On the Correlated K-Distribution With Arbitrary Fading Parameters

Petros S. Bithas, *Student Member, IEEE*, Nikos C. Sagias, *Member, IEEE*, P. Takis Mathiopoulos, *Senior Member, IEEE*, Stavros A. Kotsopoulos, and Andreas M. Maras, *Member, IEEE*

***Abstract*—The correlated bivariate K-distribution with arbitrary and not necessarily identical parameters is introduced and analyzed. Novel infinite series expressions for the joint probability density function and moments are derived for the general case where the associated bivariate distributions, i.e., Rayleigh and gamma, are both arbitrary correlated. These expressions generalize previously known analytical results obtained for identical parameter cases. Furthermore, considering independent gamma distributions, the cumulative distribution and characteristic functions are analytically obtained. Although the derived expressions can be used in a wide range of applications, this letter focuses on the performance analysis of dual branch diversity receivers. Specifically, the outage performance of dual selection diversity receivers operating over correlated K fading/shadowing channels is analytically evaluated. Moreover, for low normalized outage threshold values, closed-form expressions are obtained.**

***Index Terms*—Bivariate statistics, composite fading/shadowing channels, correlated K-distribution, outage probability, radar clutter modeling.**

I. Introduction

The composite K-distribution has been used in the past to model electromagnetic scattering from physical media, such as tropospheric propagation of radio waves, optical scintillation from the atmosphere, and various types of sea and land radar clutters [1], [2]. In recent years, this generic distribution has been also applied in the scientific field of digital communications over fading/shadowing channels [3]–[5]. The main reason for this choice is that the K-distribution can model composite fading environments where multipath (short-term) fading is superimposed on shadowing (long-term fading), e.g., land mobile satellite links [3, Ch. (2.2.3)], [6]. Such channel models exist in applications where signal envelopes follow the Rayleigh distribution and their average powers follow the gamma distribution. In all these applications of the K-distribution, it is important to further consider the effects of correlation present in various real-life practical scenarios.

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P. S. Bithas and S. A. Kotsopoulos are with the Department of Electrical and Computer Engineering, University of Patras, Rion 26442 Patras, Greece (e-mail: pbithas@space.noa.gr; kotsop@ee.upatras.gr).

N. C. Sagias and A. M. Maras are with the Department of Telecommunications Science and Technology, School of Applied Sciences and Technology, University of Peloponnese, 22100 Tripolis, Greece (e-mail: nsagias@ieee.org; amaras@uop.gr).

P. T. Mathiopoulos is with the Institute for Space Applications and Remote Sensing, National Observatory of Athens, 15236 Athens, Greece (e-mail: mathio@space.noa.gr).

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In contrast to others bivariate (correlated) distributions, e.g., Rayleigh, Nakagami-m, Weibull, and generalized-gamma [3], [7]–[9], the correlated K-distribution has not been thoroughly studied in the open research literature. For example, previous studies have considered only identical shaping and scaling parameters for the correlated K-distribution [10]–[12]. In [11], the joint probability density function (PDF) and the moments of two clutter amplitudes following the K-distribution have been derived. Extending this work, also under the assumption of identical parameters, [12] has considered correlation of the powers of the clutter amplitudes.

Motivated by the preceding, in this letter, we generalize related previous research activities by presenting novel analytical expressions for the bivariate K-distribution, with arbitrary and not necessarily identical parameters. Moreover, the proposed analysis is general enough to consider correlation in both the signal envelopes and their average powers. These expressions include the joint PDF and moments, while the cumulative distribution function (CDF) and the characteristic function (CF) are obtained for the case of uncorrelated powers. Capitalizing on the derived CDF, the outage probability (OP) at the output of dual-branch selection diversity (SD) receivers operating over correlated K fading channels is derived and its performance is evaluated.

II. Correlated K-Distribution

In the first part of this section, the joint PDF and moments of the K-distribution are presented for the generic case where the constituted distributions have different correlation coefficients. In the second part, by considering independent gamma distributions, analytical expressions for important statistical characteristics of the K-distribution are obtained.

