation between the magnetic field and the electric field in a RCS, using a self-consistent calculation for the collisionless case.

We now consider a case in which the ambient plasma parameters inside our flaring region are given by a density of $n = 10^{10}$ cm^{-3} and temperature $T = 10^7$ K, corresponding to an electron thermal velocity $V_e = 1.23 \times 10^9$ cm s^{-1} and a kinetic energy $E_T = 430$ eV. Using these typical values, we find that the Dreicer electric field is $E_{\rm D} \approx 6 \times 10^{-5}$ V cm^{-1}.

The SOC model described in the previous section connects the magnetic field strength $B(t)$ at a given time t with the derived energy-release time series $\epsilon_r(t)$ as $\epsilon_r(t) \sim B^2(t)$ (see eq. [2]). We can thus estimate the ratio of the convec-

tive electric field to the Dreicer field as

$$\frac{E(t)}{E_D} \approx 3.64 \times 10^7 n^{-1/2} \epsilon_r(t) . \quad (5)$$

Using the above relation, we are able to construct a new time series, $E(t)$, that represents the electric fields at each RCS associated with the energy-release sites. This new time series exhibits the same characteristics as the energy-release time series. Notice that for our model parameters $n = 10^{10}$ cm^{-3} and $\epsilon_r(t) \geq 0$, the electric field inside the RCS is larger than the Dreicer field. This fact is consistent with a number of arguments presented by Litvinenko (1996). These highly localized and strong electric fields will excite plasma instabilities. The particle, interacting with the electric field and plasma waves, will depart from the RCS with a final energy gain or loss, which for our purposes will be simulated by a "virtual" electric field (see below).

Each electron of the ambient Maxwellian distribution

$$f(v) = \frac{n}{(2\pi)^{1/2} V_e} \exp\left(-\frac{v^2}{2V_e^2}\right) \quad (6)$$

with initial velocity in the range $2 \leq v/V_e \leq 5$ remains inside the flaring region for a different number of time steps (N_j), selected randomly from an interval (1, N_{max}). The parameter N_{max} represents the maximum number of time steps for which the injected electron distribution remains inside the acceleration volume. Thus, N_{max} is a measure of the maximum trapping time for the energetic particles. This trapping time is associated with the environment of the local random electric fields and not the possible large-scale trapping of electrons due to magnetic field convergence and/or their pitch angle scattering.

Each injected electron enters into the acceleration volume and interacts successively with a number $N_j \leq N_{max}$ of randomly selected elements of the electric field time series. It is clear that the random selection of N_j for each electron leads to a random selection of the number of RCSs that it will experience. Following this approach, electrons with the same initial kinetic energy E_k are neither entering into the acceleration volume at the same time and at the same position nor escaping from the same place at the same instant.

We include a consideration of energy losses for the electrons by assuming that their interaction with the associated electric field can be accomplished either in or out of phase. This selection is also random. Notice that these kinds of losses are energy independent and are due to the electric field at each RCS. Our model does not consider the effect of Coulomb collisions, which are energy-dependent losses, as we initialize the tail of the Maxwellian distribution and thus the collision frequency is small.

We define an acceleration coefficient α, selected randomly to vary between 0 and 1 at each electron-RCS interaction. With this coefficient, we intend to simulate the fact that only a portion of the energy released contributes to the particle-acceleration process; the rest of this energy heats the ambient medium of the flaring region. Thus, an electron interacts with a portion of the electric field (the "virtual field") allocated at a particular RCS. The variation of the acceleration coefficient at a single electron-RCS interaction also represents the fact that each electron travels along the RCS electric field for a different distance, so that the effective acceleration length Δl is not constant. In this way, the variability of Δl can also be assigned to the coefficient α.

