On the Sum of Kappa Stochastic Variates and Applications to Equal-Gain Combining

Petros S. Bithas, *Member, IEEE,* Nikos C. Sagias, *Member, IEEE,* and Ranjan K. Mallik, *Senior Member, IEEE*

***Abstract*—In this paper we study the statistics of the sum of not necessarily identically distributed kappa, that is, $\mathcal{K}$, random variables (RV)s. Assuming half-integer values for the shaping parameters, novel closed-form expressions for the probability density function (PDF) of the sum of independent $\mathcal{K}$ RVs are obtained, while for arbitrary values of the shaping parameters, a corresponding PDF expression is derived in terms of fast converging infinite series. Furthermore, an infinite series representation for the PDF of the sum of two arbitrarily correlated $\mathcal{K}$ RVs is derived. The proposed analysis is employed to the performance analysis of equal-gain combining (EGC) receivers operating over composite fading/shadowing channels modeled by the $\mathcal{K}$ distribution. More specifically, the outage and the average bit error probabilities, as well as the average channel capacity of EGC receivers operating over such composite environment are studied. Considering different channel fading/shadowing conditions and correlation effects, various numerical performance evaluation results are presented. These results are complemented by equivalent computer simulated ones that validate the accuracy of the proposed analysis.**

***Index Terms*—Average bit error probability (ABEP), channel capacity, correlated statistics, equal gain combining (EGC), kappa fading/shadowing channels, outage probability, sum distributions.**

I. Introduction

WIRELESS communication systems are subject to severe channel impairments, including short-term fading (multipath), as well as, long-term fading (shadowing), that can seriously degrade the overall system performance [1]. One of the simplest and yet most efficient techniques that are usually used to countermeasure signal distortions due to multipath fading/shadowing is diversity. There are several diversity reception techniques used in digital communication systems, including maximal-ratio combining, equal-gain combining (EGC), and selection diversity [1]. Among them, EGC provides an intermediate solution between performance and implementation complexity. Additionally, when employing diversity techniques without sufficiently separated antennas due to space limitations, e.g., small size mobile terminals, performance degradation occurs due to fading correlation.

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P. S. Bithas and N. C. Sagias are with the Department of Telecommunications Science & Technology, School of Applied Sciences and Technology, University of Peloponnese, end of Karaiskaki street, 22100 Tripolis, Greece (e-mail: {pbithas, nsagias}@ieee.org).

R. K. Mallik is with the Department of Electrical Engineering, Indian Institute of Technology - Delhi, Hauz Khas, New Delhi 110016, India (e-mail: rkmallik@ee.iitd.ernet.in).

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Critical issues for studying the performance of diversity systems are *i*) the statistical behavior of the multipath fading and/or shadowing that depends on the radio propagation environment and *ii*) the existence or not of fading correlation. An important statistical characteristic that can describe both *i*) and *ii*) is the probability density function (PDF) of the diversity receivers output signal-to-noise ratio (SNR). However, in many cases the derivation of that PDF, in terms of tabulated functions, is a very difficult mathematical task, especially in cases where EGC receiver is considered. In the open technical literature, several approaches have been proposed for studying EGC receivers over various fading channel models, e.g., [2]–[7]. For instance, in [2], by deriving an exact expression for the PDF of the sum of two correlated Nakagami-m random variables (RV)s, the symbol error rate of several modulation schemes under EGC reception was obtained. In [3], a tight upper bound was derived for the PDF of the sum of generalized-gamma RVs, that was used for studying the performance of EGC receivers. Finally in [7], an accurate approximation for the distribution of the sum of equally correlated Nakagami-m variates was presented and used to study the performance of EGC receivers operating over Nagakami fading channels.

As a general comment it is noted that all the previously published papers deal only with short-term fading, despite the fact that in real radio propagation environments, i.e., terrestrial and satellite land-mobile link, shadowing effects play a critical role. In several works, e.g., [1], [8], the composite fading/shadowing environment is statistically modeled by lognormal-based distributions such as Rayleigh-, Nakagami- and Rice-lognormal. In order to facilitate the performance evaluation of such environments, new families of composite fading distributions have been proposed, most notably the kappa, that is $\mathcal{K}$, (and generalized-$\mathcal{K}$ ($\mathcal{K}_G$)) distributions [9]–[11]. The main advantage of these two distributions is their relatively simple mathematical form, as compared to lognormal-based ones, that allows an integrated performance analysis of digital communication systems operating over fading/shadowing channels. However, as far as the distributions of the sum of $\mathcal{K}$ (and $\mathcal{K}_G$) RVs are concerned, very few publications exist in the open technical literature, e.g., [12]–[14]. For instance, in [12], the fading PDF of EGC diversity receiver operating over free-space optical communication systems modeled by the gamma-gamma distribution, i.e., squared $\mathcal{K}_G$, was derived in terms of infinite series. All in all, to the best of the authors knowledge, the PDF of the sum of $\mathcal{K}$ and/or $\mathcal{K}_G$ distributed envelopes is not available in the open research literature. In the current work, assuming integer plus one-half values for the fading parameters, closed-form expressions for the PDF of the sum of independent $\mathcal{K}$ RVs are obtained, while for the correlated case, fast converging infinite series are assessed. Further considering identical parameters, simpler expressions are provided and employed to the performance analysis of EGC diversity receivers operating over such fading/shadowing environment.

The remainder of this paper is as follows. In Section II, expressions for the PDF, cumulative distribution function (CDF), and characteristic function (CF) of the sum of independent and correlated $\mathcal{K}$ RVs are obtained. In Section III, these results are applied to the performance analysis of EGC diversity receivers operating over independent and correlated $\mathcal{K}$ fading channels, in terms of the outage probability, average bit error probability (ABEP), and average channel capacity. In Section IV, analytical performance evaluation results are presented, while the concluding remarks can be found in Section V.

II. Statistical Analysis of the Sum of $\mathcal{K}$ RVs

Let X_ℓ ($\ell = 1, 2, \ldots, L$) represent $\mathcal{K}$-distributed RVs with PDF given by [9, eq. (2)]

$$f_{X_\ell}(x) = \frac{4x^{k_\ell}}{\Gamma(k_\ell)} \left(\frac{1}{\Omega_\ell}\right)^{(k_\ell+1)/2} K_{k_\ell-1}\left(2\frac{x}{\sqrt{\Omega_\ell}}\right) \quad (1)$$

where $k_\ell \geq 0$ is shaping parameter of the distribution, Ω_ℓ is the average fading power given as $\Omega_\ell = \mathbb{E}\langle X_\ell^2 \rangle / k_\ell$, with $\mathbb{E}\langle\cdot\rangle$ denoting expectation, $K_\alpha(\cdot)$ is the modified Bessel function of the second kind and order α [15, eq. (8.407/1)] and $\Gamma(\cdot)$ is the Gamma function [15, eq. (8.310/1)]. By using different values for k_ℓ, (1) describes various shadowing conditions, from severe shadowing, e.g., $k_\ell \to 0$, to no shadowing, e.g., $k_\ell \to \infty$. The CF of X_ℓ is given by [16, eq. (8)]

$$\begin{aligned}\Phi_{X_\ell}(\omega) = {} & \frac{\sqrt{\pi}\, 4^{k_\ell}\, \Gamma(2k_\ell)}{\Gamma(k_\ell)\,\Gamma(k_\ell+3/2)} \left(2 - j\sqrt{\Omega_\ell}\,\omega\right)^{-2k_\ell} \\ & \times {}_2F_1\left(2k_\ell, k_\ell - \frac{1}{2}; k_\ell + \frac{3}{2}; \frac{2 + j\sqrt{\Omega_\ell}\,\omega}{-2 + j\sqrt{\Omega_\ell}\,\omega}\right)\end{aligned} \quad (2)$$

where $j = \sqrt{-1}$ and ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ is the Gauss hypergeometric function [15, eq. (9.100)].