A. General Case

The K-distribution is a mixture of zero mean complex Gaussian and gamma random variables (RVs) [3]. Let R_i ($i = 1$ and 2) be the envelopes of two zero mean complex Gaussian RVs and G_i their mean square envelope values, i.e., $G_i = \mathbb{E}_{R_i}\langle x_i^2\rangle$, with $\mathbb{E}_{R_i}\langle\cdot\rangle$ denoting averaging over the distribution of R_i. Then, R_i are Rayleigh RVs with conditional, on G_i's, bivariate PDF given by

$$f_{R_1|G_1,R_2|G_2}(x_1|G_1,x_2|G_2) = \sum_{t=0}^{\infty}\frac{4\rho_R^t/(t!)^2}{(1-\rho_R)^{2t+1}} \times\prod_{i=1}^{2}\frac{x_i^{2t+1}}{G_i^{t+1}}\exp\left[-\frac{x_i^2}{(1-\rho_R)G_i}\right] \quad (1)$$

with ρ_R being the power correlation coefficient between R_1^2 and R_2^2. Considering G_1 and G_2 to be correlated gamma RVs, their joint PDF can be derived using [7, eq. (12)] as

$$f_{G_1,G_2}(x_1,x_2) = \sum_{h=0}^{\infty} \frac{(k_1)_h \rho_G^h}{h!(1-\rho_G)^{k_1+2h}} {}_1F_1\left[k_2-k_1; k_2+h; \frac{\rho_G x_2}{(1-\rho_G)\Omega_2}\right] \times \prod_{i=1}^{2} \frac{x_i^{k_i+h-1} \exp\{-x_i/[(1-\rho_G)\Omega_i]\}}{\Gamma(k_i+h)\Omega_i^{k_i+h}} \tag{2}$$

where $k_i > 0$ is the shaping parameter, $\Omega_i = \mathbb{E}_{G_i}\langle x_i^2\rangle$, ρ_G is the correlation coefficient between G_1 and G_2, ${}_1F_1(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function [13, eq. (9.210/1)], $(\alpha)_n$ is the Pochhammer symbol [14, eq. (6.1.22)], and $\Gamma(\cdot)$ is the gamma function [13, eq. (8.310/1)]. Without loss of generality, it is assumed throughout this letter that $k_2 \geq k_1$ [7].

The correlated K-distribution can be obtained by averaging the conditional on G_i's (1), over the bivariate gamma PDF (2), i.e.,

$$f_{X_1,X_2}(x,y) = \int_0^{\infty}\int_0^{\infty} f_{R_1|G_1,R_2|G_2}(x|v,y|w) f_{G_1,G_2}(v,w)\,dv\,dw. \tag{3}$$

Substituting (1) and (2) in (3), an integral of the form $\mathcal{I} = \int_0^{\infty} y^{\alpha} \exp(-A_1 y^{-1} - A_2 y) {}_1F_1(B_1; B_2; \rho_G A_2 y)dy$ needs to be solved. This integral can be solved by using the infinite series representation of ${}_1F_1(\cdot;\cdot;\cdot)$ [13, eq. (9.210/1)] and then applying [13, eq. (3.471/9)]. Hence, in the general case with different correlation coefficients and not necessarily identical statistical parameters, the joint PDF of the K-distribution can be obtained as

$$f_{X_1,X_2}(x_1,x_2) = \sum_{t,h,q=0}^{\infty} \frac{16(k_1)_h \rho_G^{h+q} \rho_R^t (k_2-k_1)_q}{h!q!(t!)^2 \Gamma(k_1+h)\Gamma(k_2+h+q)} \times \frac{1}{(1-\rho_G)^{1-k_2}} \prod_{i=1}^{2} \frac{x_i^{\tau_i} \Omega_i^{-(\tau_i+1)/2}}{[(1-\rho_R)(1-\rho_G)]^{\tau_i/2}} \times K_{\tau_i-2t-1}\left[\frac{2x_i/\sqrt{\Omega_i}}{\sqrt{(1-\rho_R)(1-\rho_G)}}\right] \tag{4}$$

where $K_v(\cdot)$ is the second kind of modified Bessel function of order v [13, eq. (8.407/1)] and $\tau_i = k_i+h+t+(i-1)q$. For $\Omega_1 = \Omega_2$ and $k_1 = k_2$, (4) simplifies to a previously known expression [12, eq. (29)], i.e., the PDF of the correlated K-distribution with identical parameters.