Under the above assumptions, the kinetic energy change of a single electron (in eV) due to interaction with an electric field $E(t)$ at a given time t is given by the relation

$$\Delta E_k = \pm \alpha e E(t) \Delta l , \quad (7)$$

where the plus or minus sign corresponds to in or out of phase interaction, respectively; e is the electron charge ($e = -1$); and $\Delta l = 10^3$ cm is the maximum effective length of the current sheet in which the electric field $E(t)$ is located.

The only free parameter in our model is the maximum number of time steps (N_{max}) that the electron distribution moving inside the flaring region will complete before escaping from the box, and thus the maximum number of RCSs it can experience. In the following section, we present our results concerning the final energy distribution of accelerated electrons by performing a parametric study with respect to our free parameter.

4. RESULTS

According to our model, an electron performs a "free flight" between electric fields of variable strength that exhibit power-law frequency distributions. At each interaction, an electron can gain or lose energy in a nonsystematic way.

In order to justify this model, the temporal evolution of the velocity (normalized to thermal) of a single electron is presented in Figure 2*a*, and the part of the electric field time series with which this particular particle interacts is shown in Figure 2*b*. Note that the sign of the velocity in Figure 2*a* corresponds to the direction of motion. Rescaled details of Figures 2*a* and 2*b* are given in Figures 2*c* and 2*d*, respectively, in order to illustrate the self-similar character of the acceleration scaling.

As the electric field exhibits large variations locally, the energy gain of a single particle undergoes dramatic changes locally as well. It is then difficult to define a mean acceleration rate along the path of the electron with small variations around this mean. We believe that this process is not a normal diffusion (i.e., normal random walks, Brownian motion). New methods of analysis have been developed lately for highly nonlinear systems, in which the accelerators are localized and have irregular strength, using the Lévy flight statistic approach (e.g., Shlesinger, Zaslavsky, & Frisch 1995; Klafter, Shlesinger, & Zumofen 1996). Developing diffusion equations for particles moving inside electric fields with fractal spatiotemporal characteristics is currently a very challenging problem.

We are interested in the electron population of the distribution given by equation (6), with initial velocity range $2 \leq v/V_e \leq 5$. We normalize the injection distribution in such a way that $f(5V_e) = 1$, and follow numerically the kinetic energy change of 1000 electrons per injected velocity bin. The final kinetic energy distributions of the accelerated electrons (those with $E_k > E_T = 430$ eV) for different realizations of the model, based on the interval (1, N_{max}), are given in Figure 3. These are averaged distributions over 10 sample runs with the same parameters. Results for $N_{max} =$ 500, 2000, 5000, 10,000, 20,000, and 40,000 are presented in Figures 3*a*–3*f*.

The kinetic energy distributions for the high-energy electrons (with $E_k/E_T > 10^3$) initially exhibit a well-defined

