On the Capacity of Generalized Fading/Shadowing Channels

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***Abstract*—In this paper the Shannon capacity of generalized-gamma (GG) fading channels is studied, under different adaptive transmission techniques. The GG is a generic distribution, which is considered to be appropriate for modeling fading/shadowing phenomena. Hence, assuming single (i.e., no diversity), as well as, dual-branch selection diversity receivers, novel expressions for the capacity under three adaptive transmission policies, namely optimal power and rate adaptation (OPRA), channel inversion with fixed rate (CIFR) and truncated CIFR (TIFR), are obtained. The derived expressions are in closed form and generalize previously presented results. The proposed analysis is accompanied by numerical results, clearly demonstrating the usefulness of the theoretical approach.**

***Index Terms*— Adaptive transmission, fading/shadowing channels, generalized-Gamma distribution, land-mobile satellite channels, selection diversity receivers, Shannon capacity.**

I. Introduction

The capacity in fading channels is, in general, a complex expression in terms of the channel variations in time and/or frequency, depending also upon the transmitter and/or receiver knowledge of the channel side information (CSI). For the various CSI assumptions that have been proposed, several definitions of the channel capacity have been provided. These definitions depend on the different employed power and rate adaptation policies and the existence, or not, of an outage probability [1]. Widely accepted adaptation techniques are the optimal power and rate adaptation (OPRA), constant power with rate adaptation (ORA), channel inversion with fixed rate (CIFR) and truncated CIFR (TIFR). In all these techniques the distribution of the channel power gain is consider to be known at both the transmitter and the receiver. Hence, it is very critical to study the above mentioned adaptation policies under different fading channel models.

In the past, the capacity has been studied for several fading distributions, e.g., [2]–[8]. In [2], the general theory for the capacity of fading channels with an average power constraint, under different CSI conditions was developed. In [3], this theory was applied to Rayleigh fading channels, assuming no diversity, as well as, selection and maximal ratio combining (MRC) diversity receivers. In [4], the results presented in [3] were generalized to Nakagami-m fading channels, while in [5] the channel capacity was studied for MRC and equal gain combining diversity receivers, assuming Nakagami-q fading channel modeling. In [6], expressions for the Shannon capacity of single-branch receivers operating over Nakagami-m, Rice and Weibull fading channels were derived. In [8], the capacity was studied for generalized-K composite fading channels, for different adaptive transmission techniques. Hence, since the general framework proposed in [2] has gained an increased interest, it is considered also here in the context of generalized-Gamma (GG) fading channels.

The composite GG distribution includes many well-known fading channel models for multipath, e.g., Rayleigh, Nakagami-m, and Weibull, as special cases, while it can also describe the Lognormal as a limiting case. Thus, due to its generic form, the GG distribution can accurately describe the behavior of multipath and shadowing fading effects, which is the typical channel model observed in land mobile satellite systems [9]. However, despite the ability of the GG distribution to characterize so many different fading/shadowing channel models, only recently it has been applied to the field of digital communications over fading channels [10]–[13]. As an example, in [13] the performance analysis of switch and stay combining diversity receivers operating over GG fading channels was studied. In the same work, the spectral efficiency was also obtained only for the ORA policy. Hence, the capacity of GG fading channels under OPRA, CIFR and TIFR policies has not been yet studied in the open technical literature and thus is the subject of this paper.

This paper is organized as follows. After this introduction, in Section II, the system and channel model is introduced. In Sections III-V, closed-form expressions for the capacity of GG fading channels are derived for the OPRA, CIFR and TIFR adaptation policies, respectively. In Section VI, several numerical evaluating results are presented and discussed, while in Section VII, the concluding remarks of this paper are provided.

II. System and Channel Model

Let us consider a single-branch (SB) receiver operating over slow varying fading channels, typically encountered in geostationary satellite channels and metropolitan areas with slow moving pedestrians [14]. The received instantaneous signal amplitudes are considered to be GG distributed. Hence, the probability density function (PDF) of the instantaneous signal-to-noise ratio (SNR) at the output of the SB receiver,