Using (4) and [13, eqs. (6.561/16) and (9.14/1)], the joint moments $\mu(n_1,n_2) = \mathbb{E}_{X_1,X_2}\langle x_1^{n_1} x_2^{n_2}\rangle$ can be expressed as

$$\mu_{X_1,X_2}(n_1,n_2) = \left[\prod_{i=1}^{2}(1-\rho_G)^{(n_i+k_2)/2}\Omega_i^{n_i/2}\Gamma\left(\frac{n_i}{2}+1\right)\right](k_1)_{\frac{n_1}{2}} \times {}_2F_1\left(-\frac{n_1}{2}, -\frac{n_2}{2}; 1; \rho_R\right) \sum_{q=0}^{\infty} \frac{(k_2-k_1)_q \rho_G^q}{q!}(k_2+q)_{\frac{n_2}{2}} \times {}_2F_1\left(k_1+\frac{n_1}{2}, k_2+\frac{n_2}{2}+q; k_2+q; \rho_G\right) \tag{5}$$

with ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ being the Gauss hypergeometric function [13, eq. (9.100)]. For the special case where $k = k_1 = k_2$, (5) can be mathematically expressed in closed form as

$$\mu_{X_1,X_2}(n_1,n_2) = \left[\prod_{i=1}^{2} \frac{\Gamma(n_i/2+1)\Gamma(k+n_i/2)\Omega_i^{n_i/2}}{\Gamma(k)}\right] \times {}_2F_1\left(-\frac{n_1}{2}, -\frac{n_2}{2}; 1; \rho_N\right) {}_2F_1\left(-\frac{n_1}{2}, -\frac{n_2}{2}; k; \rho_G\right). \tag{6}$$

The power correlation coefficient between X_1^2 and X_2^2 is defined as $\varrho \triangleq \operatorname{cov}(X_1^2, X_2^2)/[\sqrt{\operatorname{var}(X_1^2)}\sqrt{\operatorname{var}(X_2^2)}]$ [15, eq. (7.7)], where $\operatorname{cov}(\cdot,\cdot)$ is the covariance and $\operatorname{var}(\cdot)$ is the variance. Using (6) and [3, eq. (2.64)], ϱ can be obtained in a simple closed-form expression as

$$\varrho = \frac{(k+\rho_G)(1+\rho_R)-k}{k+2}. \tag{7}$$

B. Independent Gamma RVs Case

For $\rho_G = 0$, i.e., when G_1 and G_2 are independent gamma RVs, a simplified expression of (4) can be obtained as

$$f_{X_1,X_2}(x_1,x_2) = \sum_{t=0}^{\infty} \frac{16\rho_R^t/(t!)^2}{(1-\rho_R)^{(k_1+k_2)/2+t}} \times \prod_{i=1}^{2} \frac{x_i^{k_i+t}/\Gamma(k_i)}{\Omega_i^{(k_i+t+1)/2}} K_{k_i-t-1}\left[\frac{2x_i}{\sqrt{(1-\rho_R)\Omega_i}}\right]. \tag{8}$$

For $\Omega_1 = \Omega_2$ and $k_1 = k_2$, (8) further simplifies to another previous known expression [11, eq. (12)], i.e., the correlated K PDF with identical parameters.

