The Bivariate Generalized-K (K_G) Distribution and its Application to Diversity Receivers

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***Abstract*—The correlated bivariate generalized-K (K_G) distribution, with not necessarily identical shaping and scaling parameters, is introduced and studied. This composite distribution is convenient for modeling multipath/shadowing correlated fading environments when the correlations between the signal envelopes and their powers are different. Generic infinite series expressions are derived for the probability density function (PDF), the cumulative distribution function (CDF) and the joint moments. Assuming identical shaping parameters, simpler expressions for the PDF, CDF and the characteristic function (CF) are provided, while the joint moments are derived in closed form. Furthermore, the PDFs of the product and ratio of two correlated K_G random variables are obtained. Capitalizing on these theoretical expressions for the statistical characteristics of the correlated K_G distribution, the performance analysis of various diversity reception techniques, such as maximal ratio combining (MRC), equal gain combining (EGC) and selection diversity (SD), over bivariate K_G fading channels is presented. For the SD, the outage probability is studied, while for the MRC and EGC the average bit error probability is obtained. The proposed analysis is accompanied by numerical results, clearly demonstrating the usefulness of the theoretical approach as well as the appropriateness of the K_G distribution to model multipath/shadowing fading channels.**

***Index Terms*—Bit error probability, bivariate generalized-K distribution, correlated multipath-shadowing fading, equal gain combining (EGC), maximal ratio combining (MRC), outage probability, product-ratio of correlated RVs, selection diversity.**

I. INTRODUCTION

SEVERAL statistical channel models have been used in the past for the analysis of communication systems in the presence of the composite propagation environment that appears when multipath fading and shadowing occur simultaneously [1]. Well known and frequently used distributions for modeling short term fading (multipath) are the Nakagami-m, Rice, and Rayleigh distributions, while for modeling long term fading (shadowing) the lognormal distribution is typically used. It thus comes without surprise that the most commonly used distributions for modeling composite propagation environments are the Rayleigh-, Rice-, and Nakagami-lognormal (NL) [1]–[4]. Unfortunately these lognormal-based fading/shadowing channel models, due to their complex mathematical nature, are rather inconvenient for analytically evaluating the performance of digital communication systems in the presence of such fading channels. An alternative and mathematical more convenient channel model, which has been shown to accurately approximate shadowing phenomena, is the gamma distribution [5], [6]. By employing the gamma distribution, new families of composite fading distributions have been proposed, most notably the K and generalized-K (K_G) distributions [7]–[9]. The main advantage of these two distributions is their relatively simple mathematical form allowing an integrated performance analysis of digital communication systems operating in composite fading environments. Such propagation environments exist in land-mobile satellite systems [3], and in metropolitan areas with slow moving users [1]. Hence, capitalizing on the convenient mathematical form of the K_G distribution, the outage probability (OP), capacity, and several diversity reception techniques over such fading/shadowing channels have been studied in [10]–[13].

Independent of the channel fading distribution, the received diversity signals could be correlated resulting in a degradation of the achievable diversity gain [1]. Typically, such signal correlation exists in relatively small size mobile terminals, where usually the distance between the diversity antennas is small. The open research literature concerning bivariate (correlated) distributions is quite extensive, e.g., [14]–[22]. In [14] the bivariate Nakagami-m was introduced, and in [15] infinite series representations for the bivariate Rayleigh and Nakagami-m were presented. In [16] the bivariate Nakagami-m with arbitrary fading parameters was derived, while expressions for multivariate Rayleigh generated from correlated Gaussian random variables (RV)s have been obtained in [17]. In the same reference the bivariate case was also introduced and applied to the performance analysis of maximal ratio combining (MRC). In [18], the bivariate Rice was presented and based on this, in [19], the performance of dual-branch switch diversity receivers was studied. In [20] an infinite series representation for the bivariate Rice probability density function (PDF) was presented, while in [22] the multivariate Weibull distribution originated from Gaussian random processes was introduced and studied. As a general observation it is mentioned that the previously published papers, dealing with correlated fading channel models, consider only small scale

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effects despite the fact that the lognormal statistics, and hence the gamma statistics, become dominant when shadowing is present.

In the past, the correlated K-distribution has gained increased interest since it was considered as an appropriate model for electromagnetic scattering from physical media such as tropospheric propagation of radio waves, various types of radar clutter, and phenomenological description of sea clutter [23]–[25]. For example in [24], the joint PDF and moments of the bivariate K-distribution are derived, assuming identical shaping and scaling parameters. Additionally, in [25], the power of the clutter amplitudes were considered correlated, assuming again identical parameters. In [26], extending these works the bivariate K distribution with non identical parameters was studied. However, it should be noted that for the bivariate (correlated) K_G distribution very few publications exist in the open technical literature. For example in a recent paper [27], the PDF at the output of a hybrid diversity scheme, where MRC receiver is considered at the micro diversity level and selection diversity (SD) receiver at the macro diversity level, operating over identical distributed (id) K_G fading channels was presented. For this special diversity reception technique, the PDF was derived in infinite series expressions. All in all, to the best of the authors knowledge, a thorough and detailed analysis of the bivariate K_G distribution and the performance of digital communications systems over such composite fading channels, is not available in the open research literature, and thus is the subject of the current work.

Motivated by the preceding, in this paper the most important statistical properties for the bivariate K_G fading distribution with non identical shaping and scaling parameters are presented and applied to the performance analysis of diversity receivers. The remainder of the paper is organized as follows. In Section II, the PDF, cumulative distribution function (CDF), and characteristic function (CF) are derived in infinite series representation, while a closed-form expression for the joint moments is presented. In the same section, the product and the ratio of two correlated K_G RVs are also studied. In Section III, the CDF of SD, the moments generating function (MGF) of MRC and the CF of equal gain combining (EGC) operating over correlated K_G fading channels are derived. These expressions are used to obtain numerous analytical performance evaluation results presented in Section IV. The conclusions of the paper are given in Section V.