Next, important statistical characteristics for the distribution of sum of $\mathcal{K}$ RVs are studied for independent as well as correlated statistics.

A. Independent RVs

By considering independent statistics and letting k_ℓ be equal to a half-integer, the PDF of the sum of L $\mathcal{K}$ RVs is extracted in closed form. Furthermore, for arbitrary values of $k_\ell \geq 3/2$ and identically distributed (id) RVs, a fast converging infinite series expression for the corresponding PDF is provided.

1) Case of half-integers k_ℓ's:

Theorem 1: Let Z denote a RV defined as

$$Z \triangleq \sum_{\ell=1}^{L} X_\ell \quad (3)$$

where X_ℓ follows (1), with $k_\ell = \alpha_\ell + 1/2$, $\alpha_\ell \in \mathbb{N}$. The PDF of Z is given by

$$\begin{aligned}f_Z(x) = {} & \sum_{\{i_\ell\}_{\ell=1}^{L}=0}^{\{\alpha_\ell-1\}_{\ell=1}^{L}} \sum_{\{j_\ell\}_{\ell=1}^{L-1}=0}^{2} \sum_{b_{1,j_1}=0}^{A_{j_1}} \sum_{b_{2,j_2}=0}^{(2-j_2)A_3+(j_2-1)b_{1,j_1}} \cdots \\ & \sum_{b_{L-1,j_{L-1}}=0}^{(2-j_{L-1})A_L+(j_{L-1}-1)b_{L-2,j_{L-2}}} \left[\prod_{i=1}^{L-1} (-1)^{\left(b_{i,j_i}+1\right)(j_i-1)} \lambda_i^{b_{i,j_i}}\right] \prod_{\ell=1}^{L} \mathbb{S}_\ell \\ & \times \left[\prod_{i=3}^{L} \frac{A_i!\left(-A_i - b_{i-2,j_{i-2}} - 1\right)_{b_{i-2,j_{i-2}}+1}}{\left(A_i + b_{i-2,j_{i-2}} + 1\right)\lambda_{i-1}^{A_i+b_{i-2,j_{i-2}}+1}} \right. \\ & \quad \left. \times \frac{\left[A_i\left(j_{i-1} - 2\right) - b_{i-2,j_{i-2}}\left(j_{i-1} - 1\right)\right]_{b_{i-1,j_{i-1}}}}{b_{i-1,j_{i-1}}!\left(-A_i - b_{i-2,j_{i-2}}\right)_{b_{i-1,j_{i-1}}}}\right] \\ & \quad \times \frac{A_1!\left(-A_1 - A_2 - 1\right)_{A_2+1}\left(-A_{j_1}\right)_{b_{1,j_1}}}{\left(A_1 + A_2 + 1\right)\Delta_{1-2}^{A_1+A_2+1} b_{1,j_1}!\left(-A_1 - A_2\right)_{b_{1,j_1}}} \\ & \quad \times x^{b_{L-1,j_{L-1}}} \exp\left(-\phi_{L-1}x\right)\end{aligned} \quad (4)$$

where

$$\mathbb{S}_\ell = \sqrt{\pi}\frac{(\alpha_\ell + i_\ell - 1)!}{i_\ell!\,(\alpha_\ell - i_\ell - 1)!}\frac{2^{1-2i_\ell}}{\Gamma(k_\ell)}\Omega_\ell^{(2i_\ell-2k_\ell-1)/4},$$

$$\begin{aligned}&\lambda_1 = \Delta_{1-2}, && \phi_1 = \Delta_{h_1}, \\ &\lambda_2 = \Delta_{3-h_1}, && \phi_2 = (2-h_2)\,\Delta_3 + (h_2-1)\,\Delta_{h_1}, \\ &\lambda_L = \Delta_{L+1} - \phi_{L-1}, && \phi_L = (2-h_L)\,\Delta_{L+1} + (h_L-1)\,\lambda_{L-1}.\end{aligned}$$

Furthermore, in (4), $A_x = \alpha_x - i_x$, $\Delta_x = 2\,(1/\Omega_x)^{1/2}$ and $\Delta_{x-y} = 2\left[(1/\Omega_x)^{1/2} - (1/\Omega_y)^{1/2}\right]$, $(\cdot)_p$ is the Pochhammer's symbol [15, p. xliii], with $p \in \mathbb{N}$.

Proof: See the Appendix. ■

A worth noting observation that can be made from (4) is that the sum of L $\mathcal{K}$ RVs is given as a finite sum of gamma distributed RVs. Assuming identical parameters, i.e., $k_\ell = k, \alpha_\ell = \alpha, \Omega_\ell = \Omega$, (4) simplifies to

$$f_Z(x) = \mathcal{G}(L, k, \Omega, \alpha)\, x^{\Xi_L - 1} \exp\left(-2\frac{x}{\sqrt{\Omega}}\right) \quad (5)$$

where

$$\begin{aligned}\mathcal{G}(x, t, s, g) = {} & \sum_{\{i_\ell\}_{\ell=1}^{x}=0}^{g-1} B\,(g - i_1 + 1, g - i_2 + 1) \\ & \times \left[\prod_{\ell=1}^{x} \sqrt{\pi}\frac{(g + i_\ell - 1)!}{i_\ell!\,(g - i_\ell - 1)!}\frac{2^{1-2i_\ell}}{\Gamma(t)} s^{(2i_\ell-2t-1)/4}\right] \\ & \times \prod_{\ell=3}^{x} B\left(g - i_\ell + 1, (\ell - 1)\,g - \sum_{y=1}^{\ell-1} i_y + \ell - 1\right)\end{aligned} \quad (6)$$

with $\Xi_L = L\alpha - \sum_{\ell=1}^{L} i_\ell + L$ and $B(\cdot, \cdot)$ being the Beta function [15, eq. (8.380)]. It is noted that an alternative representation to (5) is given in [17]. For the special case where $k = 1/2$, (5) further simplifies to [18, eq. (13)], i.e., the Erlang distribution.

Lemma 1: For id X_ℓ's, the CDF of Z can be obtained as

$$F_Z(x) = \mathcal{G}(L, k, \Omega, \alpha)\left(\sqrt{\Omega}/2\right)^{\Xi_L} \gamma\left(\Xi_L, 2\frac{x}{\sqrt{\Omega}}\right) \quad (7)$$

TABLE I
MINIMUM NUMBER OF TERMS FOR CONVERGENCE OF (11) WITH AN ACCURACY BETTER THAN $\pm 10^{-4}$.

	$\Omega = 0$ dB		$\Omega = 20$ dB	
x	$k = 1.5$	$k = 4.5$	$k = 1.5$	$k = 4.5$
1	1	5	1	6
3	2	6	1	6
5	6	6	1	6
7	19	6	1	6

where $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function [15, eq. (8.350/1)].