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$\gamma_{\rm sb}$, is given by [13]

$$f_{\gamma_{\rm sb}}(\gamma)=\frac{\beta\,\gamma^{m\beta/2-1}}{2\,\Gamma(m)\,\left(\tau\,\overline{\gamma}\right)^{m\,\beta/2}}\exp\left[-\left(\frac{\gamma}{\tau\,\overline{\gamma}}\right)^{\beta/2}\right] \quad (1)$$

where $\beta>0$ and $m\geq 1/2$ are the distribution's shaping parameters related to the fading severity, $\overline{\gamma}$ is the average input SNR per symbol, and $\tau=\Gamma(m)/\Gamma(m+2/\beta)$, with $\Gamma(\cdot)$ being the Gamma function [15, eq. (8.310/1)]. By using different values of m and β, (1) simplifies to several important distributions for fading modeling. For example, for $\beta=2$ and $m=1$, it becomes Rayleigh, for $\beta=2$, becomes Nakagami-m, and for $m=1$, becomes Weibull. Moreover, as $\beta\rightarrow 0$ and $m\rightarrow\infty$, (1) approaches the well-known lognormal PDF. The corresponding cumulative distribution function (CDF) is given by

$$F_{\gamma_{\rm sb}}(\gamma)=1-\frac{\Gamma\left[m,(\gamma/\tau\overline{\gamma})^{\beta/2}\right]}{\Gamma(m)} \quad (2)$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete Gamma function [15, eq. (8.350/2)].

Furthermore, let $\gamma_{\rm sd}$ represent the SNR per symbol at the output of a dual-branch selection diversity (SD) receiver operating over independent and identical distributed GG fading channels. The SD receiver is one of the simplest diversity reception techniques, as only the selectively chosen single branch is processed [9]. The CDF of $\gamma_{\rm sd}$, $F_{\gamma_{\rm sd}}(\gamma)$, can be expressed as $F_{\gamma_{\rm sd}}(\gamma)=\left[F_{\gamma}(\gamma)\right]^2$. For integer values of m, by differentiating $F_{\gamma_{\rm sd}}(\gamma)$ with respect to γ and using [15, eq. (8.352/2)], the PDF of $\gamma_{\rm sd}$ can be mathematical expressed in closed form as

$$f_{\gamma_{\rm sd}}(\gamma)=\frac{\beta}{\tau\,\overline{\gamma}\,\Gamma(m)}\left\{\left(\frac{\gamma}{\tau\,\overline{\gamma}}\right)^{\beta m/2-1}\exp\left[-\left(\frac{\gamma}{\tau\,\overline{\gamma}}\right)^{\beta/2}\right]\right.$$
$$\left.-\sum_{i=0}^{m-1}\frac{1}{i!}\left(\frac{\gamma}{\tau\,\overline{\gamma}}\right)^{\beta(m+i)/2-1}\exp\left[-2\left(\frac{\gamma}{\tau\,\overline{\gamma}}\right)^{\beta/2}\right]\right\}. \quad (3)$$

III. Optimal Power and Rate Adaptation (OPRA)

The channel capacity under an average transmitting power constraint and optimal power and rate adaptation is given by [2]

$$C_{\rm opra}=B\int_{\gamma_0}^{\infty}\log_2\left(\frac{\gamma}{\gamma_0}\right)f_{\gamma}(\gamma)d\gamma \quad (4)$$

where B is the channel bandwidth and γ_0 the optimal cutoff SNR. If $\gamma<\gamma_0$ no data is transmitted and hence an outage probability occurs as $P_{\rm out}=F_{\gamma}(\gamma_0)$. Furthermore, by denoting

$$p(x)=\int_{x}^{\infty}\left(\frac{1}{x}-\frac{1}{\gamma}\right)f_{\gamma}(\gamma)d\gamma-1 \quad (5)$$

γ_0 must satisfy $p(\gamma_0)=0$.

A. SB Receiver

Substituting (1) in (5), making a change of variables and using [15, eq. (8.350/2)], the optimal cutoff function for the SB receiver can be obtained as

$$p_{\rm sb}(\gamma_0)=\sum_{k=1}^{2}\frac{(-1)^{k+1}}{\Gamma(m)\,\gamma_0^{2-k}(\tau\,\overline{\gamma})^{k-1}}\times\Gamma\left[m-2\frac{k-1}{\beta},\left(\frac{\gamma_0}{\tau\,\overline{\gamma}}\right)^{\beta/2}\right]-1. \quad (6)$$

In the above equation γ_0 cannot be obtained in closed form and hence numerical evaluation will be employed, by using any of the well-known mathematical software packages. Moreover, it can be proved that there is a unique positive value for γ_0, which satisfies $p_{\rm sb}(\gamma_0)=0$. Note that for $\beta=2$, (6) simplifies to previous known results, i.e., [4, eq. (7)].