II. Statistical Characteristics

Let Y_ℓ ($\ell = 1, 2$) represent the envelopes of Nakagami-m fading processes, with joint PDF given by [16, eq. (12)]

$$f_{Y_1,Y_2}(y_1,y_2) = 4(1-\rho_N)^{m_2}\sum_{t=0}^{\infty}\frac{(m_1)_t\rho_N^t}{t!} \times {}_1F_1\left[m_2-m_1; m_2+t; \frac{\rho_N\, m_2\, y_2^2}{W_2(1-\rho_N)}\right] \times \prod_{\ell=1}^{2}\left[\frac{m_\ell}{W_\ell(1-\rho_N)}\right]^{m_\ell+t}\frac{y_\ell^{2(m_\ell+t)-1}}{\Gamma(m_\ell+t)}\exp\left[-\frac{m_\ell\, y_\ell^2}{(1-\rho_N)W_\ell}\right] \tag{1}$$

where $m_\ell \geq 1/2$ is the Nakagami-m shaping parameter, ρ_N is the power correlation coefficient between Y_1^2 and Y_2^2, and W_ℓ is the average fading power $W_\ell = \mathbb{E}\langle Y_\ell^2\rangle$, with $\mathbb{E}\langle\cdot\rangle$ denoting expectation. Furthermore, ${}_1F_1(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function [28, eq. (9.210/1)], $(\cdot)_p$ is the Pochhammer's symbol [28, p. xliii], with $p \in \mathbb{N}$, and $\Gamma(\cdot)$ is the gamma function [28, eq. (8.310/1)]. When multipath fading is superimposed on shadowing, the powers of the multipath components, W_ℓ, randomly vary[1], modeled in the following analysis with the gamma distribution [1, Chapter 2]. Furthermore, by considering correlation between the RVs W_1 and W_2, and by using [16, eq. (12)], the PDF of the bivariate distribution that governs W_1 and W_2 can be mathematically expressed as

$$f_{W_1,W_2}(W_1,W_2) = (1-\rho_G)^{k_2}\sum_{h=0}^{\infty}\frac{(k_1)_h\rho_G^h}{h!} \times {}_1F_1\left[k_2-k_1; k_2+h; \frac{\rho_G\, W_2}{\Omega_2(1-\rho_G)}\right]\left\{\prod_{\ell=1}^{2}\frac{W_\ell^{-1}}{\Gamma(k_\ell+h)} \times \left[\frac{W_\ell}{\Omega_\ell(1-\rho_G)}\right]^{k_\ell+h}\exp\left[-\frac{W_\ell}{(1-\rho_G)\Omega_\ell}\right]\right\} \tag{2}$$

where $k_\ell > 0$ is the shaping parameter, Ω_ℓ is the average power, and ρ_G is the correlation coefficient between W_1 and W_2. By using different values of k_ℓ, (2) approximates several shadowing conditions, e.g., from severe shadowing ($k_\ell \to 0$), to no shadowing ($k_\ell \to \infty$). The bivariate K_G PDF can be obtained (see Appendix for details) as follows

$$f_{X_1,X_2}(x_1,x_2) = \sum_{t,h,b,d=0}^{\infty}\frac{16\,(m_2-m_1)_b\,(k_2-k_1)_d}{\Gamma(m_2+t+b)\,\Gamma(k_2+h+d)} \times \frac{(m_1)_t\,(k_1)_h\,\rho_N^{t+b}\rho_G^{h+d}}{t!\,h!\,b!\,d!\,\Gamma(m_1+t)\,\Gamma(k_1+h)} \times \frac{\left[\prod_{\ell=1}^{2}\left(\sqrt{m_\ell/\Omega_\ell}x_\ell\right)^{\zeta_\ell}K_{\psi_\ell}\left(2\sqrt{m_\ell/\sigma_\ell}x_\ell\right)\right]}{(1-\rho_G)^{\xi_1/2+t+h}(1-\rho_N)^{\xi_2/2+t+h}\;x_1x_2} \tag{3}$$

where

$$\begin{aligned}
\sigma_\ell &= (1-\rho_N)(1-\rho_G)\,\Omega_\ell,\\
\xi_\ell &= k_1 + (-1)^\ell k_2 + m_1 + (-1)^{\ell+1}m_2 + b + d,\\
\psi_\ell &= k_\ell + h - m_\ell - t + (\ell-1)(d-b),\\
\zeta_\ell &= k_\ell + m_\ell + t + h + (\ell-1)(b+d)
\end{aligned}$$

with $K_\nu(\cdot)$ being the second kind modified Bessel function of order ν [28, eq. (8.407/1)]. For $\Omega_1 = \Omega_2$, $k_1 = k_2$, and $m_1 = m_2 = 1$, (3) simplifies to a previously known expression, i.e., the PDF of the correlated K-distribution with identical parameters [25, eq. (29)]. Assuming identical shaping parameters, i.e., $m = m_1 = m_2$ and $k = k_1 = k_2$, using [30, eq. (19)], [28, eq. (8.445)] and following a similar procedure as for deriving (3), a simplified expression for $f_{X_1,X_2}(x_1,x_2)$

[1]This type of shadowing is also referred to as "multiplicative shadow fading" [29].

can be obtained as

$$f_{X_1,X_2}(x_1,x_2) = \frac{16}{\Gamma(m)\Gamma(k)} \sum_{b,d=0}^{\infty} \frac{m^{\xi}\,\rho_N^b\,\rho_G^d}{\Gamma(m+b)\,\Gamma(k+d)} \times \frac{\left[\prod_{\ell=1}^{2}\left(x_\ell/\sqrt{\Omega_\ell}\right)^{\xi} K_{\psi}\left(2\sqrt{m/\sigma_\ell}\,x_\ell\right)\right]}{b!d!\,(1-\rho_N)^{k+b+d}\,(1-\rho_G)^{m+b+d}\,x_1x_2} \tag{4}$$

where $\xi = k+m+b+d$, $\psi = k+d-m-b$. For $\Omega_1 = \Omega_2$, $m=1$, and $\rho_G = 0$, (4) further simplifies to another previously known expression, i.e., the special case of the PDF of the correlated K distribution with identical parameters [24, eq. (12)].

In the following subsections, important statistical properties of the correlated K_G distribution, namely the product moments, the cumulative distribution and characteristic functions, as well as, the PDFs of the product and ratio of two correlated K_G RVs, will be presented.