Using (8) and [13, eq. (6.621/3)], the joint CF of X_1 and X_2 can be expressed as

$$\Phi_{X_1,X_2}(s_1,s_2) = (1-\rho_R)\pi\left[\prod_{i=1}^{2}\frac{\Gamma(2k_i)}{\Gamma(k_i)}\right] \times \sum_{t=0}^{\infty}\rho_R^t\left[\frac{[(2t+1)!/t!]}{2^{2t-(k_1+k_2)}}\right]^2 \times \prod_{i=1}^{2}\frac{1/\Gamma(k_i+t+3/2)}{\left[js_i\sqrt{(1-\rho_R)\Omega_i}+2\right]^{2k_i}} \times {}_2F_1\left[2k_i, k_i-t-\frac{1}{2}; k_i+t+\frac{3}{2}; \frac{js_i\sqrt{(1-\rho_R)\Omega_i}-2}{js_i\sqrt{(1-\rho_R)\Omega_i}+2}\right]. \tag{9}$$

where $j = \sqrt{-1}$

By expressing $K_\alpha(\cdot)$ in (8) as in [13, eq. (9.34/4)] and using [16, eq. (07.34.21.0003.01)], the joint CDF of X_1 and X_2 can be obtained as

$$F_{X_1,X_2}(x_1,x_2) = \sum_{t=0}^{\infty}\rho_R^t\prod_{i=1}^{2}\frac{\xi_i^{(k_i+t+1)/2}(1-\rho_R)}{\Gamma(k_i)t!} \times G_{1,3}^{2,1}\left[\xi_i\Big|_{(k_i-t-1)/2,-(k_i-t-1)/2,-(k_i+t+1)/2}^{1-(k_i+t+1)/2}\right] \tag{10}$$

where $G[\cdot]$ is the Meijer's G-function [13, eq. (9.301)] and $\xi_i = x_i^2/[(1-\rho_R)\Omega_i]$. Furthermore, for non-integer values of k_i, an alternative and quite simpler expression for the joint CDF can be extracted using [16, eq. (03.04.21.0007.01)] in (8), yielding

$$F_{X_1,X_2}(x_1,x_2) = \pi^2(1-\rho_R)\sum_{t=0}^{\infty}\rho_R^t\prod_{i=1}^{2}\csc(\pi k_i) \times\left[\frac{\xi_i^{t+1}}{\Gamma(k_i)}{}_1\tilde{F}_2(t+1;t+2-k_i,t+2;\xi_i) -\frac{\xi_i^{k_i}}{t!}{}_1\tilde{F}_2(k_i;k_i-t,k_i+1;\xi_i)\right]. \quad (11)$$

In (11), ${}_p\tilde{F}_q(\cdot;\cdot;\cdot)$ represents the regularized generalized hypergeometric function [16, eq. (07.32.02.0001.01)], where p and q are integers. This function is related with the well-known generalized hypergeometric function ${}_pF_q(\cdot;\cdot;\cdot)$ [13, eq. (9.14/1)], as ${}_p\tilde{F}_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z) = {}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z)/[\prod_{k=1}^{q}\Gamma(b_k)]$ [16, eq. (07.32.26.0001.01)].

III. SD Receivers Outage Probability

In principal, the previously derived expressions can be used to obtain the performance of a wide range of diversity receivers operating over fading/shadowing channels modeled by the correlated K-distribution. However here the OP performance of an SD receiver is studied. Thus, a dual-branch SD receiver operating over an additive white Gaussian noise (AWGN) channel, subject also to flat fading, modeled by the previously discussed K-distribution, is considered. The instantaneous signal-to-noise ratio (SNR) of the ith, ($i=1$ and 2), input branch is given by

$$\gamma_i = X_i^2\frac{E_s}{N_0} \quad (12)$$

with E_s being the transmitted symbol energy, and N_0 the power spectral density of the AWGN, assumed to be identical for both branches. The corresponding average value of γ_i can be mathematically expressed as

$$\overline{\gamma}_i = \mathbb{E}_{X_i}\left\langle x_i^2\right\rangle\frac{E_s}{N_0} = k_i\Omega_i\frac{E_s}{N_0}. \quad (13)$$