Substituting (1) in (4) integral of the following form need to be solved

$$\mathcal{I}=\int_{\gamma_0}^{\infty}\gamma^{m\beta/2-1}\ln\left(\frac{\gamma}{\gamma_0}\right)\exp\left[-\left(\frac{\gamma}{\tau\,\overline{\gamma}}\right)^{\beta/2}\right]d\gamma. \quad (7)$$

This integral can be solved in closed form (see the Appendix for details), and hence $C_{\rm opra}^{\rm sb}$ can be obtained as

$$C_{\rm opra}^{\rm sb}=\frac{2\,B}{\beta\,\Gamma(m)\ln(2)}\Phi\left[\left(\frac{\gamma_0}{\tau\,\overline{\gamma}}\right)^{\beta/2},m,1\right] \quad (8)$$

where

$$\Phi(x,y,z)=\Gamma(y)\psi(y)-\frac{\beta}{2}\ln\left[\frac{2^{2(y-m)/(\beta\,z)}\gamma_0}{\tau\,\overline{\gamma}}\right]\Gamma\left(y,x\right)$$
$$+\Gamma\left(y\right)^2x^y{}_2\tilde{F}_2\left(y,y;y+1,y+1;-x\right)-\gamma\left(y,x\right)\ln\left(x\right)$$

where ${}_p\tilde{F}_q(\cdot;\cdot;\cdot)$ represents the regularized generalized hypergeometric function [16, eq. (07.32.02.0001.01)], where p,q are integers. Furthermore, $\gamma(\cdot,\cdot)$ represents the lower incomplete Gamma function [15, eq. (3.381/1)] and $\psi(\cdot)$ denotes the psi function [15, eq. (8.360)].

B. SD Receiver

Substituting (3) in (5) and following similar approach as for deriving (6), the optimal cutoff function for the SD receiver can be obtained as

$$p_{\rm sd}(\gamma_0)=\sum_{k=1}^{2}2\frac{(-1)^{k+1}\Gamma\left[m-2(k-1)/\beta,\left[\gamma_0/\left(\tau\,\overline{\gamma}\right)\right]^{\beta/2}\right]}{\Gamma(m)\,\gamma_0^{2-k}(\tau\,\overline{\gamma})^{k-1}}$$
$$+\sum_{k=1}^{2}\sum_{i=0}^{m-1}\frac{(-1)^k2^{1+2(k-1)/\beta-i-m}}{i!\gamma_0^{2-k}\Gamma(m)\left(\tau\,\overline{\gamma}\right)^{k-1}}$$
$$\times\Gamma\left[i+m-\frac{2(k-1)}{\beta},2\left(\frac{\gamma_0}{\tau\,\overline{\gamma}}\right)^{\beta/2}\right]-1. \quad (9)$$

It can also be proved that there is a unique positive γ_0 satisfying $p_{\rm sd}(\gamma_0)=0$, which is numerical evaluated. Substituting (3) in (4), identical type of integrals with that one in (7) appear.

Hence, using again the analysis presented in the Appendix, and after some mathematical manipulations, the capacity of a SD receiver under the OPRA policy, $C_{\mathrm{opra}}^{\mathrm{sd}}$, can be obtained in closed form as

$$\begin{aligned} C_{\mathrm{opra}}^{\mathrm{sd}} = \frac{4B}{\beta\,\Gamma(m)\ln(2)} \Bigg\{ \Phi\left[\left(\frac{\gamma_0}{\tau\,\overline{\gamma}}\right)^{\beta/2}, m, 1\right] \\ - \sum_{i=0}^{m-1} \frac{2^{-m-i}}{i!} \Phi\left[2\left(\frac{\gamma_0}{\tau\,\overline{\gamma}}\right)^{\beta/2}, m+i, i\right]\Bigg\}. \end{aligned} \quad (10)$$

IV. CHANNEL INVERSION WITH FIXED RATE (CIFR)

In CIFR policy the transmitter exploits the channel side information in order to maintain constant the SNR at the receiver [2]. The channel capacity employing CIFR, C_{cifr}, is given by

$$C_{\mathrm{cifr}} = B \log_2\left(1 + \frac{1}{\int_0^\infty f_\gamma(\gamma)/\gamma\, d\gamma}\right). \quad (11)$$