A. Product Moments

From the definition of the product moments of X_1 and X_2 of order n_1+n_2, $\mu_{X_1,X_2}(n_1,n_2) \triangleq \mathbb{E}\langle X_1^{n_1}X_2^{n_2}\rangle$ [31, eq. (7.18)], using (3) and by applying first [28, eq. (6.561/16)] and then the definition of the generalized hypergeometric function [28, eq. (9.14/1)], the following expression for $\mu_{X_1,X_2}(n_1,n_2)$ can be obtained

$$\begin{aligned}\mu_{X_1,X_2}(n_1,n_2) &= \left[\prod_{\ell=1}^{2}\left(\frac{\Omega_\ell}{m_\ell}\right)^{n_\ell/2}\right](m_1)_{\frac{n_1}{2}}\,(k_1)_{\frac{n_1}{2}} \\ &\times \sum_{b,d=0}^{\infty} \frac{(m_2-m_1)_b\,(k_2-k_1)_d\,(m_2+b)_{\frac{n_2}{2}}\,(k_2+d)_{\frac{n_2}{2}}}{(1-\rho_G)^{-(n_1+n_2)/2-k_2}\,(1-\rho_N)^{-(n_1+n_2)/2-m_2}} \\ &\times \frac{\rho_N^b\rho_G^d}{b!\,d!}\,{}_2F_1\left(k_1+\frac{n_1}{2}, k_2+d+\frac{n_2}{2}; k_2+d;\rho_G\right) \\ &\quad\times {}_2F_1\left(m_1+\frac{n_1}{2}, m_2+b+\frac{n_2}{2}; m_2+b;\rho_N\right)\end{aligned} \tag{5}$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss hypergeometric function [28, eq. (9.100)]. It is noted that for the special case of identical shaping parameters the above equation simplifies to the following closed-form expression

$$\begin{aligned}\mu_{X_1,X_2}(n_1,n_2) &= \left[\prod_{\ell=1}^{2}\frac{\Gamma(m+n_\ell/2)\,\Gamma(k+n_\ell/2)\,\Omega_\ell^{n_\ell/2}}{\Gamma(m)\,\Gamma(k)\,m^{n_\ell/2}}\right] \\ &\times {}_2F_1\left(-\frac{n_1}{2},-\frac{n_2}{2};m;\rho_N\right){}_2F_1\left(-\frac{n_1}{2},-\frac{n_2}{2};k;\rho_G\right).\end{aligned} \tag{6}$$

By definition, the K_G power correlation coefficient between X_1^2 and X_2^2 is given by [31, eq. (7.8)]

$$\rho \triangleq \frac{\mathbb{E}\langle X_1^2X_2^2\rangle - \mathbb{E}\langle X_1^2\rangle\mathbb{E}\langle X_2^2\rangle}{\sqrt{\mathbb{E}\langle X_1^4\rangle - \mathbb{E}^2\langle X_1^2\rangle}\sqrt{\mathbb{E}\langle X_2^4\rangle - \mathbb{E}^2\langle X_2^2\rangle}}. \tag{7}$$

Using [8, eq. (7)]

$$\mathbb{E}\langle X_i^{n_i}\rangle = \left(\frac{\Omega_i}{m}\right)^{n_i/2}\frac{\Gamma(k+n_i/2)\,\Gamma(m+n_i/2)}{\Gamma(m)\,\Gamma(k)} \tag{8}$$

and (6) in (7), after some straight forward mathematical manipulations the following closed-form expression for ρ is obtained

$$\rho = \frac{(m+\rho_N)(k+\rho_G)-mk}{(k+1)(m+1)-mk}. \tag{9}$$

It is noted that the above expression relates ρ with the correlation coefficients of Nakagami-m, ρ_N, and gamma, ρ_G, bivariate distributions.

B. Cumulative Distribution Function (CDF)

The joint CDF of X_1 and X_2 is given by $\mathcal{F}_{X_1,X_2}(x_1,x_2) = \int_0^{x_1}\int_0^{x_2} f_{X_1,X_2}(x_1,x_2)dx_2dx_1$ [31, eq. (6.6)]. Substituting (3) in this expression, representing $K_\nu(\cdot)$ as in [32, eq. (03.04.26.0006.01)] and using [33, eq. (26)] the following generic expression for the joint CDF can be obtained

$$\begin{aligned}\mathcal{F}_{X_1,X_2}(x_1,x_2) &= \sum_{t,h,b,d=0}^{\infty}\frac{(1-\rho_G)^{-\xi_1/2-t-h}}{(1-\rho_N)^{\xi_2/2+t+h}} \\ &\times \frac{\rho_N^{t+b}\,\rho_G^{h+d}\,(m_2-m_1)_b\,(k_2-k_1)_d}{t!\,h!\,b!\,d!\,\Gamma(m_1)\,\Gamma(m_2+t+b)\,\Gamma(k_1)\,\Gamma(k_2+h+d)} \\ &\times \left\{\prod_{\ell=1}^{2}\left(\sqrt{\frac{m_\ell}{\Omega_\ell}}x_\ell\right)^{\zeta_\ell}\mathcal{G}_{1,3}^{2,1}\left[\frac{m_\ell\,x_\ell^2}{\sigma_\ell}\middle|\begin{matrix}1-\zeta_\ell/2\\ \psi_\ell/2,-\psi_\ell/2,-\zeta_\ell/2\end{matrix}\right]\right\}\end{aligned} \tag{10}$$

where $\mathcal{G}_{p,q}^{m,n}[\cdot|\cdot]$ is the Meijer's G-function [28, eq. (9.301)]. Furthermore, for the identical shaping parameters case, i.e., using (4), and with the aid of [32, eq. (03.04.21.0007.01)], (10) simplifies to

$$\begin{aligned}\mathcal{F}_{X_1,X_2}(x_1,x_2) &= \frac{\pi^2}{\Gamma(m)\Gamma(k)}\sum_{b,d=0}^{\infty}\frac{m^{2(m+b)}\csc^2(\pi\psi)}{\Gamma(m+b)\Gamma(k+d)} \\ &\times \frac{\rho_N^b\,\rho_G^d\,(1-\rho_G)^{k-2(m+b)}}{b!\,d!\,(1-\rho_N)^{m+2b}}\left[\prod_{\ell=1}^{2}\left(\frac{x_\ell^2}{\Omega_\ell}\right)^{m+b}\mathcal{Q}\left(\frac{mx_\ell^2}{\sigma_\ell}\right)\right]\end{aligned} \tag{11}$$

where

$$\begin{aligned}\mathcal{Q}(x) &= \Gamma(m+b)\,{}_P\tilde{F}_Q\,(m+b;1-\psi,m+b+1;x) \\ &\quad - x^{\psi}\Gamma(k+d)\,{}_P\tilde{F}_Q\,(k+d;\psi+1,k+d+1;x)\end{aligned}$$

and ${}_P\tilde{F}_Q(\cdot;\cdot;\cdot)$ is the regularized generalized hypergeometric function [32, eq. (07.32.02.0001.01)].