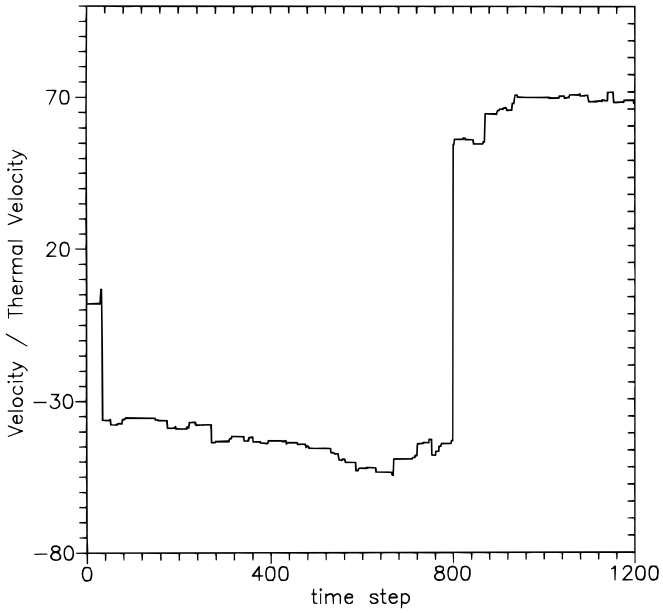

FIG. 2*a*

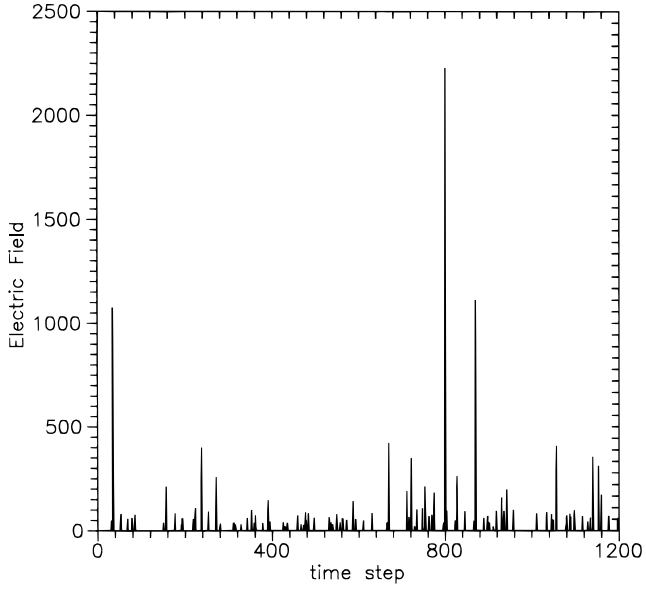

FIG. 2*b*

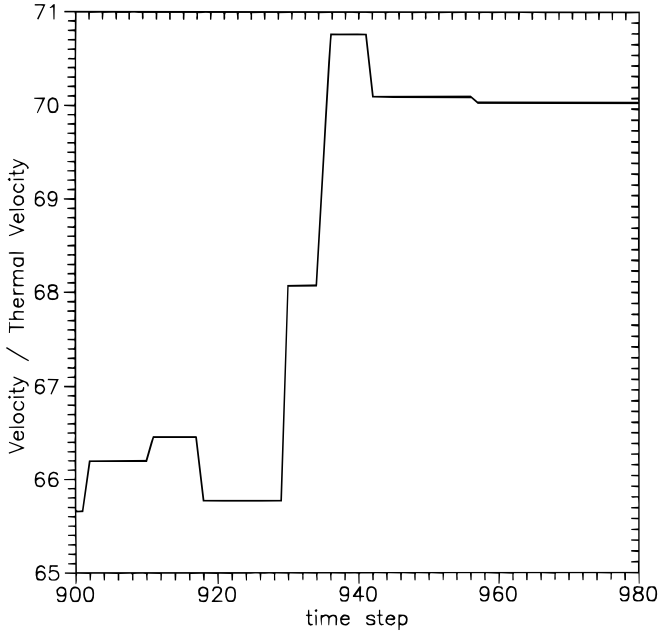

FIG. 2*c*

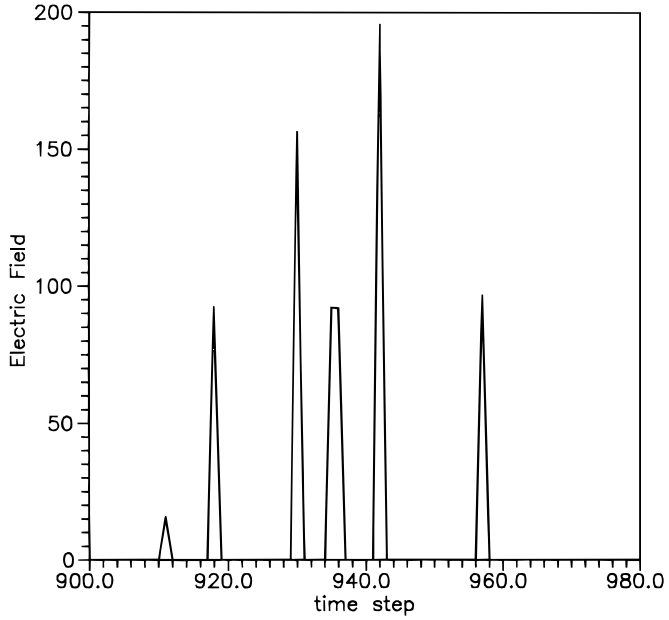

FIG. 2*d*

FIG. 2.—(*a*) Temporal evolution of the velocity (normalized to thermal) of a single electron. The sign of the velocity corresponds to the direction of motion. (*b*) Electric field time series experienced by the electron. (*c*) A rescaled detail of (*a*). (*d*) A rescaled detail of (*b*).

power-law behavior for $N_{\max} = 500$, 2000, and 5000. The power-law index has an initial value of -1.64, and as the maximum trapping time increases it becomes flatter, reaching -1.44 for $N_{\max} = 5000$. The relative error in the calculation of the above indices is less than 3%. Further increases in the trapping time affect the shape of the final distribution, which begins to diverge from the power-law behavior and starts to develop an exponential tail. For $N_{\max} = 10{,}000$, the shape of the distribution deviates significantly from both power-law and exponential-law behavior. Finally, further increase of $N_{\max}$ leads to the emergence of an exponential law. The electron distribution in this stage becomes thermal, with an effective temperature of the order of 10^5–10^6 E_T. The energetic electrons in this case remain for longer time intervals inside the acceleration volume and try to reach an equilibrium with the mean energy associated with the random electric fields.

We note that if we limit the energy range of interest to the interval $10^3 \leq E_k/E_T < 10^5$, then the energy distributions for the case of $N_{\max} \geq 10{,}000$ can also be fitted with power laws. The power-law index has an initial value of -0.91 (for $N_{\max} = 10{,}000$), which, again, becomes flatter as the maximum trapping time increases, reaching -0.48 for $N_{\max} = 40{,}000$. It is clear that for the case of $N_{\max} \geq 10{,}000$, the shape of the energy distribution depends strongly on the considered energy range.