Proof: By using (5) in the definition of the CDF, i.e., [19, eq. (4.17)], and with the aid of [15, eq. (3.351/1)], (7) is extracted. ■

Lemma 2: For id X_ℓ's, the CF of Z can be obtained as

$$\Phi_Z(\omega) = \mathcal{G}(L,k,\Omega,\alpha)\,\Gamma(\Xi_L)\left[2\left(\frac{1}{\Omega}\right)^{1/2} - j\omega\right]^{-\Xi_L}. \quad (8)$$

Proof: By using (5) in the definition of the CF, i.e., [19, eq. (5.60)], and with the aid of [15, eq. (3.351/3)], (8) is extracted. ■

2) Case of $k_\ell \geq 3/2$: Now we consider id X_ℓ's and arbitrary values of $k_\ell = k \geq 3/2$. Substituting (2) in [19, eq. (8.52)] and using first [20, eq. (07.23.17.0054.01)] and [15, eq. (9.100)], then the multinomial identity [15, eq. (0.314)] and the binomial theorem [15, eq. (1.111)], $\Phi_Z(\omega)$ can be expressed as

$$\begin{aligned}\Phi_Z(\omega) = &\left[\frac{\sqrt{\pi}\,\Gamma(2k)}{\Gamma(k)\Gamma(k+3/2)}\right]^L 2^{-2L(k-1)}\\ &\times \sum_{h=0}^{\infty}\sum_{i=0}^{h}\binom{h}{i}\delta_h(-2)^i\left(1-\frac{\sqrt{\Omega}}{2}j\,\omega\right)^{-2L-i}\end{aligned} \quad (9)$$

where

$$\begin{aligned}\delta_0 = \beta_0^L,\quad \delta_h &= \frac{1}{h\beta_0}\sum_{p=1}^{h}(pL-h+p)\,\beta_p\,\delta_{h-p},\\ \beta_h &= \frac{(-k+3/2)_h\,(2)_h}{h!\,(k+3/2)_h}.\end{aligned} \quad (10)$$

By applying the inverse Fourier transform in (9), an alternative expression for $f_Z(x)$ can be easily derived as

$$\begin{aligned}f_Z(x) = &\left[\frac{\sqrt{\pi}\,\Gamma(2k)/\Gamma(k)}{\Gamma(k+3/2)}\right]^L 2^{2L(2-k)}\sum_{h=0}^{\infty}\sum_{i=0}^{h}\binom{h}{i}\\ &\times\frac{\delta_h(-1)^i 2^{2i}}{\Gamma(2L+i)\Omega^{L+i/2}}\,x^{2L+i-1}\exp\left(-2\frac{x}{\sqrt{\Omega}}\right).\end{aligned} \quad (11)$$

The rate of convergence of the infinite series of the above expression is investigated in Table I, where we summarize the minimum number of terms, $h_{\min}$, needed to achieve an accuracy better than $\pm 10^{-4}$ for $L=2$ and several values of Ω, k, and x. It is clear that only a few terms are needed in order to achieve the target accuracy, while the required terms increase by increasing x, k and/or decreasing Ω. Similar rates of convergence have been observed for different values of L.

B. Correlated RVs

The PDF of two correlated and not id $\mathcal{K}$-distributed RVs X_1 and X_2 is given by [21, eq. (4)]

$$\begin{aligned}f_{X_1,X_2}(x_1,x_2) = &\sum_{t,h,q=0}^{\infty}\frac{16(k_2-k_1)_q(1-\rho_G)^{k_2-1}}{h!q!(t!)^2\Gamma(k_1)\Gamma(k_2+h+q)}\\ &\times\rho_G^{h+q}\rho_R^t\left[\prod_{i=1}^{2}\frac{x_i^{\tau_i}\Omega_i^{-1/2}}{(\sigma_1\Omega_i)^{\tau_i/2}}K_{\tau_i-2t-1}\left(\frac{2x_i}{\sqrt{\sigma_1\Omega_i}}\right)\right]\end{aligned} \quad (12)$$

where $\tau_\ell = k_\ell + h + t + (\ell-1)q$, $0 \leq \rho_R < 1$ is the correlation coefficient between the envelopes, $0 \leq \rho_G < 1$ is the correlation coefficient between their powers, $\sigma_x = (1-\rho_R)(1-\rho_G)x$ and $k_2 \geq k_1$.

Theorem 2: Let Z denote a RV defined as

$$Z \triangleq X_1 + X_2 \quad (13)$$

where the joint PDF of X_1 and X_2 is given by (12), with $k_\ell = \alpha_\ell + 1/2$, $\alpha_\ell \in \mathbb{N}$. The PDF of Z is given by

$$\begin{aligned}f_Z(x) = &\sum_{t,h,q=0}^{\infty}\sum_{i_1=0}^{|\alpha_1+h-t-1|}\sum_{i_2=0}^{|\alpha_2+h-t+q-1|}\frac{\rho_R^t\rho_G^{h+q}(k_2-k_1)_q}{h!q!t!^2\Gamma(k_1)/4\pi}\\ &\times\frac{B(\tau_2-i_2+1/2,\tau_1-i_1+1/2)(1-\rho_G)^{k_2-1/2}}{\sigma_1^{(\tau_{1,2}-i_{1,2})/2}\Gamma(k_2+h+q)(1-\rho_R)^{-1/2}}\\ &\times\left[\prod_{\ell=1}^{2}\frac{(|\tau_\ell-2t|+i_\ell-1/2)!}{(|\tau_\ell-2t-1|-i_\ell-1/2)!}\frac{\Omega_\ell^{(i_\ell-\tau_\ell-1/2)/2}}{i_\ell!2^{2i_\ell}}\right]\\ &\times z^{\tau_{1,2}-i_{1,2}}\exp\left(\frac{-2x}{\sqrt{\sigma\Omega_2}}\right)\\ &\times {}_1F_1\left(\tau_1-i_1+\frac{1}{2};1+\tau_{1,2}-i_{1,2};\sum_{\ell=1}^{2}\frac{2(-1)^{\ell+1}}{\sqrt{\sigma\Omega_\ell}}x\right)\end{aligned} \quad (14)$$

where $|\cdot|$ denotes absolute value, $\theta_{1,2} = \theta_1 + \theta_2$, ${}_1F_1(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function [15, eq. (9.210/1)].

Proof: Using [20, eq. (03.04.03.0004.01)] in (12) and then applying [19, eq. (6.40)], an integral as in (A-2) appears. This integral can be solved in closed form with the aid of [15, eq. (3.383/1)], and hence, after some mathematical manipulations, (14) can be extracted. ■

Assuming identical parameters, (14) simplifies to the following expression

$$\begin{aligned}f_Z(x) = &\sum_{n,h=0}^{\infty}\sum_{i_1,i_2=0}^{|\alpha+n-h-1|}\mathcal{H}\frac{\sigma_\Omega^{(i_{1,2}-1)/2-n-h}}{\Omega^k(1-\rho_R)^{k-1}}\\ &\times\left[\prod_{\ell=1}^{2}\frac{(|\tau-2h-1|+i_\ell-1/2)!}{i_\ell!\,(|\tau-2h-1|-i_\ell-1/2)!}\right]\\ &\times x^{2\tau-i_{1,2}}\exp\left(-\frac{2x}{\sqrt{\sigma_\Omega}}\right)\end{aligned} \quad (15)$$

where

$$\mathcal{H} = \frac{\rho_R^h\rho_G^n\,\pi B(\tau-i_2+1/2,\tau-i_1+1/2)}{\Gamma(k)\,\Gamma(k+n)\,h!^2\,n!\,2^{2(i_{1,2}-1)}},\ \tau = k+n+h.$$

Lemma 3: For id X_ℓ's, the CDF of Z can be obtained as

$$F_Z(x) = \sum_{n,h=0}^{\infty} \sum_{i_1,i_2=0}^{|\alpha+n-h-1|} \mathcal{H} \frac{(1-\rho_R)(1-\rho_G)^k}{2^{2\tau+1-i_{1,2}}} \times \left[\prod_{\ell=1}^{2} \frac{(|\tau-2h-1|+i_\ell-1/2)!}{i_\ell!\,(|\tau-2h-1|-i_\ell-1/2)!}\right] \times \gamma\left(1+2\tau-i_{1,2}, \frac{2x}{\sqrt{\sigma_\Omega}}\right). \tag{16}$$