C. Characteristic Function (CF)

Since the joint CF of X_1 and X_2 is given by $\Phi_{X_1,X_2}(s_1,s_2) = \mathbb{E}\langle\exp[-j(s_1X_1+s_2X_2)]\rangle$ [31, eq. (7.23)], where $j=\sqrt{-1}$, and by using (4) and [28, eq. (6.621/3)], the following expression for the joint CF can be obtained

$$\begin{aligned}\Phi_{X_1,X_2}(s_1,s_2) &= \frac{\pi\,(1-\rho_N)^m\,(1-\rho_G)^k}{\Gamma(m)\Gamma(k)}\sum_{b,d=0}^{\infty}\frac{m^{2(k+d)}}{\Gamma(m+b)} \\ &\times \frac{2^{4(\psi+1)}\rho_N^b\rho_G^d}{\Gamma(k+d)b!d!}\left\{\prod_{\ell=1}^{2}\frac{\Gamma[2(m+b)]\,\Gamma[2(k+d)]}{\left(js_\ell\sqrt{\sigma_\ell}+2\sqrt{m}\right)^{2(k+d)}\Gamma(\xi+1/2)}\right. \\ &\left.\times\,{}_2F_1\left[2(k+d),\psi+\frac{1}{2};\xi+\frac{1}{2};\frac{js_\ell\sqrt{\sigma_\ell}-2\sqrt{m}}{js_\ell\sqrt{\sigma_\ell}+2\sqrt{m}}\right]\right\}.\end{aligned} \tag{12}$$

D. Product and Ratio of Two Correlated K_G RVs

From the statistical point of view, in order to present a thorough analysis of the bivariate K_G distribution, the PDF of the product and the ratio of two correlated K_G RVs will be studied.

1) Product: Let $\mathcal{A}$ denote a RV defined as $\mathcal{A} \triangleq X_1X_2$. The PDF of $\mathcal{A}$ is given by $f_{\mathcal{A}}(x) = \int_0^\infty f_{X_1,X_2}(x_1, x/x_1)/|x_1|dx_1$, [31, eq. (6.74)], with $|\cdot|$ denoting absolute value. Substituting (4) in this expression, representing again $K_\nu(\cdot)$ as in [32, eq. (03.04.26.0006.01)], using [33, eq. (21)] and after some mathematical manipulations, the following expression for $f_{\mathcal{A}}(x)$ can be obtained

$$
\begin{aligned}
f_{\mathcal{A}}(x) = &\frac{2}{\Gamma(m)\Gamma(k)}\sum_{b,d=0}^{\infty}\frac{m^\xi\,\rho_N^b\,\rho_G^d}{\Gamma(m+b)\,\Gamma(k+d)\,b!d!}\\
&\times\frac{(1-\rho_G)^{-m-b-d}\;x^{\xi-1}}{(1-\rho_N)^{k+b+d}\,(\Omega_1\Omega_2)^{\xi/2}}\\
&\times\mathcal{G}_{4,0}^{0,4}\left[\frac{\sigma_1\,\sigma_2}{(m\,x)^2}\middle|\begin{matrix}1-\psi/2,1+\psi/2,1-\psi/2,1+\psi/2\\ -\end{matrix}\right].
\end{aligned}\tag{13}
$$

2) Ratio: Let $\mathcal{B}$ denote a RV defined as $\mathcal{B} \triangleq X_1/X_2$. The PDF of $\mathcal{B}$ is given by $f_{\mathcal{B}}(x) = \int_0^\infty |x_2| f_{X_1,X_2}(x_2x, x_2)dx_2$ [31, eq. (6.43)]. Substituting (4) in this expression and using [28, eq. (6.576/4)], the following expression for $f_{\mathcal{B}}(x)$ can be obtained

$$
\begin{aligned}
f_{\mathcal{B}}(x) = &\frac{2}{\Gamma(m)\Gamma(k)}\sum_{b,d=0}^{\infty}\frac{\rho_N^b\,\rho_G^d\,(1-\rho_N)^m\,(1-\rho_G)^k}{b!d!\Omega_1^{-k-d}\Omega_2^{k+d}\Gamma[2\xi]}\\
&\times\frac{\Gamma[2(k+d)]\,\Gamma[2(m+b)]\,\Gamma^2(\xi)}{\Gamma(m+b)\,\Gamma(k+d)}\\
&\times x^{-2(k+d)-1}{}_2F_1\left[2(k+d),\xi;2\xi;1-\frac{\Omega_1}{\Omega_2}x^{-2}\right].
\end{aligned}\tag{14}
$$

It should be noted that the last two expressions are useful for several communication theory research areas such as multi-hop transmissions [34], and diversity reception techniques [35]. For example in [35], the product of generalized-gamma RVs has been used to obtain upper bounds for the OP and average bit error probability (ABEP) of EGC diversity receivers operating over such fading channels.