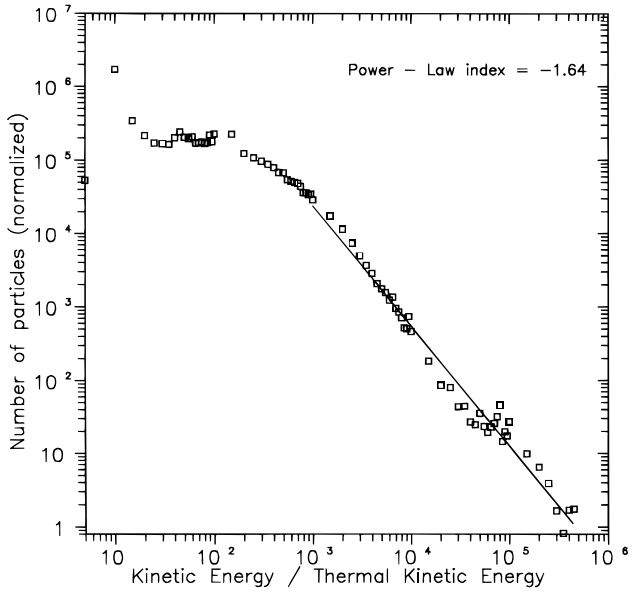

FIG. 3*a*

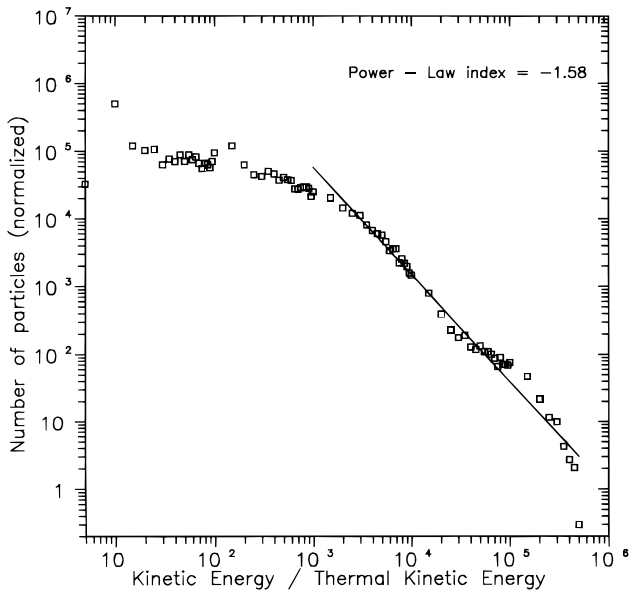

FIG. 3*b*

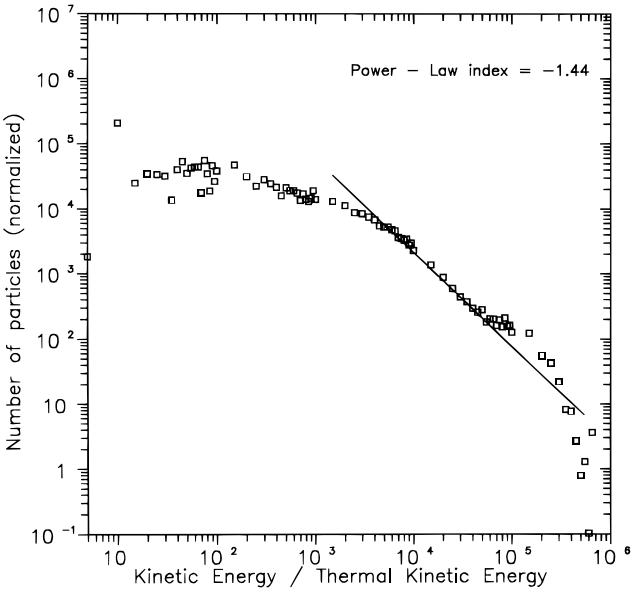

FIG. 3*c*

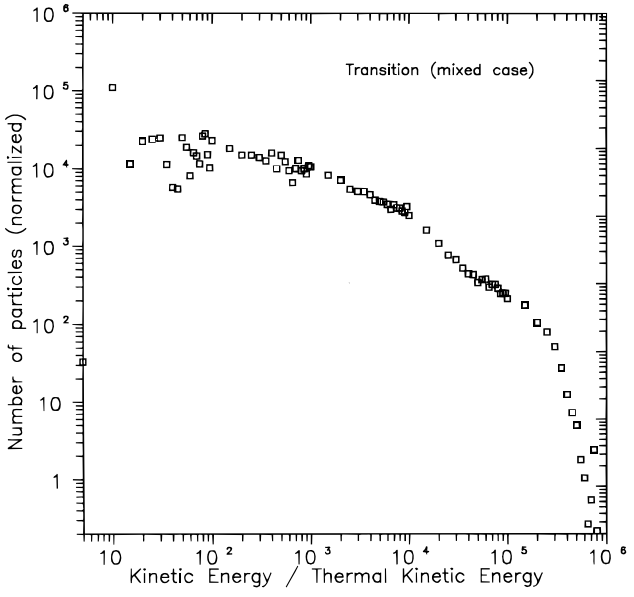

FIG. 3*d*

FIG. 3.—(*a–f*) Numerically evaluated kinetic energy distribution of accelerated electrons (average of 10 sample runs) as a function of the maximum trapping time. (*a*) $N_{max} = 500$; (*b*) $N_{max} = 2000$; (*c*) $N_{max} = 5000$; (*d*) $N_{max} = 10{,}000$; (*e*) $N_{max} = 20{,}000$; (*f*) $N_{max} = 40{,}000$.