Proof: By using (15) in the definition of the CDF, i.e., [19, eq. (4.17)], and with the aid of [15, eq. (3.351/1)], (16) is extracted. ■

Lemma 4: For id X_ℓ's, the CF of Z can be obtained as

$$\Phi_Z(x) = \sum_{n,h=0}^{\infty} \sum_{i_1,i_2=0}^{|\alpha+n-h-1|} \mathcal{H} \frac{\sigma_\Omega^{(i_{1,2}-1)/2-n-h}}{\Omega^k (1-\rho_R)^{k-1}} \times \left[\prod_{\ell=1}^{2} \frac{(|\tau-2h-1|+i_\ell-1/2)!}{i_\ell!\,(|\tau-2h-1|-i_\ell-1/2)!}\right] \times \frac{(2\tau-i_{1,2})!}{\left(\frac{2}{\sqrt{\sigma_\Omega}}-j\omega\right)^{1+2\tau-i_{1,2}}}. \tag{17}$$

Proof: By using (15) in the definition of the CF, i.e., [19, eq. (5.60)], and with the aid of [15, eq. (3.351/3)], (17) can be obtained. ■

III. Application to EGC Receivers

Let us consider an EGC receiver operating over $\mathcal{K}$-distributed fading/shadowing channels. The equivalent baseband received signal at the ℓth antenna is expressed as $z_\ell = s h_\ell + n_\ell$, where s is the transmitted complex symbol with energy $E_s = \mathbb{E}\langle |s|^2 \rangle$, n_ℓ is the complex additive white Gaussian noise (AWGN) with single sided power spectral density N_0 assumed identical to all branches, and h_ℓ is the channel complex gain, i.e., $X_\ell = |h_\ell|$. Furthermore, by considering ideal phase estimation, only the distributed fading envelope affects the received signal. The instantaneous SNR per symbol at the ℓth input branch, γ_ℓ, and the corresponding average SNR, $\overline{\gamma}_\ell$, can be expressed as

$$\gamma_\ell = X_\ell^2 \frac{E_s}{N_0} \tag{18a}$$

$$\overline{\gamma}_\ell = \mathbb{E}\langle X_\ell^2 \rangle \frac{E_s}{N_0} = \Omega_\ell\, k \frac{E_s}{N_0} \tag{18b}$$

respectively. In the following subsections, important statistical metrics of the EGC diversity receiver output SNR will be presented and then applied to its performance analysis.

A. EGC Statistical Properties

The total SNR at the output of an EGC receiver is given by [1, eq. (9.51)]

$$\gamma_{\text{egc}} = \frac{E_s \left(\sum_{\ell=1}^{L} X_\ell\right)^2}{L N_0} = \frac{E_s Z^2}{L N_0}. \tag{19}$$

Applying (18) and (19) to the expressions provided in Section II, important statistical properties of the EGC output SNR can be easily obtained as follows.

1) Independent $\mathcal{K}$ Fading: Applying (19) in (5), the PDF of the EGC output SNR can be obtained as

$$f_{\gamma_{\text{egc}}}(\gamma) = \mathcal{G}\left(L, k, \frac{\overline{\gamma}}{k}, \alpha\right) \frac{L}{2} (L\gamma)^{\Xi_L/2-1} \exp\left(-2\sqrt{\frac{kL\gamma}{\overline{\gamma}}}\right) \tag{20}$$

while the corresponding CDF can be derived from (20) as

$$F_{\gamma_{\text{egc}}}(\gamma) = \mathcal{G}\left(L, k, \frac{\overline{\gamma}}{k}, \alpha\right) \left(2\sqrt{\frac{k}{\overline{\gamma}}}\right)^{-\Xi_L} \gamma\left(\Xi_L, 2\sqrt{\frac{kL\gamma}{\overline{\gamma}}}\right). \tag{21}$$

2) Correlated $\mathcal{K}$ Fading: Applying (19) in (15), the PDF of the EGC output SNR can be obtained as

$$f_{\gamma_{\text{egc}}}(\gamma) = \sum_{n,h=0}^{\infty} \sum_{i_1,i_2=0}^{|\alpha+n-h-1|} \mathcal{H} \frac{(1-\rho_R)^{1-k}}{(\sigma_{\overline{\gamma}}/k)^{n+h-(i_{1,2}-1)/2}} \times \left(\frac{k}{\overline{\gamma}}\right)^k \left[\prod_{\ell=1}^{2} \frac{(|\tau-2h-1|+i_\ell-1/2)!}{i_\ell!\,(|\tau-2h-1|-i_\ell-1/2)!}\right] \times (2\gamma)^{\tau-(i_{1,2}+1)/2} \exp\left(-2\sqrt{\frac{2k\gamma}{\sigma_{\overline{\gamma}}}}\right) \tag{22}$$

while the corresponding CDF can be derived from (22) as

$$F_{\gamma_{\text{egc}}}(\gamma) = \sum_{n,h=0}^{\infty} \sum_{i_1,i_2=0}^{|\alpha+n-h-1|} \mathcal{H} \frac{(1-\rho_R)(1-\rho_G)^k}{2^{2\tau+1-i_{1,2}}} \times \left[\prod_{\ell=1}^{2} \frac{(|\tau-2h-1|+i_\ell-1/2)!}{i_\ell!\,(|\tau-2h-1|-i_\ell-1/2)!}\right] \times \gamma\left(1+2\tau-i_{1,2}, 2\sqrt{\frac{2k\gamma}{\sigma_{\overline{\gamma}}}}\right). \tag{23}$$

B. Performance Analysis of EGC

In this subsection the performance of EGC receivers operating over composite $\mathcal{K}$ distributed fading channels is presented in terms of ABEP ($\overline{P}_{\text{be}}$), outage probability and the average channel capacity ($\overline{C}$).

1) Average Symbol Error Probability (ASEP): The ASEP, $\overline{P}_{\text{se}}$, can be evaluated directly by averaging the conditional symbol error probability, $P_e(\gamma)$, over the PDF of γ_{egc}, i.e., $\overline{P}_{\text{se}} = \int_0^\infty P_e(\gamma)\, f_{\gamma_{\text{egc}}}(\gamma) \mathrm{d}\gamma$. This yields: For binary phase shift keying (BPSK), square M-quadrature amplitude modulation (QAM) and for high values of the average input SNR, $P_e(\gamma) = \mathcal{A}\text{erfc}(\sqrt{\mathcal{B}\gamma})$, where erfc$(\cdot)$ is the complementary error function [15, eq. (8.250/4)] and $\mathcal{A}, \mathcal{B}$ constants depending on the specific modulation scheme, i.e., for BPSK $\mathcal{A} = 1/2$ and $\mathcal{B} = 1$ [22]. By making a change of variables and using [20, eq. (06.27.21.0133.01)], the ASEP for EGC assuming independent K fading channels can be expressed in closed form as

$$\overline{P}_{\rm se} = \mathcal{G}\left(L, k, \frac{\overline{\gamma}}{k}, \alpha\right) \frac{\mathcal{A}}{\sqrt{\pi}} \left(\frac{L}{\mathcal{B}}\right)^{\Xi_L/2} \left\{ \Gamma\left(\frac{\Xi_L+1}{2}\right) \times \frac{1}{\Xi_L} {}_pF_q\left(\frac{\Xi_L+1}{2}, \frac{\Xi_L}{2}; \frac{1}{2}, \frac{\Xi_L+2}{2}; \frac{kL}{\mathcal{B}\overline{\gamma}}\right) - \frac{2}{\Xi_L+1}\left(\frac{kL}{\mathcal{B}\overline{\gamma}}\right)^{1/2} \Gamma\left(\frac{\Xi_L}{2}+1\right) \times {}_pF_q\left(\frac{\Xi_L+1}{2}, \frac{\Xi_L}{2}+1; \frac{3}{2}, \frac{\Xi_L+3}{2}; \frac{kL}{\mathcal{B}\overline{\gamma}}\right)\right\} \quad (24)$$

