Correspondence

Capacity of Correlated Generalized Gamma Fading With Dual-Branch Selection Diversity

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***Abstract*—The capacity of correlated generalized Gamma (GG) fading channels with dual-branch selection diversity (SD) reception and channel side information (CSI) available at both the transmitter and the receiver is studied and analyzed. In particular, the capacity is evaluated under four adaptive transmission policies defined according to the operational CSI: 1) optimal power and rate adaptation (OPRA); 2) constant power with optimal rate adaptation (ORA); 3) channel inversion with fixed rate (CIFR); and 4) truncated CIFR (TIFR). Under these four adaptation policies, fast converging infinite series expressions for the capacity are derived and evaluated. Furthermore, for the simplifying cases of independent GG fading channels and/or single-channel reception (i.e., no diversity), closed-form expressions have been obtained. Previously published expressions are shown to be special cases of the derived expressions. Performance evaluation results obtained by means of numerical analysis clearly demonstrate the usefulness and accuracy of the analysis.**

***Index Terms*—Adaptive transmission techniques, channel side information (CSI), correlated generalized Gamma (GG) fading, selection diversity (SD) receivers, Shannon capacity.**

I. Introduction

Obtaining expressions for the capacity in fading channels is not a trivial task since such derivations depend on the channel variations in time and/or frequency, as well as on the transmitter and/or receiver knowledge of the channel side information (CSI) [1]. For the various CSI assumptions that have been considered in the past, several definitions for the Shannon channel capacity[1] have been proposed. These definitions depend on a number of parameters, including the various power and rate adaptation policies for the transmitter, as well as the existence or absence of an outage probability [1]. Widely accepted adaptation policies include optimal power and rate adaptation (OPRA), constant power with optimal rate adaptation (ORA), channel inversion with fixed rate (CIFR), and truncated CIFR (TIFR) [1], [2]. For these policies, the CSI is considered to be known to both the transmitter and receiver sides. Clearly, the channel fading conditions play an important role in accurately determining the theoretical performance limits of the capacity.

In the past, the problem of estimating the capacity in the presence of *independent* fading channels has received considerable attention (e.g., [2]–[8]). In [2], a general theoretical framework for estimating the capacity of fading channels with an average power constraint and using CSI at both the transmitter and the receiver was proposed. In [3], this theory was applied to Rayleigh fading channels, also considering selection diversity (SD) and maximal ratio combining (MRC) receivers. Furthermore, it was applied to other independent fading channels, including Nakagami-m [4], Hoyt [5], Weibull [6], and generalized-K [7], employing various diversity-reception techniques. However, the assumption of statistical independence is not always valid, particularly in practical applications where the diversity antennas are not sufficiently separated from each other [9, ch. 9]. For correlated fading channels, the diversity gain is usually overestimated, and the non-negligible correlation effects become dominant. Hence, in the past, the impact of correlation to the capacity of fading channels has been studied for many of the previously mentioned distributions (e.g., [6], [10]–[12]). For example, in [10], by considering correlated Rayleigh fading channels, closed-form expressions for the capacity of MRC diversity receivers, under the various adaptation policies previously mentioned, are presented. In [12], assuming correlated Nakagami-m fading channels and ORA policy, infinite series representations have been provided for the capacities of several diversity-reception techniques.

Another important fading distribution, which has recently witness an increased interest in the modeling of mobile communication channels, is the generalized Gamma (GG) distribution [13]–[17]. The composite GG distribution includes many well-known distributions as special cases, e.g., Rayleigh, Nakagami-m, and Weibull, whereas it has the important advantage of also describing the lognormal distribution as a limiting case [18]. Clearly, the generic form of the GG distribution allows the accurate modeling of several multipath fading channels and shadowing conditions [18], which appear in many indoor and outdoor propagation environments [9, ch. 2]. Hence, in the context of single receivers (SRs) operating over GG fading channels, several important performance metrics have been studied, e.g., the bit error rate of various modulation formats [13]. Furthermore, in [14], the performance of several diversity reception techniques, including MRC, SD, and equal gain combining, operating over GG fading channels has been studied, whereas in [15], the performance of generalized selection combining was analyzed. However, on the subject of GG fading channel capacity, very few publications exist in the open technical literature. For example, in [16], the GG fading capacity with switch-and-stay combining diversity reception was analyzed and evaluated. Nevertheless, to the best of the authors' knowledge, a thorough analytical study of the GG fading channel capacity, e.g., under the OPRA, ORA, CIFR, and TIFR adaptation policies, has not been previously considered in the open technical literature. Thus, the purpose of this correspondence is to present an analytical performance study of the capacity for *correlated* GG fading channels with dual-branch SD reception and different transmission adaptation policies.

The remainder of this correspondence is organized as follows: In Section II, the system and channel model is introduced. In Section III, assuming integer values for m, infinite series expressions for the capacity of correlated GG fading channels with dual-branch SD receivers are derived for OPRA, ORA, CIFR, and TIFR adaptation policies. For independent GG fading channels, closed-form expressions for the SD, as well as single (i.e., no diversity) receivers, are also presented. In Section IV, several numerical evaluated results are presented and discussed, whereas in Section V, concluding remarks can be found.