The maximum energy reached by the electrons is of the order of several MeV. This value is connected with the selection of the maximum effective acceleration length Δl, which on the other hand does not influence the overall behavior of the distributions. There is a close connection between the structure and the nature of the energy-release $[\epsilon_r(t)]$ or the electric field $[E(t)]$ time series and the resulting energy distributions. This is obvious, since if we assume a random-type time series (white noise), our distributions do not exhibit any kind of structure (power or exponential). In Figure 4, the final kinetic energy distribution for the case of random time series and for $N_{max} = 500$, averaged over 10 sample runs, is presented.

5. SUMMARY AND DISCUSSION

The statistical flare model presents a new way to study eruptive phenomena in the solar atmosphere. It incorporates two basic elements: (1) the fragmentation of the energy-release process and (2) global aspects of the energy flow from the convection zone to the outer parts of the stellar atmosphere. Considering active regions as driven dissipative systems with many interacting elements changes our strategy for the study of acceleration and transport of high-energy particles. Such an approach to the acceleration of energetic particles has been used in the past (see Anastasiadis & Vlahos 1991, 1994). The acceleration of particles has previously been found to be due to the presence of a

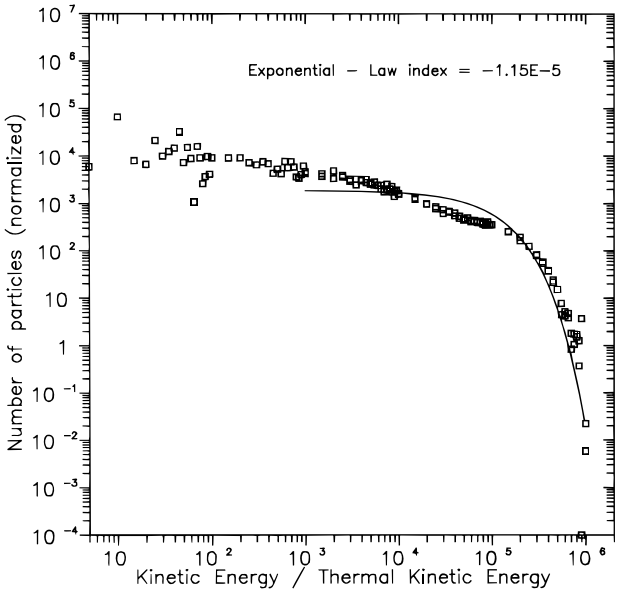

FIG. 3e

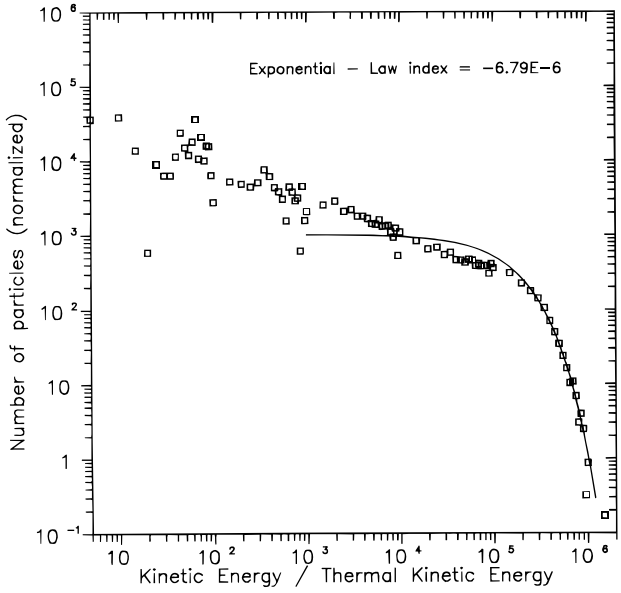

FIG. 3f

number of shock waves distributed randomly inside an active region, with no consideration of the spatiotemporal evolution of the energy-release process. In the present work, the concept of self-organized criticality (SOC) is used to improve our understanding of the amounts of energy released during solar flare events and the ways in which it is released.