III. Dual Branch Diversity Receivers Statistics

Let us consider a dual-branch diversity receiver operating over correlated K_G fading/shadowing channels. The equivalent baseband received signal at the ℓth ($\ell = 1$ and 2) antenna is expressed as $z_\ell = sh_\ell + n_\ell$, where s is the transmitted complex symbol with energy $E_s = \mathbb{E}\langle|s|^2\rangle$, n_ℓ is the complex additive white Gaussian noise (AWGN) with single sided power spectral density N_0 assumed identical to both branches, and h_ℓ is the channel complex gain, i.e., $X_\ell = |h_\ell|$. Similar to other works, e.g., [1, Chapter 9],[22], the n_ℓ's are assumed to be uncorrelated. Furthermore, by considering ideal phase estimation, only the distributed fading envelope affects the received signal. The instantaneous signal-to-noise ratio (SNR) per symbol at the ℓth input branch, γ_ℓ, and the corresponding average SNR, $\overline{\gamma}_\ell$, can be expressed as

$$\gamma_\ell = X_\ell^2\frac{E_s}{N_0}\tag{15a}$$

$$\overline{\gamma}_\ell = \mathbb{E}\langle X_\ell^2\rangle\frac{E_s}{N_0} = \Omega_\ell\,k\frac{E_s}{N_0}\tag{15b}$$

respectively. In the following subsections, important statistical metrics for the diversity receivers under consideration, namely SD, MRC and EGC, will be presented.

A. Selection Diversity (SD)

Since the instantaneous SNR at the output of the SD receiver is $\gamma_{\rm sd} = \max(\gamma_1,\gamma_2)$ its CDF can be expressed as $\mathcal{F}_{\gamma_{\rm sd}}(\gamma) = \mathcal{F}_{\gamma_1,\gamma_2}(\gamma,\gamma)$ [31, eq. (6.54)]. Using this equation and by making a change of variables in (11), of the form given in (15), $\mathcal{F}_{\gamma_{\rm sd}}(\gamma)$ can be expressed as

$$
\begin{aligned}
\mathcal{F}_{\gamma_{\rm sd}}(\gamma) = &\frac{\pi^2}{\Gamma(m)\Gamma(k)}\sum_{b,d=0}^{\infty}\frac{(mk)^{2(m+b)}\csc^2(\pi\psi)\,\rho_N^b\,\rho_G^d}{\Gamma(m+b)\Gamma(k+d)\,b!\,d!}\\
&\times\frac{(1-\rho_G)^{k-2(m+b)}}{(1-\rho_N)^{m+2b}}\left[\prod_{\ell=1}^{2}\left(\frac{\gamma}{\overline{\gamma}_\ell}\right)^{m+b}\mathcal{Q}\left(\frac{mk\gamma}{\sigma_{\gamma_\ell}}\right)\right]
\end{aligned}\tag{16}
$$

where $\sigma_{\gamma_\ell} = (1-\rho_N)(1-\rho_G)\overline{\gamma}_\ell$.

The OP is defined as the probability that the SD output SNR falls below a predetermined outage threshold $\gamma_{\rm th}$. By employing (16), the OP of dual-branch SD can be obtained by replacing γ with $\gamma_{\rm th}$ in (16) as $P_{\rm out} = \mathcal{F}_{\gamma_{\rm sd}}(\gamma_{\rm th})$.

B. Maximal Ratio Combining (MRC) and Equal Gain Combining (EGC)

1) Moments of the Output SNR: The instantaneous output SNR per symbol of EGC and MRC receivers can be expressed as

$$\gamma_{\rm out} = \theta_{\delta,1}\frac{E_s}{N_0}\left(\sum_{i=1}^{2}X_i^{-\delta+2}\right)^{\delta+1}\tag{17}$$

where $\theta_{\delta,a} = (2^{-a}-1)\,\delta+1$, $a\in\mathbb{N}$. Clearly from (17) for $\delta = 0$ and $\delta = 1$, $\gamma_{\rm out}$ represents the output for MRC and EGC diversity receivers, respectively. The nth-order moment of $\gamma_{\rm out}$ can be obtained by averaging $\gamma_{\rm out}^n$, i.e., $\mu_n = \mathbb{E}\langle\gamma_{\rm out}^n\rangle$, yielding

$$\mu_n = \theta_{\delta,n}\left(\frac{E_s}{N_0}\right)^n\mathbb{E}\left\langle\left(X_1^{-\delta+2}+X_2^{-\delta+2}\right)^{n(\delta+1)}\right\rangle.\tag{18}$$

Hence, using the binomial identity [28, eq. (1.111)], making a change of variables in (6) and after some straight forward mathematical manipulations, μ_n at the output of the MRC and EGC diversity receivers can be obtained in closed form as

$$
\begin{aligned}
\mu_n = &\theta_{\delta,n}\sum_{i=0}^{n(\delta+1)}\binom{n(\delta+1)}{i}\frac{\Gamma(k+n-i_\delta)\,\Gamma(m+n-i_\delta)}{\Gamma(m)^2\,\Gamma(k)^2\,(k\,m)^n}\\
&\times\frac{\Gamma(m+i_\delta)\,\Gamma(k+i_\delta)\,{}_2F_1(m+i_\delta,m+n-i_\delta;m;\rho_N)}{(1-\rho_N)^{-m-n}\,(1-\rho_G)^{-k-n}}\\
&\times\overline{\gamma}_1^{\,i_\delta}\overline{\gamma}_2^{\,n-i_\delta}\,{}_2F_1(k+i_\delta,k+n-i_\delta;k;\rho_G)
\end{aligned}\tag{19}
$$

where $i_\delta = i/(\delta+1)$.

TABLE I

NUMBER OF TERMS, N_τ, FOR CONVERGENCE OF (16) IN RANGE OF $\pm 0.5\,\%$.

			$\overline{\gamma}=-5\,$dB	$\overline{\gamma}=0\,$dB	$\overline{\gamma}=5\,$dB
$\rho_N=0.2$	$\rho_G=0.2$	$k=1, m=1$	2	2	3
		$k=3, m=4$	3	5	5
	$\rho_G=0.7$	$k=1, m=1$	4	7	11
		$k=3, m=4$	14	20	23
$\rho_N=0.7$	$\rho_G=0.2$	$k=1, m=1$	7	9	11
		$k=3, m=4$	14	20	22
	$\rho_G=0.7$	$k=1, m=1$	9	11	14
		$k=3, m=4$	17	22	25

By employing (19) useful performance metrics, including the average output SNR and the amount of fading (AF), for the EGC and MRC receivers can be derived in closed form. The AF is unified measure of the fading severity of a particular channel model [1]. As well known, using the first and second order moments of (19), the AF can be expressed in closed form as $\mathrm{AF}=\mu_2/\mu_1^2-1$.