Manuscript received August 8, 2008; revised January 15, 2009 and April 2, 2009. First published June 16, 2009; current version published November 11, 2009. This work was performed within the framework of the Satellite Network of Excellence (SatNEx-II) under Project IST-027393 and the Network of Excellence funded by the European Commission under the FP6 program. The review of this paper was coordinated by Dr. C. Cozzo.

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Digital Object Identifier 10.1109/TVT.2009.2025594

[1]Similar to other publications (e.g., [2] and [3]) and for the conciseness of the presentation, what is known as *Shannon channel capacity* will be referred to here simply as *capacity*.

II. System and Channel Model

Let us consider a dual-branch diversity receiver operating over identically distributed GG fading channels. The probability density function (pdf) of the instantaneous SNR per symbol γ_ℓ at the ℓth ($\ell = 1$ and 2) input branch is given by [16]

$$f_{\gamma_\ell}(\gamma) = \frac{\beta\,\gamma^{m\beta/2-1}}{2\,\Gamma(m)(\tau\overline{\gamma})^{m\beta/2}} \exp\left[-\left(\frac{\gamma}{\tau\overline{\gamma}}\right)^{\beta/2}\right] \tag{1}$$

where $\beta > 0$ and $m \geq 1/2$ are the distribution's shaping parameters related to the fading severity, $\overline{\gamma}$ is the average input SNR per symbol, and $\tau = \Gamma(m)/\Gamma(m+2/\beta)$, with $\Gamma(\cdot)$ being the Gamma function [19, eq. (8.310/1)]. For different values of m and β, (1) simplifies to several important distributions used in fading channel modeling, e.g., Rayleigh, Nakagami-m, and Weibull [16]. Its corresponding cumulative distribution function (cdf) can be expressed as

$$\mathcal{F}_{\gamma_\ell}(\gamma) = 1 - \frac{\Gamma\left[m, (\gamma/(\tau\overline{\gamma}))^{\beta/2}\right]}{\Gamma(m)} \tag{2}$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete Gamma function [19, eq. (8.350/2)].

Furthermore, let $\gamma_{\rm sd}$ represent the SNR per symbol at the output of a dual-branch SD receiver operating over correlated and identical distributed GG fading channels. As is well known, SD is a low-complexity diversity reception technique since only one received signal from the selectively chosen branch is processed [9, ch. 9]. Using [20, eq. (3)] and following the transformations presented in [21], the following expression for the cdf of $\gamma_{\rm sd}$, i.e., $\mathcal{F}_{\gamma_{\rm sd}}(\gamma)$, can be obtained:

$$\mathcal{F}_{\gamma_{\rm sd}}(\gamma) = (1-\rho)^m \sum_{k=0}^{\infty} \frac{\rho^k}{\Gamma(m)\Gamma(m_k)k!} \gamma\left[m_k, \frac{1}{1-\rho}\left(\frac{\gamma}{\tau\overline{\gamma}}\right)^{\beta/2}\right]^2 \tag{3}$$

where $\gamma(\cdot,\cdot)$ is the lower incomplete Gamma function [19, eq. (8.350/1)], $m_x = m + x$, and $0 \leq \rho \leq 1$ is the distribution's correlation coefficient. Since the mathematical representation of (3) is as in [20, eq. (3)], the error in truncating the infinite series in (3) can be upper bounded by following a procedure similar to that proposed in [20]. In the case of independent GG random variables (RVs), i.e., $\rho = 0$, (3) simplifies to $\mathcal{F}_{\gamma_{\rm sd}}(\gamma) = [\mathcal{F}_{\gamma_\ell}(\gamma)]^2$. Assuming integer values of m, by differentiating (3) with respect to γ and using first [19, eq. (8.356/4)] and then [19, eq. (8.352/1)], the pdf of $\gamma_{\rm sd}$ can mathematically be expressed as

$$f_{\gamma_{\rm sd}}(\gamma) = \frac{\beta}{\Gamma(m)} \sum_{k=0}^{\infty} \frac{[\rho/(1-\rho)]^k}{k!(\tau\overline{\gamma})^{\beta m_k/2}} \left\{\gamma^{\beta m_k/2-1} \exp\left[-\frac{1}{1-\rho}\left(\frac{\gamma}{\tau\overline{\gamma}}\right)^{\beta/2}\right]\right.$$
$$\left. - \sum_{t=0}^{m_k-1} \frac{\gamma^{\beta m_{k+t}/2-1}/t!}{(1-\rho)^t(\tau\overline{\gamma})^{t\beta/2}} \exp\left[-\frac{2}{1-\rho}\left(\frac{\gamma}{\tau\overline{\gamma}}\right)^{\beta/2}\right]\right\}. \tag{4}$$

For independent GG RVs, (4) simplifies in the following closed-form expression:

$$f_{\gamma_{\rm sd}}(\gamma) = \frac{\beta}{\tau\overline{\gamma}\Gamma(m)} \left(\frac{\gamma}{\tau\overline{\gamma}}\right)^{\beta m/2-1} \left\{\exp\left[-\left(\frac{\gamma}{\tau\overline{\gamma}}\right)^{\beta/2}\right]\right.$$
$$\left. - \sum_{i=0}^{m-1} \frac{1}{i!}\left(\frac{\gamma}{\tau\overline{\gamma}}\right)^{\beta i/2} \exp\left[-2\left(\frac{\gamma}{\tau\overline{\gamma}}\right)^{\beta/2}\right]\right\}. \tag{5}$$

III. Capacity Analysis

In this section, the capacity of correlated and/or independent GG fading channels with single or SD reception will be presented under different adaptive transmission policies. Similar to [3], it is assumed that, for all the considered adaptation policies, there exist perfect channel estimation and an error-free delayless feedback path.