A. SB Receiver

Substituting (1) in (11) and using [15, eq. (3.351/3)], the capacity of a SB receiver under the CIFR technique can be obtained as

$$C_{\mathrm{cifr}}^{\mathrm{sb}} = B \log_2\left[1 + \frac{1}{\Theta_1\,(m,0)}\right] \quad (12)$$

where

$$\Theta_1(x,y) = \frac{\Gamma\,(x - 2/\beta)}{\Gamma(m)\tau\,\overline{\gamma}\,2^{(x-2/\beta)y}}.$$

By setting $\beta = 2$ and $m = 1$ in the above equation, it simplifies to previous known results [4, eq. (29)] and [17, eq. (23)], respectively.

B. SD Receiver

In the case of SD receiver, substituting (3) in (11), using again [15, eq. (3.351/3)], the capacity following CIFR policy can be obtained in closed form as

$$C_{\mathrm{cifr}}^{\mathrm{sd}} = B \log_2\left[1 + \frac{1}{2\left[\Theta_1(m,0) - \sum\limits_{i=0}^{m-1}\Theta_1\,(m+i,1)\,/i!\right]}\right]. \quad (13)$$

By setting $m = 1$ in (13), it simplifies to previous derived expression [17, eq. (22)], for $\rho = 0$.

V. TRUNCATED CHANNEL INVERSION WITH FIXED RATE (TIFR)

The CIFR technique is very simple to implement, however it exhibits a large capacity penalty in extreme fading environments, e.g., assuming Rayleigh fading $C_{\mathrm{cifr}} = 0$ [2]. An alternative approach is to consider a truncated CIFR policy, usually referred as TIFR, where the channel fading is inverted only above the fixed cutoff γ_0. In this case the capacity can be obtained as

$$C_{\mathrm{tifr}} = B \log_2\left[1 + \frac{1}{\int_{\gamma_0}^\infty f_\gamma(\gamma)/\gamma\, d\gamma}\right](1 - P_{\mathrm{out}}). \quad (14)$$

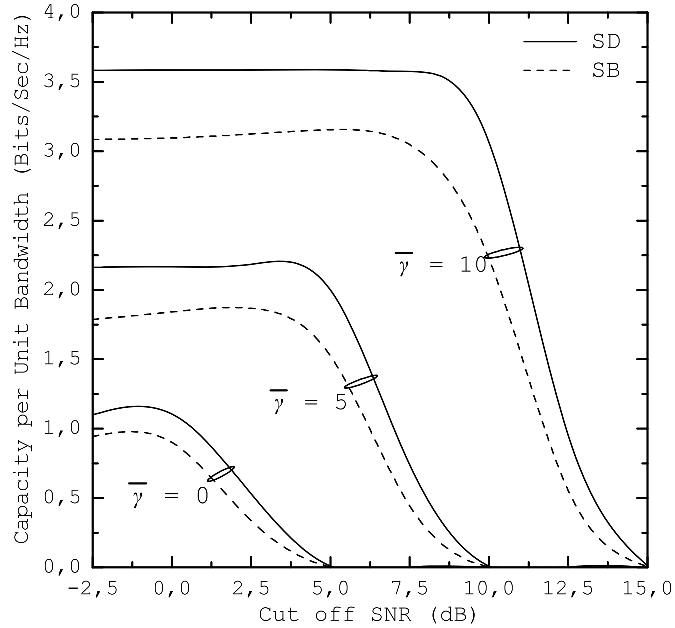

Fig. 1. Normalized average channel capacity under TIFR policy versus cutoff SNR for several values of $\overline{\gamma}$ and $m = 2, \beta = 3$.

A. SB Receiver

Substituting (1) in (14), making a change of variables and using [15, eq. (8.350/2)], the capacity of a single receiver under the TIFR policy can be obtained as

$$C_{\mathrm{tifr}}^{\mathrm{sb}} = B \log_2\left[1 + \frac{1}{\Theta_2\,(m,0)}\right] \frac{\Gamma\left[m, \left(\frac{\gamma_0}{\tau\,\overline{\gamma}}\right)^{\beta/2}\right]}{\Gamma(m)} \quad (15)$$

where

$$\Theta_2(x,y) = \frac{\Gamma\left[x - 2/\beta, 2^y\,(\gamma_0/\tau\,\overline{\gamma})^{\beta/2}\right]}{\Gamma(m)\tau\,\overline{\gamma}\,2^{(x-2/\beta)y}}.$$

Setting $\beta = 2$, (15) simplifies to [4, eq. (30)], while by setting $m = 1$ it simplifies to [17, eq. (26)].