Moreover, the instantaneous SNR at the output of the SD receiver, γ_{sd}, is obtained by selecting the $\max(\gamma_1,\gamma_2)$ and the CDF of γ_{sd} is given by $F_{\gamma_{\mathrm{sd}}}(\gamma) = F_{\gamma_1,\gamma_2}(\gamma,\gamma)$ [15, eq. (6.54)]. Clearly from (12) and (13), $X_i^2/\Omega_i = k_i\gamma_i/\overline{\gamma}_i$, and thus with the aid of this relationship, it is not difficult to recognize that from (11), $F_{\gamma_1,\gamma_2}(\gamma_1,\gamma_2)$ can be obtained. Hence, by denoting the predefined outage threshold as γ_{th}, the OP at the output of SD can be easily obtained as $P_{\mathrm{out}}(\gamma_{\mathrm{th}}) = F_{\gamma_{\mathrm{sd}}}(\gamma_{\mathrm{th}})$, leading to

$$P_{\mathrm{out}}(\gamma_{\mathrm{th}}) = \pi^2(1-\rho_R)\sum_{t=0}^{\infty}\rho_R^t\prod_{i=1}^{2}\csc(\pi k_i) \times\left[\frac{\zeta_i^{t+1}}{\Gamma(k_i)}{}_1\tilde{F}_2(t+1;t+2-k_i,t+2;\zeta_i) -\frac{\zeta_i^{k_i}}{t!}{}_1\tilde{F}_2(k_i;k_i-t,k_i+1;\zeta_i)\right] \quad (14)$$

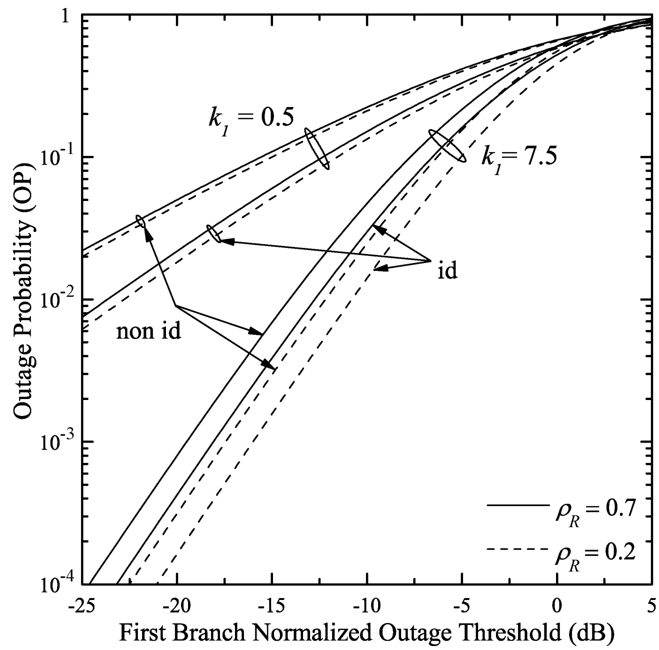

Fig. 1. Outage probability (P_{out}) at the output of a dual-branch SD receiver versus the first branch normalized outage threshold, $\gamma_{\mathrm{th}}/\overline{\gamma}_1$, for several values of k_1 and ρ_R.

with $\zeta_i = k_i\gamma_{\mathrm{th}}/[(1-\rho_R)\overline{\gamma}_i]$. For small values of ζ_i, i.e., $\zeta_i \le 10^{-1}$, using [16, eqs. (07.32.06.0003.01) and (01.10.26.0025.01)], it can be easily shown that (14) reduces to

$$P_{\mathrm{out}}(\gamma_{\mathrm{th}}) = (1-\rho_R)\sum_{t=0}^{\infty}\rho_R^t\prod_{i=1}^{2}\zeta_i^{t+1}\frac{\Gamma(1-k_i)}{t!} \times\left[\frac{1}{\Gamma(t+2-k_i)(t+1)} - \frac{\zeta_i^{k_i-t-1}}{\Gamma(k_i-t)k_i}\right]. \quad (15)$$