A. OPRA

The capacity under an average transmitting power constraint and the OPRA policy of a fading channel with the received SNR pdf $f_\gamma(\gamma)$ is defined as [2]

$$C_{\rm opra} \triangleq B \int_{\gamma_0}^{\infty} \log_2\left(\frac{\gamma}{\gamma_0}\right) f_\gamma(\gamma)\,d\gamma \tag{6}$$

where B (in hertz) is the channel bandwidth, and γ_0 is the optimal cutoff SNR. For $\gamma < \gamma_0$, no data are transmitted; hence, an outage probability occurs as $P_{\rm out} = \mathcal{F}_\gamma(\gamma_0)$. To obtain the optimal cutoff SNR, γ_0 must satisfy $p(\gamma_0) = 0$, where $p(\cdot)$ is defined as [2]

$$p(x) \triangleq \int_x^{\infty} \left(\frac{1}{x} - \frac{1}{\gamma}\right) f_\gamma(\gamma)\,d\gamma - 1. \tag{7}$$

To obtain $p(x)$ for SD reception, substituting (4) in (7), making a change of variables, and using [19, eq. (8.350/2)] and some cumbersome but straightforward mathematical manipulations yield

$$p_{\rm sd}(\gamma_0) = \frac{2(1-\rho)^m}{\Gamma(m)} \sum_{k=0}^{\infty} \frac{\rho^k}{k!}$$
$$\times \left\{\sum_{\ell=1}^{2} \frac{(-1)^{\ell+1}(\tau\overline{\gamma})^{1-\ell}}{\gamma_0^{2-\ell}(1-\rho)^{2(\ell-1)/\beta}}\right.$$
$$\times \Gamma\left[m_k - 2\frac{\ell-1}{\beta}, \frac{1}{1-\rho}\left(\frac{\gamma_0}{\tau\overline{\gamma}}\right)^{\beta/2}\right]$$
$$- \sum_{t=0}^{m_k-1} \sum_{\ell=1}^{2} \frac{(-1)^{\ell+1}(\tau\overline{\gamma})^{1-\ell}(1-\rho)^{2(1-\ell)/\beta}}{t!2^{m_{k+t}-2(\ell-1)/\beta}\gamma_0^{2-\ell}}$$
$$\left.\times \Gamma\left[m_{k+t} - 2\frac{\ell-1}{\beta}, \frac{2}{1-\rho}\left(\frac{\gamma_0}{\tau\overline{\gamma}}\right)^{\beta/2}\right]\right\} - 1. \tag{8}$$

Using (8) is very difficult—if not impossible—to obtain a closed-form expression for γ_0. Nevertheless, numerical evaluation techniques have confirmed that, by solving (8), there is a unique positive value for γ_0 satisfying $p_{\rm sd}(\gamma_0) = 0$ that takes values from $\gamma_0 \in [0, 1]$.

For the special case of independent GG RVs, substituting (5) into (7) and following a procedure similar to that used in deriving (8), the following closed-form expression for $p_{\rm sd}(\gamma_0)$ is obtained:

$$p_{\rm sd}(\gamma_0) = 2\sum_{\ell=1}^{2} \left\{\frac{(-1)^{\ell+1}\gamma_0^{\ell-2}}{\Gamma(m)(\tau\,\overline{\gamma})^{\ell-1}} \Gamma\left[m - 2\frac{\ell-1}{\beta}, \left(\frac{\gamma_0}{\tau\overline{\gamma}}\right)^{\beta/2}\right]\right.$$
$$\left. + \sum_{i=0}^{m-1} \frac{(-1)^\ell 2^{2(\ell-1)/\beta - m_i}}{i!\gamma_0^{2-\ell}\Gamma(m)\,(\tau\overline{\gamma})^{\ell-1}} \Gamma\left[m_i - 2\frac{\ell-1}{\beta}, 2\left(\frac{\gamma_0}{\tau\overline{\gamma}}\right)^{\beta/2}\right]\right\} - 1. \tag{9}$$

Furthermore, for the case of an SR, i.e., $L = 1$, by substituting (1) into (7), the following closed-form expression for $p_{\rm sr}(\gamma_0)$ can be obtained:

$$p_{\rm sr}(\gamma_0)=\sum_{\ell=1}^{2}\frac{(-1)^{\ell+1}\gamma_0^{\ell-2}}{\Gamma(m)(\tau\overline{\gamma})^{\ell-1}}\Gamma\left[m-2\frac{\ell-1}{\beta},\left(\frac{\gamma_0}{\tau\overline{\gamma}}\right)^{\beta/2}\right]-1. \quad (10)$$

Note that, for $\beta=2$, (10) simplifies to previously reported results, i.e., [4, eq. (7)]. Numerical evaluations of (9) and (10) have also confirmed the existence of a unique positive γ_0 satisfying $p_{\rm sd}(\gamma_0)=0$ and $p_{\rm sr}(\gamma_0)=0$, respectively.