B. SD Receiver

In the case of SD receiver, by substituting (3) in (14), and using [15, eq. (8.350/2)], the capacity following the TIFR policy can be obtained in closed form as

$$\begin{aligned} C_{\mathrm{tifr}}^{\mathrm{sd}} = B \log_2\left[1 + \frac{1}{2\left[\Theta_2(m,0) - \sum\limits_{i=0}^{m-1}\Theta_2(m+i,1)/i!\right]}\right] \\ \times \left\{1 - \left[1 - \frac{\Gamma\left[m, (\gamma_0/(\tau\overline{\gamma}))^{\beta/2}\right]}{\Gamma(m)}\right]^2\right\}. \end{aligned} \quad (16)$$

Note that by setting $m = 1$ in (16), it simplifies to [17, eq. (25)], for $\rho = 0$.

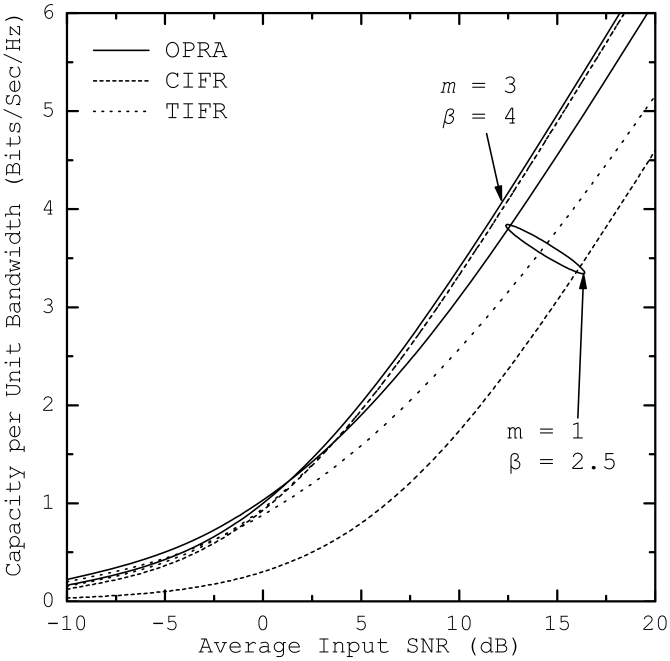

Fig. 2. Normalized channel capacity of SB receivers versus the average input SNR for different adaptation policies.

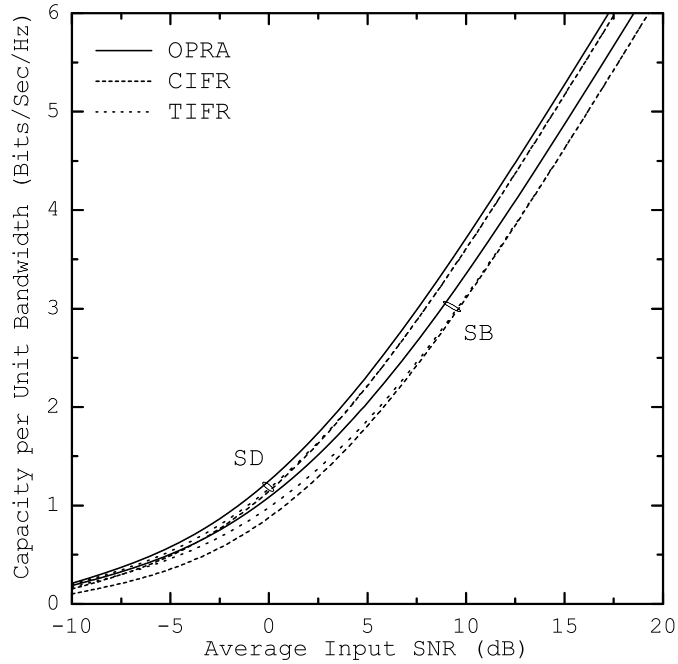

Fig. 3. Normalized channel capacity of SD receiver versus the average input SNR for different adaptation policies.