where ${}_pF_q(\cdot)$ is the generalized hypergeometric function [15, eq. (9.14/1)]. For correlated $\mathcal{K}$ fading channels, the ASEP for EGC can be expressed as

$$\overline{P}_{\rm se} = \sum_{n,h=0}^{\infty} \sum_{i_1,i_2=0}^{|\alpha+n-h-1|} \frac{\mathcal{H}\,(2/\mathcal{B})^{\tau-(i_{1,2}-1)/2}}{(\sigma\overline{\gamma}/k)^{n+h-(i_{1,2}-1)/2}(1-\rho_R)^{k-1}} \times \left[\prod_{\ell=1}^{2} \frac{(|\tau-2h-1|+i_\ell-1/2)!}{i_\ell!\,(|\tau-2h-1|-i_\ell-1/2)!}\right] \times \left(\frac{k}{\overline{\gamma}}\right)^k \frac{\mathcal{A}}{\sqrt{\pi}} \left\{ \frac{\Gamma(\tau+1-i_{1,2}/2)}{2\tau-i_{1,2}+1} \times {}_pF_q\left(\tau+1-\frac{i_{1,2}}{2}, \tau-\frac{i_{1,2}-1}{2}; \frac{1}{2}, \tau-\frac{i_{1,2}-3}{2}; \frac{2k/\mathcal{B}}{\sigma\overline{\gamma}}\right) - {}_pF_q\left(\tau+1-\frac{i_{1,2}}{2}, \tau-\frac{i_{1,2}}{2}+\frac{3}{2}; \frac{3}{2}, \tau+2-\frac{i_{1,2}}{2}; \frac{2k/\mathcal{B}}{\sigma\overline{\gamma}}\right) \times 2\sqrt{\frac{2k}{\mathcal{B}\sigma\overline{\gamma}}} \frac{\Gamma[\tau-(i_{1,2}-3)/2]}{2\tau-i_{1,2}+2} \right\}. \quad (25)$$

High SNR approximation: Considering high values for the average input SNR, i.e., $\overline{\gamma} \geq 25$ dB, and by using (24) and [20, eq. (07.31.06.0003.01)], the ASEP can be approximated as

$$\overline{P}_{\rm se} \simeq \mathcal{G}\left(L, k, \frac{\overline{\gamma}}{k}, \alpha\right) \frac{\mathcal{A}}{\sqrt{\pi}} \left(\frac{L}{\mathcal{B}}\right)^{\Xi_L/2} \left\{ \Gamma\left(\frac{\Xi_L+1}{2}\right) \times \frac{1}{\Xi_L} - \frac{2\,[kL/(\mathcal{B}\overline{\gamma})]^{1/2}}{\Xi_L+1} \Gamma\left(\frac{\Xi_L}{2}+1\right)\right\}. \quad (26)$$

It is interesting to note that for $\overline{\gamma} \to \infty$, the second term inside the curly brackets can be neglected. Furthermore, using (6), $\mathcal{G}(L, k, \overline{\gamma}/k, \alpha)$ can be written as a polynomial with respect to $\overline{\gamma}$ with its dominant term (the one with the maximum exponent) being raised to $-L$. Hence, $\overline{P}_{\rm se}$ is proportional to $\overline{\gamma}^{-L}$, yielding that the diversity order is equal to the number of EGC branches.

Also, for high SNR, (25) can be approximated as

$$\overline{P}_{\rm se} \simeq \sum_{n,h=0}^{\infty} \sum_{i_1,i_2=0}^{|\alpha+n-h-1|} \frac{\mathcal{H}\,(1-\rho_R)^{1-k}}{(\sigma\overline{\gamma}/k)^{n+h-(i_{1,2}-1)/2}} \left(\frac{k}{\overline{\gamma}}\right)^k \times \left[\prod_{\ell=1}^{2} \frac{(|\tau-2h-1|+i_\ell-1/2)!}{i_\ell!\,(|\tau-2h-1|-i_\ell-1/2)!}\right] \times \frac{\mathcal{A}}{\sqrt{\pi}} \left(\frac{2}{\mathcal{B}}\right)^{\tau-(i_{1,2}-1)/2} \left\{ \frac{\Gamma(\tau+1-i_{1,2}/2)}{2\tau-i_{1,2}+1} - \sqrt{\frac{8k}{\mathcal{B}\,\sigma\overline{\gamma}}} \frac{\Gamma[\tau-(i_{1,2}-3)/2]}{2\tau-i_{1,2}+2} \right\}. \quad (27)$$

2) Outage Probability: By using (21) and (23), for independent and correlated fading conditions respectively, the outage probability of the EGC receiver, defined as the probability that the received SNR falls below a given threshold, $\gamma_{\rm th}$, can be simply obtained as $P_{\rm out} = F_{\gamma_{\rm egc}}(\gamma_{\rm th})$.

High SNR approximation: For high values of $\overline{\gamma}$, i.e., low values of the normalized outage threshold, $\gamma_{\rm th}/\overline{\gamma}$, and using [20, eq. (06.06.06.0004.01)], $P_{\rm out}$ can be approximated as

$$P_{\rm out} \simeq \mathcal{G}\left(L, k, \frac{\overline{\gamma}}{k}, \alpha\right) \frac{(L\gamma_{\rm th})^{\Xi_L/2}}{\Xi_L} \quad (28)$$

while for the correlated case $P_{\rm out}$ can be approximated as

$$P_{\rm out} \simeq \sum_{n,h=0}^{\infty} \sum_{i_1,i_2=0}^{|\alpha+n-h-1|} \frac{\mathcal{H}\,(1-\rho_R)(1-\rho_G)^k}{2^{2\tau+1-i_{1,2}}} \times \left[\prod_{\ell=1}^{2} \frac{(|\tau-2h-1|+i_\ell-1/2)!}{i_\ell!\,(|\tau-2h-1|-i_\ell-1/2)!}\right] \frac{\left(\sqrt{\frac{8k\gamma_{\rm th}}{\sigma\overline{\gamma}}}\right)^{1+2\tau-i_{1,2}}}{1+2\tau-i_{1,2}}. \quad (29)$$

3) Average Channel Capacity: The average channel capacity, in the Shannon's sense, is defined as $\overline{C} \triangleq B_w \int_0^\infty \log_2(1+\gamma) f_{\gamma_{\rm egc}}(\gamma)\, {\rm d}\gamma$, [23], where B_w is signal bandwidth. Hence, using (20), making a change of variables of the form $x = \gamma^{1/2}$, and following a procedure similar to that in [24, App. B] the following integral needs to be solved $\int_0^\infty x^v \exp(-\mu x)/\left(x^2+1\right) {\rm d}x$. This integral can be solved with the aid of [15, eq. (3.389/6)], and consequently using this solution, the capacity of L-branch EGC for independent $\mathcal{K}$ fading channels can be expressed in closed form as

$$\overline{C} = \mathcal{G}\left(L, k, \frac{\overline{\gamma}}{k}, \alpha\right) \frac{B_w L^{\Xi_L/2}}{\ln 2} \sum_{m=1}^{\Xi_L} \frac{(\Xi_L-1)!}{(\Xi_L-m)!} \times \left[2\left(\frac{kL}{\overline{\gamma}}\right)^{1/2}\right]^{-m} \Gamma(\Xi_L-m+1) \times \sum_{h=1}^{2} \exp\left[(-1)^{1+h} j \left(2\left(\frac{kL}{\overline{\gamma}}\right)^{1/2} + \frac{(\Xi_L-m)\pi}{2}\right)\right] \times \Gamma\left[m-\Xi_L, (-1)^{h+1} 2j \left(\frac{kL}{\overline{\gamma}}\right)^{1/2}\right]. \quad (30)$$