To obtain the capacity with OPRA and SD reception, (4) is substituted in (6), which results in integrals of the following form:

$$\mathcal{I}_{\mathcal{A}}=\int_{c}^{\infty}\gamma^{a\,b-1}\log_2\left(\frac{\gamma}{c}\right)\exp(-d\gamma^b)\,d\gamma \quad (11)$$

where $a\in\mathbb{N}$, and $b,c,d\in\Re^+$, with $c,d\neq 0$. By making a change of variables of the form $x=\gamma^b$ and following a procedure similar to that in [3, App. A], the integral of (11) can be solved as follows:

$$\mathcal{I}_{\mathcal{A}}=\frac{c^{ab}}{b^2\ln 2}\sum_{k=1}^{a}\frac{(a-1)!}{(a-k)!}(dc^b)^{-a}\Gamma(a-k,dc^b). \quad (12)$$

Hence, using (12), the capacity under the OPRA policy and SD receivers can be derived as

$$\begin{aligned}C_{\rm opra}^{\rm sd}=&\frac{4B(1-\rho)^m}{\beta\ln(2)\Gamma(m)}\sum_{k=0}^{\infty}\frac{\rho^k}{k!}\\&\times\left\{\sum_{n=1}^{m_k}\frac{(m_k-1)!}{(m_k-n)!}\Gamma\left[m_k-n,\frac{1}{1-\rho}\left(\frac{\gamma_0}{\tau\overline{\gamma}}\right)^{\beta/2}\right]\right.\\&-\sum_{t=0}^{m_k-1}\frac{1}{t!\,2^{m_{k+t}}}\sum_{n=1}^{m_{k+t}}\frac{(m_{k+t}-1)!}{(m_{k+t}-n)!}\\&\left.\times\Gamma\left[m_{k+t}-n,\frac{2}{1-\rho}\left(\frac{\gamma_0}{\tau\overline{\gamma}}\right)^{\beta/2}\right]\right\}. \quad (13)\end{aligned}$$

For independent GG RVs, by substituting (5) into (6) again, integrals of the form presented in (11) appear. Hence, using the previously presented approach, after some mathematical manipulation, $C_{\rm opra}^{\rm sd}$ can be obtained in closed form as

$$\begin{aligned}C_{\rm opra}^{\rm sd}=&\frac{4B/\ln(2)}{\beta\,\Gamma(m)}\left\{\sum_{n=1}^{m}\frac{(m-1)!}{(m-n)!}\Gamma\left[m-n,\left(\frac{\gamma_0}{\tau\overline{\gamma}}\right)^{\beta/2}\right]\right.\\&-\sum_{t=0}^{m-1}\frac{1}{t!\,2^{m_t}}\sum_{n=1}^{m_t}\frac{(m_t-1)!}{(m_t-n)!}\\&\left.\times\Gamma\left[m_t-n,2\left(\frac{\gamma_0}{\tau\overline{\gamma}}\right)^{\beta/2}\right]\right\}. \quad (14)\end{aligned}$$

For $L=1$, the capacity under the OPRA policy $C_{\rm opra}^{\rm sr}$ simplifies to the following simple closed-form expression:

$$C_{\rm opra}^{\rm sr}=\frac{2B/\ln(2)}{\beta\Gamma(m)}\sum_{n=1}^{m}\frac{(m-1)!}{(m-n)!}\Gamma\left[m-n,\left(\frac{\gamma_0}{\tau\overline{\gamma}}\right)^{\beta/2}\right]. \quad (15)$$

B. ORA

The capacity under a constant transmitting power and ORA policy is defined as [2]

$$C_{\rm ora}\triangleq B\int_0^{\infty}\log_2(1+\gamma)f_\gamma(\gamma)\,d\gamma. \quad (16)$$

From (16), it is evident that the capacity under the ORA policy is identical to the capacity defined in [22, eq. (1)]; therefore, the transmitter CSI knowledge does not influence the capacity. Substituting (4) in (16), integrals of the following form appear:

$$\mathcal{I}_{\mathcal{B}}=\int_0^{\infty}\gamma^{ab-1}\ln(1+\gamma)\exp(-d\gamma^b)\,d\gamma. \quad (17)$$

By expressing $\exp(\cdot)$ and $\ln(\cdot)$ as in [23, eq. (01.03.26.0004.01)] and [23, eq. (01.04.26.0003.01)], respectively, and using [24], $\mathcal{I}_B$ can be solved in closed form. Using this solution and after some mathematical manipulations, the capacity of the SD receiver under the ORA policy $C_{\rm ora}^{\rm sd}$ can be expressed as

$$\begin{aligned}C_{\rm ora}^{\rm sd}=&\frac{\beta B}{\ln(2)\Gamma(m)}\sum_{k=0}^{\infty}\frac{\rho^k}{k!(1-\rho)^k(\tau\overline{\gamma})^{\beta m_k/2}}\\&\times\left\{\mathcal{D}\left[\frac{1}{(1-\rho)(\tau\overline{\gamma})^{\beta/2}},\frac{\beta m_k}{2}\right]\right.\\&\left.-\sum_{t=0}^{m_k-1}\frac{(1-\rho)^{-t}}{t!(\tau\overline{\gamma})^{\beta t/2}}\mathcal{D}\left[\frac{2}{(1-\rho)(\tau\overline{\gamma})^{\beta/2}},\frac{\beta m_{k+t}}{2}\right]\right\}\end{aligned} \quad (18)$$