VI. Numerical Results and Discussion

In this section various numerical evaluated results have been obtained using the previous derived analysis for SB (no diversity), as well as, dual-branch SD receivers. These results include capacity comparisons of different adaptation policies and GG fading conditions.

In Fig. 1, considering SB (using (15)) and SD (using (16)) receivers, the normalized average capacity under the TIFR policy, $C_{\rm tifr}/B$, is plotted as a function of cutoff SNR, γ_0, for several values of the average input SNR, $\overline{\gamma}$. Moreover, it is assumed that $m = 2$ and $\beta = 3$. It is depicted that $C_{\rm tifr}/B$ improves by employing diversity reception and/or increasing $\overline{\gamma}$. Furthermore, in all cases $C_{\rm tifr}/B$ is maximized for specific values of γ_0 and the difference between the performances of SB and SD receivers increases as $\overline{\gamma}$ increase. In Fig. 2, the normalized channel capacity is plotted for the various adaptation policies presented in this paper, namely $C^{\rm sb}_{\rm opra}/B$ (using (8)), $C^{\rm sb}_{\rm cifr}/B$ (using (12)), and $C^{\rm sb}_{\rm tifr}/B$ (using (15)), as a function of $\overline{\gamma}$ for SB receiver and different GG fading conditions. Employing OPRA yields always the highest capacity, while CIFR the lowest. However, the differences among the normalized capacities of all the adaptation policies diminish as $\overline{\gamma}$ and/or m, β increase. Note that for increased values of m, β, i.e., reduced fading severity, and $\overline{\gamma}$, $C^{\rm sb}_{\rm cifr}/B$ and $C^{\rm sb}_{\rm tifr}/B$ become equal, due to the fact that the probability of outage is very low.

In Fig. 3, the normalized channel capacity for SD reception is plotted as a function of $\overline{\gamma}$, assuming OPRA, $C^{\rm sd}_{\rm opra}/B$ (using (10)), CIFR, $C^{\rm sd}_{\rm cifr}/B$ (using (13)), and TIFR, $C^{\rm sd}_{\rm tifr}/B$ (using (16)), adaptation policies and $m = 2$, $\beta = 3$. For comparison purposes, plots for the corresponding adaptation policies of SB receivers are also included. Note that CIFR and TIFR capacities, for SD receiver, become almost equal for smaller values of the average input SNR, i.e., $\overline{\gamma} = 0$, as compared to a single receiver.

VII. Conclusions

Considering different adaptation policies, novel closed-form expressions were obtained for the Shannon capacity of GG fading/shadowing channels. In particular, the channel capacity of this composite distribution was studied for OPRA, CIFR and TIFR adaptation policies assuming SB and dual-branch SD receivers. The derived formulae are quite general as they simplify to previous obtained results. For these adaptation policies, selected numerical evaluated results were presented, assuming different GG fading/shadowing conditions and different receiver structures. In all cases, it is depicted that the capacity increases as channel conditions improve, i.e., m, β increase, and/or diversity reception is employed.

Appendix
Evaluation of (7)

Starting with (7), making a change of variables of the form $x = (\gamma/\gamma_0)^{\beta/2}$, and after some mathematical manipulations, yields

$$\mathcal{I} = \frac{4\gamma_0^{m\beta/2}}{\beta^2} \int_1^\infty x^{m-1} \ln(x) \exp\left[-\left(\frac{\gamma_0}{\tau\,\overline{\gamma}}\right)^{\beta/2} x\right] dx. \tag{A-1}$$

Integral of this form can be solved in closed form, with the aid of [15, eq. (4.358/1)], as

$$\mathcal{I} = \frac{4\gamma_0^{m\beta/2}}{\beta^2}\frac{\vartheta}{\vartheta m}\left\{\left(\frac{\gamma_0}{\tau\,\overline{\gamma}}\right)^{-m\beta/2}\Gamma\left[m,\left(\frac{\gamma_0}{\tau\,\overline{\gamma}}\right)^{\beta/2}\right]\right\} \tag{A-2}$$

The differentiation in (A-2) can be performed by applying [16, eq. (06.06.20.0001.01)], and hence after some mathematical manipulations finally yields (8).

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