2) Moments Generating and Characteristic Functions:

a) Maximal Ratio Combining (MRC): Using (15) to make a change of variables in (4), the bivariate K_G PDF of γ_ℓ can be expressed as

$$f_{\gamma_1,\gamma_2}(\gamma_1,\gamma_2)=\frac{4}{\Gamma(m)\Gamma(k)}\sum_{b,d=0}^{\infty}\frac{(mk)^{\xi}\,\rho_N^b\,\rho_G^d}{\Gamma\left(m+b\right)\Gamma\left(k+d\right)}\times\frac{\left(1-\rho_G\right)^{-m-b-d}}{b!d!\left(1-\rho_N\right)^{k+b+d}}\left[\prod_{\ell=1}^{2}\frac{\gamma_\ell^{\xi/2-1}}{\overline{\gamma}_\ell^{\xi/2}}K_\psi\left(2\sqrt{\frac{k\,m}{\sigma_{\gamma_\ell}}}\gamma_\ell^{1/2}\right)\right]. \tag{20}$$

The instantaneous SNR at the output of the MRC receiver is $\gamma_{\mathrm{mrc}}=\gamma_1+\gamma_2$. Since, the MGF of γ_{mrc} is defined as $\mathcal{M}_{\gamma_{\mathrm{mrc}}}(s)\triangleq\mathcal{M}_{\gamma_1,\gamma_2}(s,s)$, using (20), with the aid of [28, eq. (6.643/3)] and after some straight forward simplifications $\mathcal{M}_{\gamma_{\mathrm{mrc}}}(s)$ is obtained as

$$\mathcal{M}_{\gamma_{\mathrm{mrc}}}(s)=\frac{1}{\Gamma(m)\Gamma(k)}\sum_{b,d=0}^{\infty}\frac{(mk)^{\xi-1}\,\Gamma(k+d)}{b!\,d!\left(1-\rho_N\right)^{k+b+d-1}}\times\frac{\Gamma(m+b)\,\rho_N^b\,\rho_G^d\,s^{-(\xi-1)}}{\left(1-\rho_G\right)^{m+b+d-1}\left(\overline{\gamma}_1\overline{\gamma}_2\right)^{(\xi-1)/2}}\times\left[\prod_{\ell=1}^{2}\exp\left(\frac{mk}{2s\sigma_{\gamma_\ell}}\right)\mathrm{W}_{-(\xi-1)/2,\psi/2}\left(\frac{mk}{s\sigma_{\gamma_\ell}}\right)\right] \tag{21}$$

where $\mathrm{W}_{\lambda,\mu}\left(\cdot\right)$ is the Whittaker function [28, eq. (9.220)].

Using (21) and following the MGF-based approach, the ABEP of the MRC output SNR can be readily evaluated for a variety of modulation schemes, e.g., phase shift keying (PSK) and quadrature amplitude modulation (QAM), [1]. More specifically, the ABEP can be calculated: *i)* directly for non-coherent differential binary PSK (DBPSK), i.e., $P_{\mathrm{be}}=0.5\mathcal{M}_{\gamma_{\mathrm{mrc}}}(1)$; and *ii)* via numerical integration for Gray encoded M-ary PSK, i.e., $P_{\mathrm{be}}=\left[1/\left(\pi\log_2 M\right)\right]\int_0^{\pi-\pi/M}\mathcal{M}_{\gamma_{\mathrm{mrc}}}\left[\log_2 M\sin^2\left(\pi/M\right)/\sin^2\phi\right]d\phi$.

b) Equal Gain Combining (EGC): The instantaneous output signal envelope of a dual-branch EGC receiver is given by $X_{\mathrm{egc}}=\left(X_1+X_2\right)\sqrt{E_s/(2N_0)}$ and the CF of X_1+X_2 can be derived using [31, eq. (7.29)] as $\Phi_{X_1+X_2}(s)=\Phi_{X_1,X_2}(s,s)$. Thus, using $\Phi_{X_{\mathrm{egc}}}(s)=\Phi_{X_1+X_2}\left(s\sqrt{E_s/(2N_0)}\right)$ in conjunction with (12), the CF of X_{egc}, is derived as

$$\Phi_{X_{\mathrm{egc}}}(s)=\frac{\pi\left(1-\rho_N\right)^m\left(1-\rho_G\right)^k}{\Gamma(m)\Gamma(k)}\sum_{b,d=0}^{\infty}\frac{2^{4(\psi+1)}}{b!\,d!}\times\frac{(2km)^{2(k+d)}\,\rho_N^b\rho_G^d\,\Gamma\left[2(m+b)\right]^2\Gamma\left[2(k+d)\right]^2}{\Gamma(m+b)\Gamma(k+d)\,\Gamma\left(\xi+1/2\right)^2}\times\left\{\prod_{\ell=1}^{2}\frac{{}_2F_1\left[2(k+d),\psi+\frac{1}{2};\xi+\frac{1}{2};\frac{js\sqrt{\sigma_{\gamma_\ell}}-2\sqrt{2km}}{js\sqrt{\sigma_{\gamma_\ell}}+2\sqrt{2km}}\right]}{\left(js\sqrt{\sigma_{\gamma_\ell}}+2\sqrt{2km}\right)^{2(k+d)}}\right\}. \tag{22}$$

Using (22) and employing the Parseval's theorem approach [36], the average symbol error rate (ASER) can be studied. Hence, the ASER of several M-ary modulation schemes with predetection EGC is given by $P_s=(1/\pi)\int_0^\infty\Re\left\{G^*(s)\Phi_{X_{\mathrm{egc}}}(s)\right\}ds$, where $G^*(s)$ is the complex conjugate of the Fourier transform of the conditional error probability. Although, in principal using the Parseval's approach the ASER can be studied for many modulation schemes, here we focus on DBPSK, where $G(s)=0.5\left[\left(\sqrt{\pi}/2\right)\exp\left(-s^2/4\right)+j\left(s/2\right){}_1F_1\left(1;3/2;-s^2/4\right)\right]$.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section various numerical performance evaluation results, which have been obtained using the previous analysis will be presented. In particular, the results for the three diversity receiver structures, employing various modulation schemes and under different correlated K_G fading and shadowing conditions will be presented and analyzed.