with

$$\begin{aligned}\mathcal{D}(x,y)=&\frac{\sqrt{n}}{\lambda(2\pi)^{\lambda+(n-3)/2}}\\&\times G_{2\lambda,n+2\lambda}^{n+2\lambda,\lambda}\left[\left(\frac{x}{n}\right)^n\middle|\begin{matrix}\Delta(\lambda,-y), & \Delta(\lambda,1-y)\\ \Delta(n,0),\Delta(\lambda,-y), & \Delta(\lambda,-y)\end{matrix}\right]\end{aligned}$$

where $G(\cdot)$ is Meijer's G-function [19, eq. (9.301)], $\Delta(x,y)=y/x,(y+1)/x,\ldots,(y+x-1)/x$, and λ and n are positive integers, with $\lambda/n=\beta/2$ [8]. It is noted that the Meijer's G-functions appearing in (18) converge fast. For independent GG RVs, by substituting (5) into (16) and following a procedure similar to that used in deriving (18), $C_{\rm ora}^{\rm sd}$ is obtained in closed form as

$$\begin{aligned}C_{\rm ora}^{\rm sd}=&\frac{\beta B/\ln(2)}{\Gamma(m)(\tau\overline{\gamma})^{\beta m/2}}\left\{\mathcal{D}\left[\frac{1}{(\tau\overline{\gamma})^{\beta/2}},\frac{\beta m}{2}\right]\right.\\&\left.-\sum_{i=0}^{m-1}\frac{1}{i!(\tau\overline{\gamma})^{\beta i/2}}\mathcal{D}\left[\frac{2}{(\tau\overline{\gamma})^{\beta/2}},\frac{\beta m_i}{2}\right]\right\}. \quad (19)\end{aligned}$$

For $m=1$, the preceding equation simplifies to the previous derived results, i.e., the capacity with the ORA policy of SD receivers operating over Weibull fading channels [6, eq. (11) for $\delta=1$]. The capacity under the ORA policy of SR operating over GG fading channels was presented in [16, eq. (9)].

C. CIFR

The CIFR policy requires that the transmitter exploits the CSI so that a constant SNR is maintained at the receiver [2]. The capacity employing the CIFR technique $C_{\rm cifr}$ is defined as

$$C_{\rm cifr}\triangleq B\log_2\left(1+\frac{1}{\int_0^{\infty}f_\gamma(\gamma)/\gamma d\gamma}\right). \quad (20)$$

Substituting (4) into (20) and using [19, eq. (3.351/3)] and the definition of the generalized hypergeometric function [19, eq. (9.14/1)], the following expression for the capacity of an SD receiver $C_{\rm cifr}^{\rm sd}$ can be obtained:

$$C_{\rm cifr}^{\rm sd} = B\log_2\left\{1+\frac{\Gamma(m)\tau\overline{\gamma}}{2(1-\rho)^{m-2/\beta}}\left[{}_1F_0\left(m-\frac{2}{\beta};\,;\rho\right)\right.\right.$$
$$\left.\left.\times\,\Gamma\left(m-\frac{2}{\beta}\right)-\sum_{k=0}^{\infty}\frac{\rho^k}{k!}\sum_{t=0}^{m_k-1}\frac{\mathcal{P}(m_{k+t},1)}{t!}\right]^{-1}\right\} \quad (21)$$

where

$$\mathcal{P}(x,y)=\frac{\Gamma(x-2/\beta)}{2^{(x-2/\beta)y}}.$$

Assuming independent GG RVs, substituting (5) in (20), and using again [19, eq. (3.351/3)], $C_{\rm cifr}^{\rm sd}$ can be obtained in closed form as

$$C_{\rm cifr}^{\rm sd} = B\log_2\left\{1+\frac{\Gamma(m)\tau\overline{\gamma}/2}{\left[\mathcal{P}(m,0)-\sum\limits_{i=0}^{m-1}\frac{\mathcal{P}(m_i,1)}{i!}\right]}\right\}. \quad (22)$$

For $m=1$, (22) simplifies to a previously known expression, i.e., the capacity with the CIFR policy of SD receivers operating over Weibull fading channels [6, eq. (22) for $\delta=1$]. Furthermore, the capacity of an SR under the CIFR policy $C_{\rm cifr}^{\rm sr}$ can also be obtained in closed form as

$$C_{\rm cifr}^{\rm sr} = B\log_2\left[1+\frac{\Gamma(m)\tau\overline{\gamma}}{\mathcal{P}(m,0)}\right]. \quad (23)$$

For $\beta=2$, (23) further simplifies to another previously known expression, i.e., the capacity with the CIFR policy and Nakagami fading [4, eq. (29)], whereas, for $m=1$, it simplifies to the corresponding capacity for the Weibull fading channel [6, eq. (23)].