Furthermore, by using the definition of the generalized hypergeometric function ${}_pF_q(\cdot;\cdot;\cdot)$ [13, eq. (9.14/1)], (15) can be obtained in closed form as

$$\begin{aligned}P_{\mathrm{out}}(\gamma_{\mathrm{th}}) =& (1-\rho_R)\\ &\times\Bigg[\frac{{}_0F_3(;1,1-k_1,1-k_2;\zeta_1\zeta_2\rho_R)-1}{\rho_R}\\ &+\frac{\zeta_1\zeta_2^{k_2}}{1-k_1}\frac{\Gamma(-k_2)}{\Gamma(k_2)}{}_1F_2(1-k_2;2,2-k_1;-\zeta_1\rho_R)\\ &+\frac{\zeta_2\zeta_1^{k_1}}{1-k_2}\frac{\Gamma(-k_1)}{\Gamma(k_1)}{}_1F_2(1-k_1;2,2-k_2;-\zeta_2\rho_R)\\ &+{}_2F_1(1-k_1,1-k_2;1;\rho_R)\prod_{i=1}^{2}\zeta_i^{k_i}\frac{\Gamma(1-k_i)}{\Gamma(k_i+1)}\Bigg].\end{aligned} \quad (16)$$

IV. Numerical Results

Using the previous presented analysis, the OP performance of a dual-branch SD receiver operating over correlated K fading channels has been numerically evaluated in terms of the OP (see Fig. 1) and a series convergence rate (see Table I). In Fig. 1, the

TABLE I
NUMBER OF TERMS REQUIRED FOR CONVERGENCE OF (10) WITH ACCURACY $\pm 1‰$

	$\rho_R = 0.2$		$\rho_R = 0.7$	
$\overline{\gamma}$(dB)	$k=1$	$k=4$	$k=1$	$k=4$
-15	3	1	7	2
-10	4	3	10	3
-5	5	4	13	6
0	7	5	17	11
5	9	8	21	18

OP performance is plotted as a function of the first branch normalized outage threshold, $\gamma_{\rm th}/\overline{\gamma}_1$, for several values of k_i, ρ_R and for both identical (id) and non-id correlated K fading channels. In the case where unbalanced, non-identical, correlated K fading conditions are considered, it is assumed $\overline{\gamma}_2 = \overline{\gamma}_1/\sqrt{e}$ and $k_2 = k_1/2$. It is noted that these values of $\overline{\gamma}_i$ follow the well-established exponential decaying power delay profile [17]. As expected, the obtained performance results show that as $\gamma_{\rm th}/\overline{\gamma}_1$ decreases, the outage performance improves, while the performance also improves as ρ_R decreases and/or k_1 increases. As also observed in [8], the best performance is obtained when identical fading channels are considered.

Table I summarizes the number of terms required in (10) to achieve accuracy better than $1\%_{0}$ after the truncation of the infinite series. In particular, in this table, the number of required terms versus $\overline{\gamma} = \overline{\gamma}_1 = \overline{\gamma}_2$, for $\rho_R = 0.2, 0.7$, and $k = k_1 = k_2$, is presented. As depicted, the required terms increase as $\overline{\gamma}$ and/or ρ_R increase and/or k decreases, similarly to other works [8], [18], [19]. It should be noted that similar rates of convergence have been obtained for all the infinite series expressions presented in this paper, i.e., (4)–(9).

V. CONCLUSIONS

The bivariate K-distribution with arbitrary and not necessarily identical shaping and scaling parameters was presented and studied. Assuming correlation in both the signal envelopes and their average powers, generic infinite series expressions for the most important statistical metrics of this composite distribution were obtained. The derived expressions simplify to previous reported results for special cases. Furthermore, capitalizing on these expressions, the OP performance of dual-branch SD receivers operating over correlated K fading/shadowing channels is analytical studied.

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