Firstly, the rate of convergence of the previously derived infinite series expressions has been investigated. In Table I, the minimum number of terms N_τ, i.e., after the truncation of the infinite series, needed in (16) to achieve an accuracy better than $\pm 0.5\%$ is summarized. In this table, the number of terms versus $\overline{\gamma}$[2] are presented for several values of ρ_N, ρ_G

[2]For the shake of simplicity it is assumed $\overline{\gamma}_1=\overline{\gamma}_2=\overline{\gamma}$.

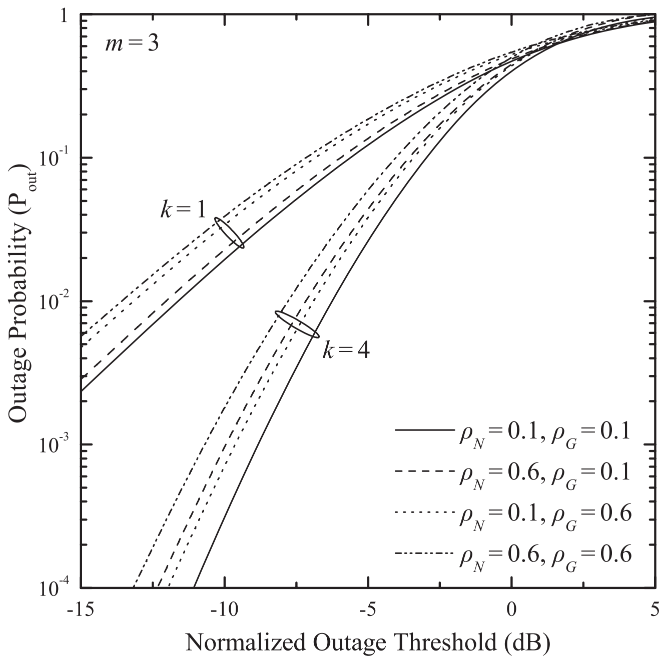

Fig. 1. SD performance: $P_{\rm out}$ versus $\gamma_{\rm th}/\overline{\gamma}$ for several values of ρ_N, ρ_G and m, k.

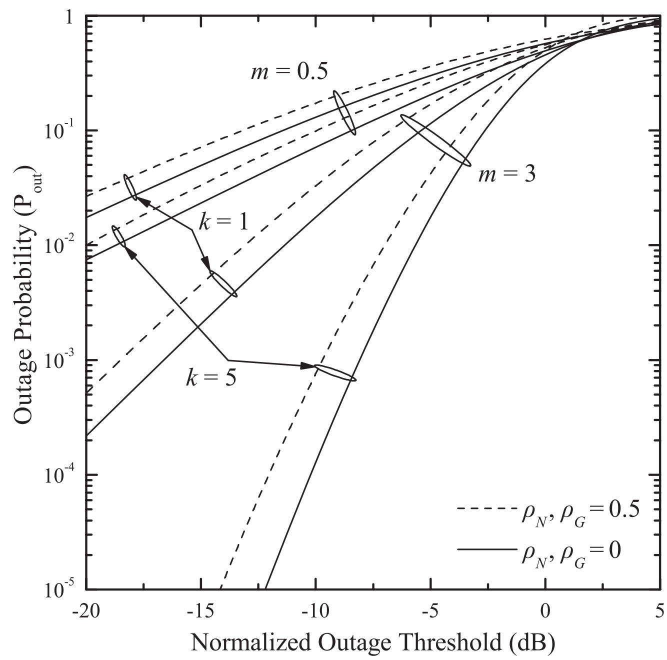

Fig. 2. SD performance: $P_{\rm out}$ versus $\gamma_{\rm th}/\overline{\gamma}$ for several values of k, m, with and without correlation.

and m, k. Clearly, N_τ increases as $\overline{\gamma}$ and/or ρ_N, ρ_G and/or m, k increase, with the latter ones having a more significant impact on the number of terms. It should be noted that N_τ is smaller as compared to the rate of convergence of other infinite series representation distributions, e.g., the bivariate Rayleigh, Nakagami-m [15], Rice [20], and Weibull [22]. For the other infinite series equations also used to obtain performance results, i.e., (21) (for MRC ABEP) and (22) (for EGC ABEP), similar rates of convergence have been observed.

In Figs. 1 and 2, the OP of a SD receiver is plotted as a function of the normalized outage threshold, $\gamma_{\rm th}/\overline{\gamma}$, for several values of ρ_N, ρ_G and m, k. In all cases it is depicted that as ρ_N, ρ_G decrease and/or m, k increase, the OP performance improves. In Fig. 1, it is interesting to note that for relatively good channel fading and shadowing conditions, i.e., for $m = 3$ and $k = 4$, the two middle lines are very close to each other and the system performance is slightly better for $\rho_N = 0.1$, $\rho_G = 0.6$ as compared to the case for $\rho_N = 0.6$, $\rho_G = 0.1$. On the contrary when the channel is subject to severe shadowing, i.e., for $k = 1$, the performance is better when $\rho_N = 0.6$, $\rho_G = 0.1$. This happens because there is an increased diversity gain due to the low correlation of the powers, i.e., $\rho_G = 0.1$, which diminishes the consequences of shadowing. In Fig. 2, it is interesting to note that for pre-Rayleigh fading conditions, i.e., for $m = 0.5$, the system performance is not acceptable, even for light shadowing conditions, i.e., $k = 5$. However, the performance improves significantly when small scale fading effects decrease, i.e., for $m = 3$.

In Fig. 3, the ABEP of DBPSK signals with MRC and EGC diversity receivers is plotted as a function of the first branch average input SNR per bit[3] $\overline{\gamma}_{1,b} = \overline{\gamma}_1$. These receivers operate in id and non id K_G fading channels with $\rho_N = \rho_G = 0.2$ or 0.7. As compared to non id and EGC, the ABEP performance is slightly better when id fading conditions and MRC diversity reception are employed, respectively. Moreover, the ABEP performance clearly improves when the correlations decrease. Finally, in Fig. 4, the ABEP performance of MRC diversity receiver is plotted as a function of the average input SNR per bit, $\overline{\gamma}_b = \overline{\gamma}/\log_2 M$, for Gray encoded M-ary PSK ($M = 2, 8, 16$) for several values of m, $\rho_N = \rho_G = 0.3$ and $k = 2$. As expected, in all cases the best ABEP performance is observed for BPSK signals, whereas the worst performance is for 16-PSK signals. It is interesting to note that for pre-Rayleigh multipath conditions, e.g., $m = 0.5$, the ABEP performance is very bad, even for increased values of the average input SNR per bit, i.e., for $\overline{\gamma}_b = 20$.