In this study, we propose that electrons are accelerated by randomly placed DC electric fields inside an evolving active region. We use the statistical flare model to simulate the spatiotemporal evolution of the energy released in active regions, and we subsequently trace electrons inside this highly structured environment.

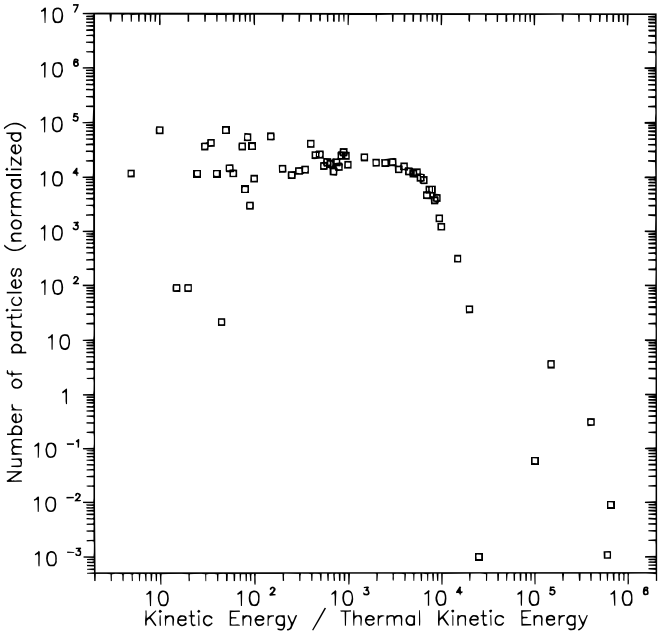

FIG. 4.—Same as Fig. 3, but for random-type time series and N_{max} = 500

Inside the energy-release region, the electric fields form a spatiotemporal structure with fractal geometry. The injected electrons move inside this environment, either sensing an electric field with varying strength locally or flying long distances without any interaction. This movement departs radically from the well-known random walk that occurs with uniformly placed electric fields or with very strong local electric fields. The free parameter in this study is the maximum number of interactions with the electric fields allowed for each electron (N_{max}). Assuming that it takes a certain mean time for each electron-electric field interaction, by increasing N_{max} we increase, on the average, the time that the injected distribution remains inside the energy-release region (the trapping time). It is clear that N_{max} is only a rough measure of the trapping time.

Our main conclusions are as follows:

1. There is a close connection between the turbulent driver inside the convection zone (the power law in the loading process), the frequency distribution of flares (power-law behavior), and the energy distribution of accelerated particles.

2. The well-known debate over the several types of solar flares, i.e., nonthermal (showing power-law behavior for electrons above tens of keV), thermal (having an exponential distribution), or hybrid, may in fact reflect a relation between types of solar flares and the maximum trapping time of the energetic particles inside the active region. For short trapping times, the distribution of energetic electrons exhibits well-defined power-law behavior. As the trapping time increases, the power-law distribution becomes flatter, until an exponential tail starts to develop (the transition region). Further increase of the trapping time leads to exponential distribution of the energetic electrons.

It is obvious that the differences between our estimated energy distributions and the ones derived by analysis of the observational data are due to the fact that no transport of the electrons is taken into account in this article. Our numerically calculated energy distribution can be con-

sidered as an injection-source distribution in a diffusion transport equation that studies the global evolution of the accelerated particles.

The goal of our study was to answer the following question: can the SOC-derived fragmentation of the energy release reproduce the observed inferred energy distributions? We have shown that this is possible. Many questions remain open; a future study will address (1) a more detailed connection of our study with the time evolution of the energy-release process and (2) including transport of energetic particles in our model.

We believe that these concepts are just at the beginning stages of their development. A new and highly promising field of research is now emerging that differs from the localized acceleration approach. Study of the gross characteristics of the global system as well as detailed study of local characteristics is necessary for a complete understanding of a complex problem such as the dynamics of an active region.

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