D. TIFR

Although the CIFR technique is very simple to implement, it exhibits an increased capacity penalty in severe fading environments, e.g., assuming Rayleigh fading $C_{\rm cifr}=0$ [2]. An alternative approach is to consider a TIFR policy, which is usually referred to as TIFR [1], where the channel fading is inverted only above the fixed cutoff γ_0. Mathematically, this capacity is defined as

$$C_{\rm tifr} \triangleq B\log_2\left[1+\frac{1}{\int_{\gamma_0}^{\infty} f_\gamma(\gamma)/\gamma d\gamma}\right](1-P_{\rm out}). \quad (24)$$

Substituting (4) in (24), making a change of variables, and using [19, eq. (8.350/2)], the capacity of an SD receiver under the TIFR policy $C_{\rm tifr}^{\rm sd}$ can be obtained as

$$C_{\rm tifr}^{\rm sd}=B\log_2\left\{1+\left[2\sum_{k=0}^{\infty}\frac{\rho^k/k!}{(1-\rho)^k}\left(\mathcal{H}\left(m_k,\frac{1}{1-\rho}\right)\right.\right.\right.$$
$$\left.\left.\left.-\sum_{t=0}^{m_k-1}\frac{1}{t!(1-\rho)^t}\mathcal{H}\left(m_{k+t},\frac{2}{1-r}\right)\right)\right]^{-1}\right\}(1-P_{\rm out}) \quad (25)$$

where

$$\mathcal{H}(x,y)=\frac{\Gamma\left[x-2/\beta,y\left(\gamma_0/(\tau\overline{\gamma})\right)^{\beta/2}\right]}{\Gamma(m)\tau\,\overline{\gamma}y^{(x-2/\beta)}}$$

and $P_{\rm out}$ is given by setting $\gamma=\gamma_0$ in (3). For independent GG RVs, by substituting (5) in (24), following a procedure similar to that in deriving (25), and using the corresponding expression for the cdf of $\gamma_{\rm sd}$, $C_{\rm tifr}^{\rm sd}$ can be obtained in closed form as

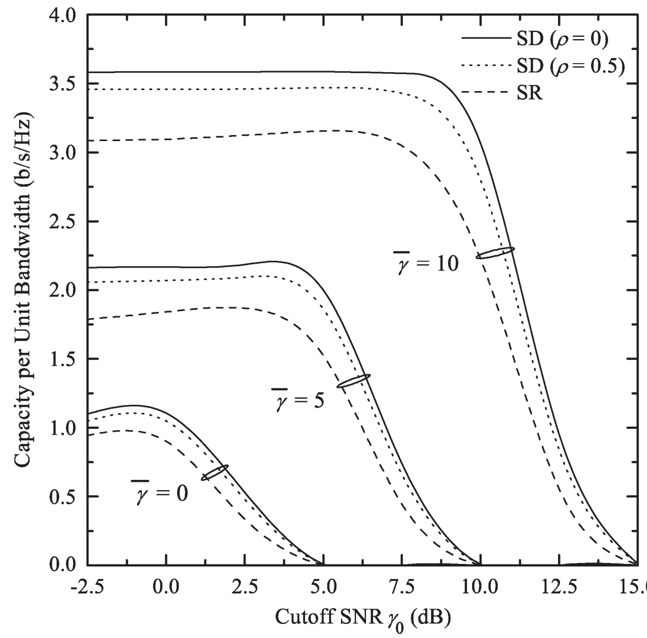

Fig. 1. Capacity per unit bandwidth under the TIFR policy versus the cutoff SNR for several values of $\overline{\gamma}$ with SR and SD reception.

$$C_{\rm tifr}^{\rm sd} = B\log_2\left[1+\frac{1}{2}\left[\mathcal{H}(m,1)-\sum_{i=0}^{m-1}\frac{\mathcal{H}(m_i,2)}{i!}\right]^{-1}\right.$$
$$\left.\times\left[1-\left(1-\frac{\Gamma\left[m,\left(\gamma_0/(\tau\overline{\gamma})\right)^{\beta/2}\right]}{\Gamma(m)}\right)^2\right]\right]. \quad (26)$$

Note that by setting $m=1$ in (26), it simplifies to [6, eq. (25)] for $\delta=1$.

Furthermore, for SR, the capacity under the TIFR policy $C_{\rm tifr}^{\rm sr}$ simplifies to the following closed-form expression:

$$C_{\rm tifr}^{\rm sr}=\frac{B}{\Gamma(m)}\log_2\left[1+\mathcal{H}(m,1)^{-1}\right]\Gamma\left[m,\left(\frac{\gamma_0}{\tau\overline{\gamma}}\right)^{\beta/2}\right]. \quad (27)$$

For $\beta=2$, the preceding expression further simplifies to $C_{\rm tifr}^{\rm sr}$ with Nakagami fading [4, eq. (30)], whereas, by setting $m=1$, it simplifies to a previously derived equivalent expression for Weibull fading channels [6, eq. (26)].

IV. Performance Evaluation Results and Discussion

In this section, various performance evaluation results for the capacity obtained using the previous analysis for dual-branch SD and SR operating over correlated or independent GG fading channels will be presented and analyzed. These results also focus on capacity performance comparisons between the different transmission adaptation policies under various GG fading channel conditions.

In Fig. 1, the capacity per unit bandwidth[2] under the TIFR policy $\widehat{C}_{\rm tifr}$ (in bits per second per hertz) is plotted as a function of the cutoff SNR γ_0 for several values of the average input SNR $\overline{\gamma}$ and for $m=2$ and $\beta=3$. In the same figure, $\widehat{C}_{\rm tifr}$ is also plotted for the SR receiver, using (27), and the SD receiver with $\rho=0$ and $\rho=0.5$ using (26) and (25), respectively. As expected, by increasing $\overline{\gamma}$ and employing

[2]For conciseness of presentation, $\widehat{C}$ will denote the capacity per unit bandwidth, i.e., C/B.