V. Conclusions

In this paper the arbitrary correlated K_G distribution with non identical parameters was introduced and studied. The most important statistical metrics of this composite fading/shadowing channel model, namely the PDF, CDF, joint moments, and CF have been presented. The obtained expression for the PDF generalize previous reported results. Furthermore, the PDFs of the product and the ratio of two correlated K_G RVs have been also derived. Capitalizing on the theoretical results, the performance of dual-branch selection, MRC and EGC diversity receivers operating over bivariate K_G fading channels, has been analyzed. Several numerically evaluated results were presented, indicating the influence of fading, shadowing and correlation to the performance of the diversity receivers.

[3]Similar to [37], when non identical scaling parameters are considered, it is assumed that $\overline{\gamma}_2 = \overline{\gamma}_1/\sqrt{e}$.

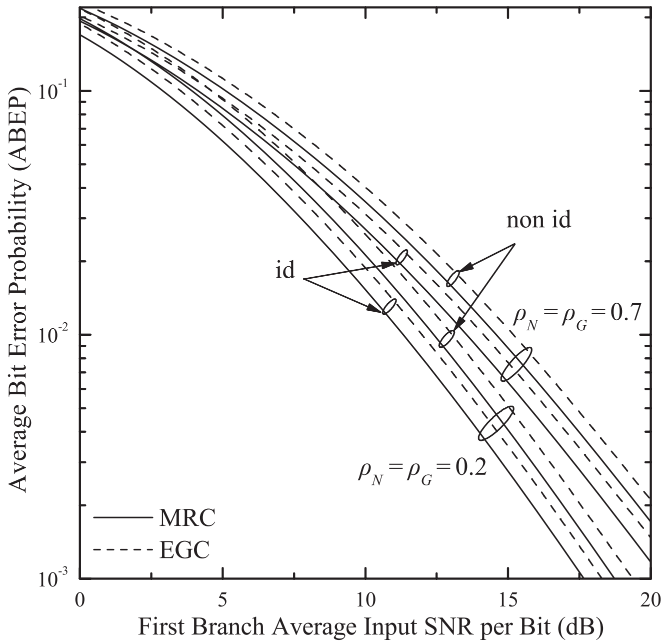

Fig. 3. MRC and EGC performance for DBPSK signals: ABEP versus $\overline{\gamma}_1$ for id and non id fading conditions and $\rho_N, \rho_G = 0.2$ or 0.7.

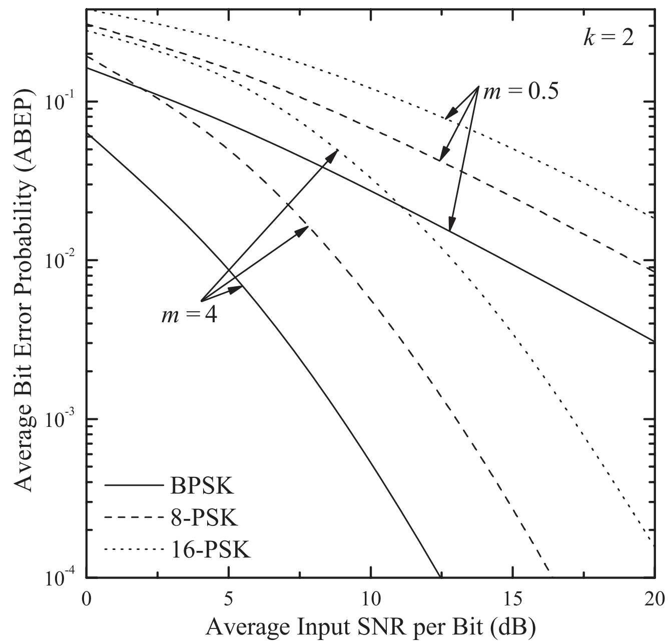

Fig. 4. MRC performance for M-PSK signals: ABEP versus $\overline{\gamma}_b$ for $m = 0.5$ or $m = 4$ and $k = 2$.

APPENDIX
DERIVATION OF (3)

Since W_ℓ's are considered as RVs, (1) is conditioned on W_ℓ's and the total probability theorem may be applied [31, eq. (7.44)]. Hence, the combined fading and shadowing bivariate K_G distribution can be obtained by averaging (1) with respect to W_ℓ's as

$$f_{X_1,X_2}(x_1,x_2) = \int_0^\infty \int_0^\infty f_{Y_1|W_1,Y_2|W_2}(y_1|W_1, y_2|W_2) \times f_{W_1,W_2}(W_1,W_2) dW_2 dW_1. \quad \text{(A-1)}$$

Substituting (1) and (2) in (A-1), integrals of the following form appear

$$\mathcal{I} = \int_0^\infty y^\alpha \exp\left(-A_1 y^{-1} - A_2 y\right) \times {}_1F_1(B_1; B_2; \rho_N\, A_1\, y^{-1})\, {}_1F_1(C_1; C_2; \rho_G\, A_2\, y) dy \quad \text{(A-2)}$$

where $A_i, B_i, C_i (\text{for } i = 1, 2) \in \Re^+$ and $a \in \Re$. Such integrals are very difficult, if not impossible, to be solved in closed from. An alternative and mathematically more convenient approach would be to employ the infinite series representation of the form [28, eq. (9.210/1)]

$${}_1F_1(a; c; y) = \sum_{b=0}^{\infty} \frac{(a)_b y^b}{(c)_b b!}. \quad \text{(A-3)}$$

With the aid of (A-3) and using [28, eq. (3.471/9)], the integrals in (A-2) can be solved, and hence after some mathematical manipulations, (3) is obtained.

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