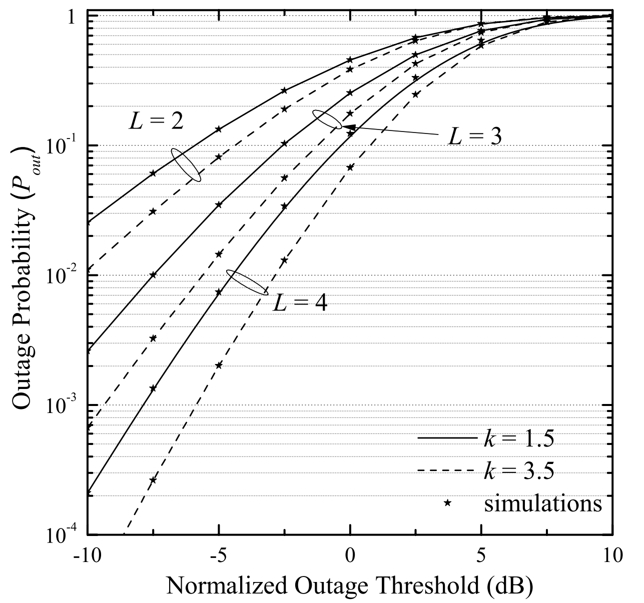

Fig. 1. $P_{\rm out}$ of EGC receiver versus $\gamma_{\rm th}/\overline{\gamma}$ for several values of L and k.

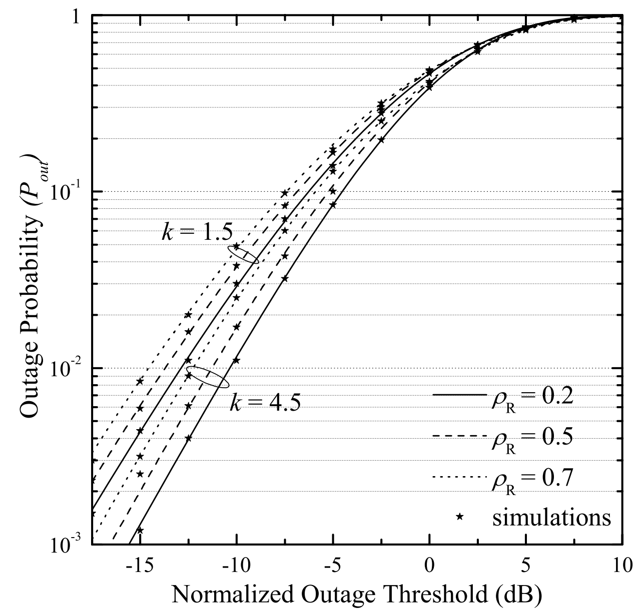

Fig. 2. $P_{\rm out}$ of EGC receiver versus $\gamma_{\rm th}/\overline{\gamma}$ for several values of ρ_R and k.

Furthermore, for correlated $\mathcal{K}$ fading channels, the average channel capacity with EGC can be obtained as

$$\begin{aligned}\overline{C} = &\sum_{n,h=0}^{\infty} \sum_{i_1,i_2=0}^{|\alpha+n-h-1|} \frac{B_w \mathcal{H}\left(\sigma_{\overline{\gamma}}/2k\right)^{(i_{1,2}-1)/2-n-h}}{(1-\rho_R)^{k-1}\ln 2} \\ &\times \left[\prod_{\ell=1}^{2} \frac{(|\tau-2h-1|+i_\ell-1/2)!}{i_\ell!\,(|\tau-2h-1|-i_\ell-1/2)!}\right] \left(\frac{2k}{\overline{\gamma}}\right)^{k} \sum_{m=1}^{2\tau-i_{1,2}+1} \\ &\times \frac{(2\tau-i_{1,2})!}{(2\tau-i_{1,2}+1-m)!} \left(\frac{\sigma_{\overline{\gamma}}}{8k}\right)^{m/2} \Gamma\left(2\tau-i_{1,2}+2-m\right) \\ &\times \sum_{p=1}^{2} \exp\left[(-1)^{1+p} j\pi \left(\sqrt{\frac{8k}{\pi^2 \sigma_{\overline{\gamma}}}} + \frac{1-m+2\tau-i_{1,2}}{2}\right)\right] \\ &\times \Gamma\left[m-1-2\tau-i_{1,2}, (-1)^{p+1} 2j \sqrt{\frac{2k}{\sigma_{\overline{\gamma}}}}\right]. \end{aligned} \tag{31}$$

IV. Numerical Results and Discussion

In this section, using the previously derived theoretical analysis for the performance of EGC receiver, representative numerical evaluated results for the $P_{\rm out}$, ABEP, and $\overline{C}$ are presented and discussed. These performances have been investigated under a wide range of $\mathcal{K}$ fading and shadowing conditions. Furthermore, it should be noted that for all the considered ranges of values of the correlated statistics, quick rates of convergence of the infinite series have been observed.

In Fig. 1, assuming independent fading conditions and using (21), the outage probability, $P_{\rm out}$, is plotted as a function of the normalized outage threshold, $\gamma_{\rm th}/\overline{\gamma}$, for various values of L and k. It is depicted that $P_{\rm out}$ improves by increasing L and/or k. Similar behavior is also observed in Fig. 2, where by considering correlated fading conditions and using (23), $P_{\rm out}$ is plotted as a function of $\gamma_{\rm th}/\overline{\gamma}$ for $\rho_G = 0$ and several values of k, ρ_R. The $P_{\rm out}$ improves by increasing k and/or decreasing ρ_R. In Fig. 3, assuming the BPSK modulation scheme and using (24), the ABEP is plotted as a function of the average input SNR, $\overline{\gamma}$, for several values of k and L. As expected the ABEP improves by increasing the diversity order and/or improving the fading/shadowing conditions.

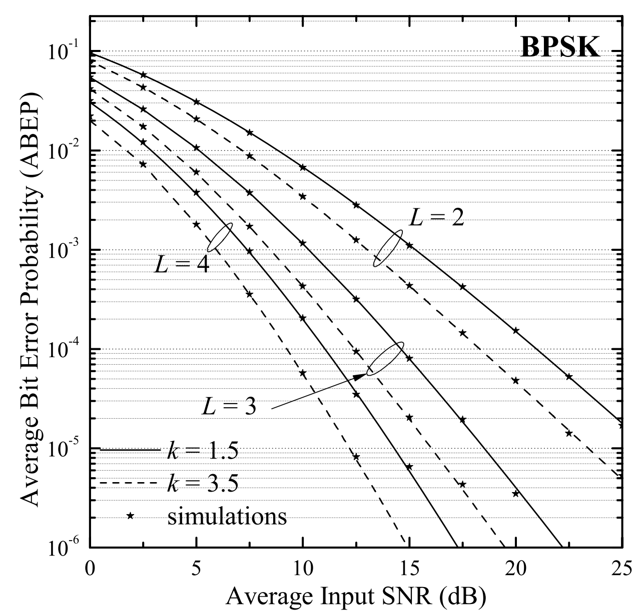

Fig. 3. ABEP of EGC receiver versus $\overline{\gamma}$ for several values of L and k.