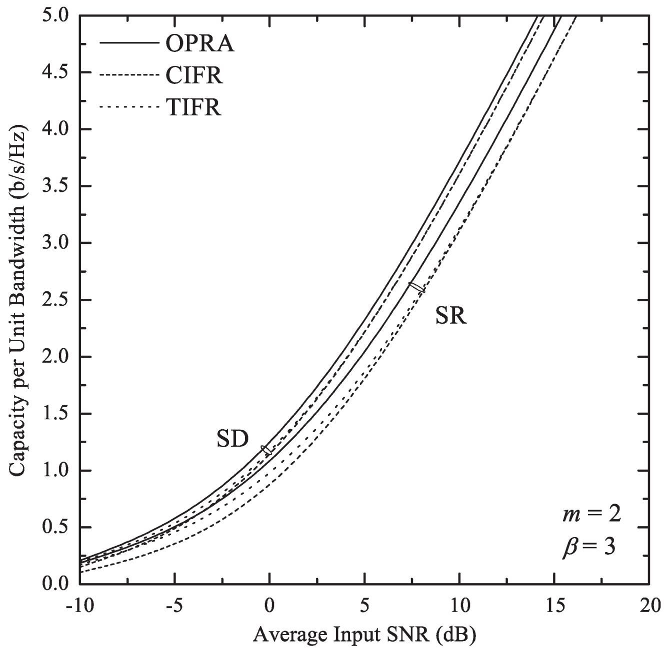

Fig. 2. Capacity per unit bandwidth under different adaptation policies versus $\overline{\gamma}$ with SR and SD reception.

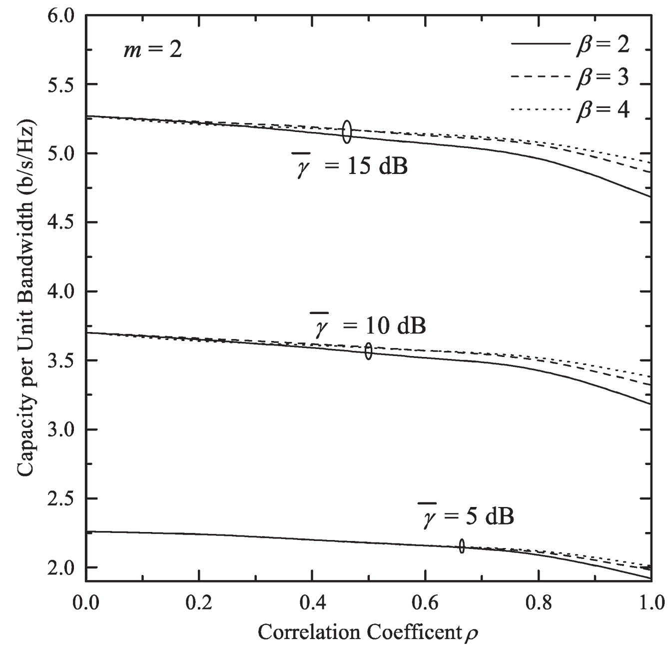

Fig. 4. Capacity per unit bandwidth under the OPRA policy versus ρ for several values of β and $\overline{\gamma}$.

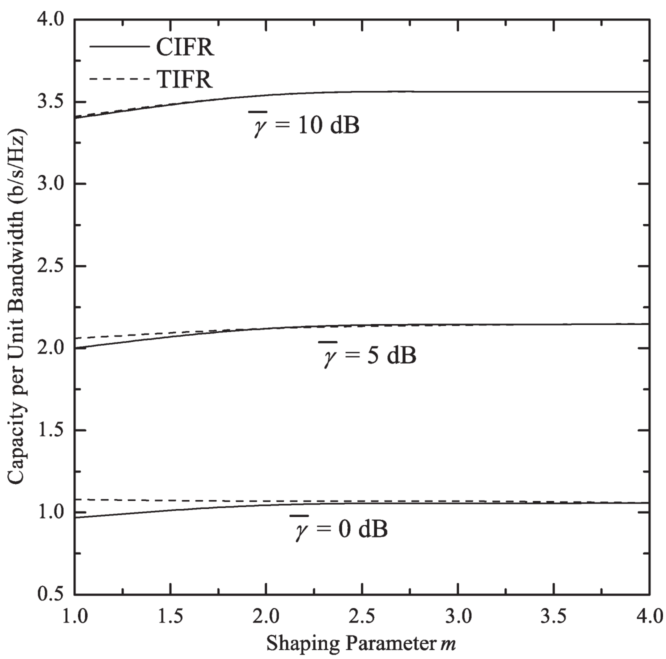

Fig. 3. Capacity per unit bandwidth under the TIFR and CIFR policies versus m for different values of $\overline{\gamma}$ and $\rho = 0.2$.

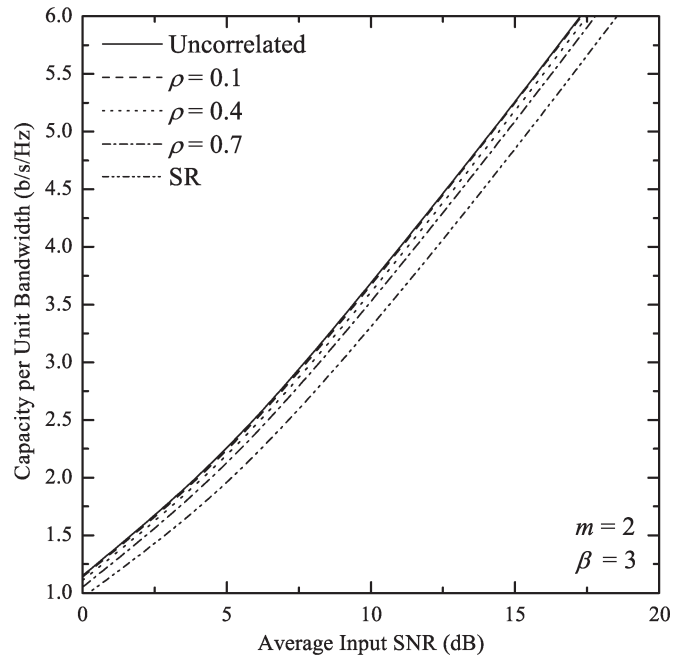

Fig. 5. Capacity per unit bandwidth under the ORA policy versus $\overline{\gamma}$ for several values of ρ.

diversity, $\widehat{C}_{\rm tifr}$ improves, even when correlation exists, whereas $\widehat{C}_{\rm tifr}$ is maximized for specific values of γ_0. In Fig. 2, the $\widehat{C}$ of independent GG fading channels with SD reception is plotted as a function of $\overline{\gamma}$, considering OPRA, CIFR, and TIFR adaptation policies, with the aid of (14), (22), and (26), respectively, and for $m = 2$ and $\beta = 3$. For comparison purposes, plots for the corresponding adaptation policies of SR are also included for OPRA [using (15)], CIFR [using (23)], and TIFR [using (27)] adaptation policies. It is depicted that, for the same fading conditions, OPRA achieves the highest capacity, whereas CIFR achieves the lowest capacity, although these differences are not significant. Note also that the CIFR and TIFR capacities for the SD receiver become almost identical for smaller values of the average input SNR, i.e., $\overline{\gamma} = 0$, compared with the SR.

In Fig. 3, the $\widehat{C}$ for correlated $(\rho = 0.2)$ SD reception is plotted as a function of the shaping parameter m for the CIFR and TIFR policies by employing (21) and (25), respectively, and for $\beta = 3$ and several values of $\overline{\gamma}$. As expected, as the channel fading conditions improve, i.e., m and/or $\overline{\gamma}$ increase, both capacities increase, with $\overline{\gamma}$ having a significant impact on the performance. It is interesting to note that, as m and/or $\overline{\gamma}$ increase, the differences between the TIFR and CIFR adaptation policies diminish since $P_{\rm out}$ in (24) also decreases. In Fig. 4, $\widehat{C}_{\rm opra}^{\rm sd}$ is plotted, using (13), as a function of the correlation coefficient for several values of $\overline{\gamma}$ and/or β with $m = 2$. Similar to the other policies, $\widehat{C}_{\rm opra}^{\rm sd}$ increases as ρ decreases, and/or $\overline{\gamma}$ and β increase. For different values of β, the differences among the curves become more distinct for higher values of ρ and $\overline{\gamma}$.

TABLE I
MINIMUM NUMBER OF TERMS N_τ IN (21) REQUIRED FOR OBTAINING ACCURACY BETTER THAN $\pm 1‰$

$\overline{\gamma}$ (dB)	$\rho = 0.2$				$\rho = 0.7$			
	$m = 1$		$m = 4$		$m = 1$		$m = 4$	
	$\beta = 2$	$\beta = 5$	$\beta = 2$	$\beta = 5$	$\beta = 2$	$\beta = 5$	$\beta = 2$	$\beta = 5$
0	3	3	6	6	12	16	30	32
5	3	3	6	6	12	15	29	31
10	3	3	5	5	11	14	28	30
15	2	3	5	5	10	13	27	29
20	2	3	5	5	10	13	26	28

In Fig. 5, the $\widehat{C}_{\rm ora}$ of correlated [using (18)] and independent [using (19)] GG fading channels with SD reception is plotted as a function of $\overline{\gamma}$. For comparison purposes, the corresponding values of SR are also included using [16, eq. (9)]. It is interesting to note that, even for lower diversity gain, i.e., higher values of the correlation coefficient ($\rho = 0.7$), the $\widehat{C}_{\rm ora}$ improves when diversity is employed. For all cases considered, the rate of convergence of the infinite series expressions has also been investigated. As a typical example, Table I presents the minimum number of terms N_τ required in (21) to obtain an accuracy that is better than $\pm 1‰$. Clearly, by decreasing $\overline{\gamma}$ and/or increasing m, β, or ρ, N_τ increases. The strong influence of ρ on N_τ is also noted. It is also clear that only a relatively small number of terms are necessary to achieve such an excellent accuracy. Similar rates of convergence have been found for the other three infinite series expressions used to obtain the various performance evaluation results presented in this section, i.e., (13), (18), and (25).

V. CONCLUSION

Novel infinite series expressions with fast converging properties have been derived to evaluate the capacity of correlated GG fading channels with dual-branch SD receivers. More specifically, for this composite fading environment, the capacity was studied for OPRA, ORA, CIFR, and TIFR transmission adaptation policies. For independent GG fading conditions, convenient closed-form expressions have also been derived for the capacity of SD and SR. It has been shown that previously published analytical results are special cases of these novel expressions. For the considered adaptation policies, selected numerical evaluated results have been presented for different correlated GG fading conditions and several receiver structures.

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