In Fig. 4, using (30), the normalized average channel capacity, $\widehat{C} = \overline{C}/B_w$, is plotted as a function of the number of branches, L, for several values of k and $\overline{\gamma}$. It is interesting to note that $\widehat{C}$ increases more rapidly for low values of L and high values of $\overline{\gamma}$, while for high values of k, $\widehat{C}$ does not significantly improve. Finally, in Fig. 5, $\widehat{C}$ is plotted as a function of the correlation coefficient ρ_R, for $k = 1.5$, $\rho_G = 0.2$ and several values of $\overline{\gamma}$. Clearly, $\widehat{C}$ improves by increasing $\overline{\gamma}$, while $\widehat{C}$ decreases rapidly only for high values of ρ_R. For comparison purposes, we have run Monte Carlo

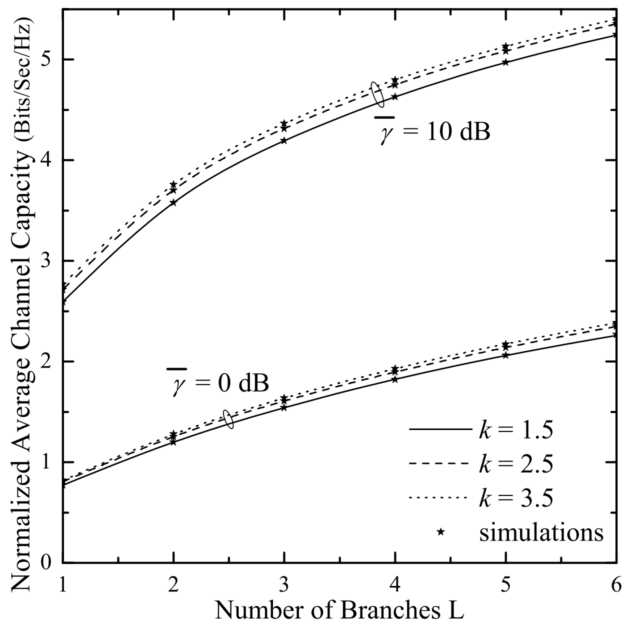

Fig. 4. $\widehat{C}$ of EGC receiver versus L for several values of $\overline{\gamma}$ and k.

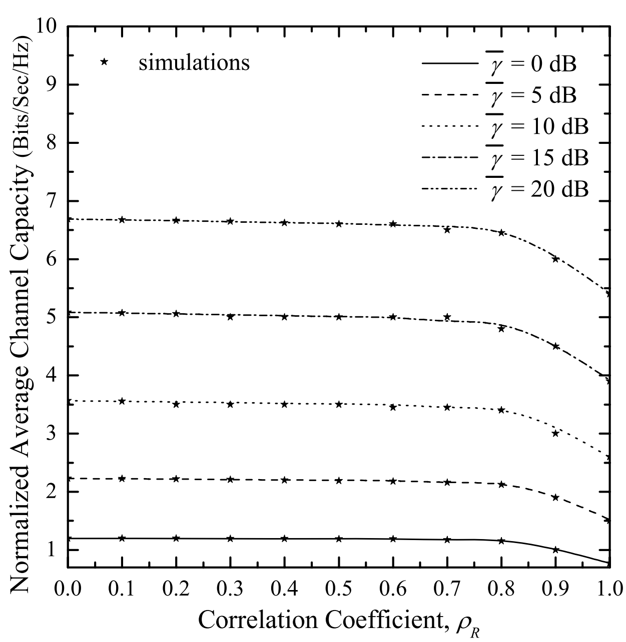

Fig. 5. $\widehat{C}$ of EGC receiver versus ρ_R for several values of $\overline{\gamma}$.

simulations with more than 10^{24} samples[1] and these results are also included in Figs. 1–5, verifying in all cases the validity of the proposed theoretical approach.

V. Conclusions

In this paper important statistical properties of the sum of $\mathcal{K}$ RVs, independent or not, are studied. More specifically, a closed-form expression for the PDF of the sum of not id $\mathcal{K}$ variates is derived, while for the correlated case the PDF is expressed with the aid of fast converging infinite series. Considering id RVs, simplified expressions are obtained for the PDF, while the CDF and CF are also derived in terms of tabulated functions. Capitalizing on the theoretical results, the performance of EGC diversity receiver operating over $\mathcal{K}$ composite fading channel, has been analyzed, in terms of the ABEP, P_{out} and $\widehat{C}$. Various numerical performance evaluation results complemented by equivalent computer simulation ones have been presented, indicating the influence of fading, shadowing and correlation to the performance of the EGC receiver.

Appendix
Proof of Theorem 1

In order to prove Theorem 1, a similar approach as in [22] is followed. Assuming $k_\ell = \alpha_\ell + 1/2$, with $\alpha_\ell \in \mathbb{N}$ and using [15, eq. (8.468)] in (1), the PDF of X_ℓ can be expressed as

$$f_{X_\ell}(x) = \underbrace{\sum_{i_\ell=0}^{\alpha_\ell-1} \frac{\sqrt{\pi}\,(\alpha_\ell+i_\ell-1)!}{i_\ell!\,(\alpha_\ell-i_\ell-1)!}\frac{2^{1-2i_\ell}}{\Gamma(k_\ell)}\Omega_\ell^{\frac{2i_\ell-2k_\ell-1}{4}}}_{\mathbb{S}_\ell} \times x^{\alpha_\ell-i_\ell}\exp\left(-2\frac{x}{\sqrt{\Omega_\ell}}\right). \quad \text{(A-1)}$$

For $Z_1 = X_1 + X_2$, using (A-1) in [19, eq. (6.40)], the PDF of Z_1 can be evaluated as

$$f_{Z_1}(z) = \sum_{i_1=0}^{\alpha_1-1}\sum_{i_2=0}^{\alpha_2-1}\left(\prod_{\ell=1}^{2}\mathbb{S}_\ell\right)\exp\left(-2\frac{z}{\sqrt{\Omega_1}}\right) \times \int_0^z x^{\alpha_2-i_2}(z-x)^{\alpha_1-i_1}\exp\left[\left(\frac{2}{\sqrt{\Omega_1}}-\frac{2}{\sqrt{\Omega_2}}\right)x\right]\mathrm{d}x. \quad \text{(A-2)}$$

The integral in (A-2) can be solved in closed form with the aid of [15, eq. (3.383/1)], and hence, (A-2) yields

$$f_{Z_1}(z) = \sum_{i_1=0}^{\alpha_1-1}\sum_{i_2=0}^{\alpha_2-1}\left(\prod_{\ell=1}^{2}\mathbb{S}_\ell\right)B(\alpha_1-i_1+1,\alpha_2-i_2+1) \times z^{\alpha_1+\alpha_2-i_{1,2}+1}\exp\left(-2\frac{z}{\sqrt{\Omega_1}}\right) \times {}_1F_1\left[\alpha_2-i_2+1;\alpha_{1,2}-i_{1,2}+2;\left(\frac{2}{\sqrt{\Omega_1}}-\frac{2}{\sqrt{\Omega_2}}\right)z\right] \quad \text{(A-3)}$$

where $B(\cdot,\cdot)$ is the Beta function and ${}_1F_1(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function. Furthermore, using [20, eq. (07.20.03.0024.01)] in (A-3) and after some mathematical manipulations, $f_{Z_1}(z)$ can be obtained in closed form as

$$f_{Z_1}(z) = \sum_{i_1=0}^{\alpha_1-1}\sum_{i_2=0}^{\alpha_2-1}\sum_{j_1=0}^{2}\sum_{b_{1,j_1}=0}^{A_{j_1}}\left(\prod_{\ell=1}^{2}\mathbb{S}_\ell\right)(A_1)! \times \frac{(-A_1-A_2-1)_{A_2+1}\,\Delta_{1-2}^{b_{1,j_1}}\,(-A_{j_1})_{b_{1,j_1}}}{(A_1+A_2+1)\,b_{1,j_1}!\,(-A_1-A_2)_{b_{1,j_1}}} \times \frac{(-1)^{(b_{1,j_1}+1)(j_1-1)}}{\Delta_{1-2}^{A_1+A_2+1}} z^{b_{1,j_1}}\exp(-\Delta_{j_1}z) \quad \text{(A-4)}$$

where $A_x = \alpha_x - i_x$, $\Delta_x = 2(1/\Omega_x)^{1/2}$ and $\Delta_{x-y} = 2\left[(1/\Omega_x)^{1/2}-(1/\Omega_y)^{1/2}\right]$. For $Z_2 = Z_1 + X_3$, the PDF of Z_2 can be evaluated, using (A-4) as

$$f_{Z_2}(z_2) = \int_0^z f_{Z_1}(x) f_{X_3}(z-x)\mathrm{d}z. \quad \text{(A-5)}$$

[1]Correlated $\mathcal{K}$ samples are generated by multiplying correlated Rayleigh samples with correlated Nakagami-m samples.

$$f_{Z_2}(z) = \sum_{\{i_\ell\}_{\ell=1}^3=0}^{\{\alpha_\ell-1\}_{\ell=1}^3} \sum_{\{j_\ell\}_{\ell=1}^2=0}^{2} \sum_{b_{1,j_1}=0}^{A_{j_1}} \sum_{b_{2,j_2}=0}^{(2-j_2)A_3+(j_2-1)b_{1,j_1}} \left(\prod_{\ell=1}^3 \mathbb{S}_\ell\right) \frac{(A_1)!\,(-A_1-A_2-1)_{A_2+1}\,(-A_{j_1})_{b_{1,j_1}}}{\Delta_{1-2}^{A_1+A_2+1}\,(A_1+A_2+1)\,(-A_1-A_2)_{b_{1,j_1}}}$$
$$\times \frac{(A_3+b_{1,j_1})!\,(-A_3-b_{1,j_1}-1)_{b_{1,j_1}+1}\,B\left(A_3+1, b_{1,j_1}+1\right)}{b_{1,j_1}!^2} \frac{\Delta_{1-2}^{b_{1,j_1}}\,\Delta_{3-j_1}^{b_{2,j_2}}}{\Delta_{3-j_1}^{A_3+b_{1,j_1}+1}} \left[\prod_{\ell=1}^{2} (-1)^{\left(b_{\ell,j_\ell}+1\right)(j_\ell-1)}\right] \quad \text{(A-6)}$$
$$\times \frac{\left[A_3\,(j_2-2) - b_{1,j_1}\,(j_2-1)\right]_{b_{2,j_2}}}{b_{2,j_2}!\,(-A_3-b_{1,j_1})_{b_{2,j_2}}} z^{b_{2,j_2}} \exp\left[-2\left(\frac{2-j_2}{\sqrt{\Omega_3}} + \frac{j_2-1}{\sqrt{\Omega_{j_1}}}\right) z\right].$$

Hence, following a similar procedure as the one used for deriving (A-4), the PDF of Z_2 can be obtained as (A-6), as shown on the top of this page.

Finally, by repeating the same procedure L times, the generalized form of the PDF of the sum of L $\mathcal{K}$ RVs can be obtained as in (4).

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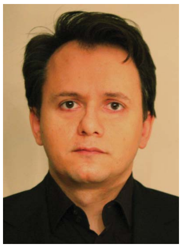
Petros S. Bithas (S'04-M'09) received the B.S. in electrical engineering from the Department of Electrical and Computer Engineering of the University of Patras, Greece, in 2003. From the same Department he received the Ph.D. degree with specialization in "Wireless Communication Systems" in 2009. During 2004-2009, he was research assistant at the Institute for Space Applications and Remote Sensing of the National Observatory of Athens, Greece. Since the October of 2009 he is affiliated with the Department of Electronics Engineering of the Technological Educational Institute of Piraeus as a Lab Instructor. Furthermore, since the November of 2010 he is also collaborating with the Department of Digital Systems, University of Piraeus in large-scale integration projects. P. S. Bithas serves on the Editorial Board of AEÜ *International Journal of Electronics and Communications* (ELSEVIER). Furthermore, he acts as a reviewer for various international journals (including various IEEE Transactions) and numerous of IEEE conferences. He has published (or have been accepted for publication) more than twenty articles in international scientific journals and in the proceedings of international conferences. His current research interests include fading channels modeling, cooperative diversity, mobile communications systems and cognitive radio networks.



Nikos C. Sagias (S'03-M'05) was born in Athens, Greece in 1974. He received the BSc degree from the Department of Physics (DoP) of the University of Athens (UoA), Greece in 1998. The MSc and PhD degrees in Telecommunication Engineering were received both from the UoA in 2000 and 2005, respectively. Since 2001, he has been involved in various National and European Research & Development projects for the Institute of Space Applications and Remote Sensing of the National Observatory of Athens, Greece. During 2006-2008, was a research associate at the Institute of Informatics and Telecommunications of the National Centre for Scientific Research-"Demokritos", Athens, Greece. Currently, he is an Assistant Professor at the Department of Telecommunications Science and Technology of the University of Peloponnese, Tripoli, Greece.

Dr. Sagias research interests are in the research area of wireless digital communications, and more specifically in MIMO and cooperative diversity systems, fading channels, and communication theory. In his record, he has over forty (40) papers in prestigious international journals and more than twenty (20) in the proceedings of world recognized conferences. He has been included in the Editorial Boards of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, *Journal of Electrical and Computer Engineering*, and the *IETE Technical Review*, while he acts as a TPC member for various IEEE conferences (GLOBECOM'08, VTC'08F, VTC'09F, VTC'09S, etc). He is a co-recipient of the best paper award in communications in the 3rd International Symposium on Communications, Control and Signal Processing (ISCCSP), Malta, March 2008. He is a member of the IEEE and IEEE Communications Society as well as the Hellenic Physicists Association.

Ranjan K. Mallik (S'88-M'93-SM'02) received the B.Tech. degree from the Indian Institute of Technology, Kanpur, in 1987 and the M.S. and Ph.D. degrees from the University of Southern California, Los Angeles, in 1988 and 1992, respectively, all in Electrical Engineering. From August 1992 to November 1994, he was a scientist at the Defence Electronics Research Laboratory, Hyderabad, India, working on missile and EW projects. From November 1994 to January 1996, he was a faculty member of the Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology, Kharagpur. From January 1996 to December 1998, he was with the faculty of the Department of Electronics and Communication Engineering, Indian Institute of Technology, Guwahati. Since December 1998, he has been with the faculty of the Department of Electrical Engineering, Indian Institute of Technology, Delhi, where he is currently a Professor. His research interests are in diversity combining and channel modeling for wireless communications, space-time systems, cooperative communications, multiple-access systems, difference equations, and linear algebra.

Dr. Mallik is a member of Eta Kappa Nu. He is also a member of the IEEE Communications, Information Theory, and Vehicular Technology Societies, the American Mathematical Society, and the International Linear Algebra Society, a fellow of the Indian National Academy of Engineering, the Indian National Science Academy, The National Academy of Sciences, India, Allahabad, The Institution of Engineering and Technology, U.K., and The Institution of Electronics and Telecommunication Engineers, India, a life member of the Indian Society for Technical Education, and an associate member of The Institution of Engineers (India). He is an Area Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and an Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS. He is a recipient of the Hari Om Ashram Prerit Dr. Vikram Sarabhai Research Award in the field of Electronics, Telematics, Informatics, and Automation, and of the Shanti Swarup Bhatnagar Prize in Engineering Sciences.