A Square-Root Adaptive V-BLAST Algorithm for Fast Time-Varying MIMO Channels

Athanasios A. Rontogiannis, *Member, IEEE*, Vassilis Kekatos, *Student Member, IEEE*, and Kostas Berberidis, *Member, IEEE*

***Abstract*—Among the methods that have been proposed for a multiple-input multiple-output (MIMO) receiver, the V-BLAST algorithm provides a good compromise between transmission rate, achievable diversity, and decoding complexity. In this letter, we derive a new adaptive V-BLAST-type equalization scheme for fast time-varying, flat-fading MIMO channels. The proposed equalizer stems from the Cholesky factorization of the MIMO system's output data autocorrelation matrix, and the equalizer filters are updated in time using numerically robust unitary Givens rotations. The new square-root algorithm exhibits identical performance to a recently proposed V-BLAST adaptive algorithm, offering at the same time noticeable reduction in computational complexity. Moreover, as expected due to its square-root form and verified by simulations, the algorithm exhibits particularly favorable numerical behavior.**

***Index Terms*—Adaptive equalization, multiple-input multiple-output (MIMO), time-varying channels, V-BLAST.**

I. Introduction

MULTIPLE-INPUT multiple-output (MIMO) wireless communications technology has gained considerable attention in recent years due to its potential to significantly increase throughput compared to traditional single-input single-output (SISO) technology. A widely used architecture for reliable symbol detection in flat-fading MIMO systems is the V-BLAST architecture [1]. In a V-BLAST receiver, the symbol corresponding to the stream with the highest signal-to-noise ratio (SNR) is detected first. Then the contribution of the detected symbol is subtracted from the output vector. This procedure is repeated for the remaining symbols by always selecting the strongest signal among the undetected ones. Assuming a known MIMO channel, computationally efficient implementations of the V-BLAST technique have been proposed in [2]–[4]. In a fast time-varying environment, however, the MIMO channel must be estimated quite frequently before the V-BLAST method is applied, resulting in an undesirable increase of the computational complexity.

By taking advantage of the equivalence between the V-BLAST receiver and the generalized decision feedback equalizer (DFE) [5], an architecture appropriate for *adaptive* equalization of flat-fading MIMO channels has been presented in [6] (and recently in [7]). To the best of our knowledge, the algorithm proposed in [6] is the only V-BLAST-type scheme in which not only the equalizer taps but also the symbol detection ordering are updated recursively in time. As a result, a significant reduction in computational complexity is achieved compared to previously derived V-BLAST schemes.

Motivated by the work in [6], we have developed a new adaptive V-BLAST algorithm for flat, time-varying MIMO channels. In the proposed algorithm, a suitable transformation of the equalizer filters used in [6] is employed, which stems from the Cholesky factorization of the equalizer's input autocorrelation matrix. It turns out that the proposed algorithm offers considerable computational savings compared to the method described in [6] but with the same performance. Moreover, its numerical behavior is expected to be quite robust, since it is based on a square root of the system's autocorrelation matrix.

II. Problem Formulation and Preliminaries

Let us consider a MIMO flat-fading system with M transmit and N receive antennas and $M \leq N$. The $N \times 1$ received vector $\mathbf{y}(n)$ at time n is expressed as

$$\mathbf{y}(n) = \mathbf{H}(n)\mathbf{d}(n) + \mathbf{v}(n) \quad (1)$$

where $\mathbf{H}(n)$ is the $N \times M$ channel matrix whose entries are the complex attenuation factors of the corresponding NM subchannels, $\mathbf{d}(n)$ contains the M independent symbols transmitted by the M antennas at time n, and $\mathbf{v}(n)$ denotes white Gaussian noise. Adaptive V-BLAST-type equalization of the system described in (1) can be performed using the architecture shown in Fig. 1 [6]. In Fig. 1, it is assumed that the symbol detection ordering is known *a priori* and is denoted as $\{k_1, k_2, \ldots, k_M\}$, where $k_i \in \{1, 2, \ldots, M\}$. The equalizer is comprised of M serially connected conventional DFE structures. As far as the ith DFE is concerned, the feedforward filter input coincides with the received vector $\mathbf{y}(n)$, while the feedback filter input is the vector of already detected symbols from the previous DFEs. Thus, the output of the ith DFE is given as follows:

$$\tilde{d}_{k_i}(n) = \mathbf{w}_i^H(n)\mathbf{y}_i(n) \quad (2)$$

where

$$\mathbf{w}_i(n) = \begin{cases} \mathbf{w}_{f,i}(n), & i = 1 \\ \left[\mathbf{w}_{f,i}^T(n) \quad \mathbf{w}_{b,i}^T(n)\right]^T, & i = 2, \ldots, M \end{cases} \quad (3)$$

$$\mathbf{y}_i(n) = \begin{cases} \mathbf{y}(n), & i = 1 \\ \left[\mathbf{y}^T(n) \quad \hat{\mathbf{d}}_{k_{i-1}}^T(n)\right]^T, & i = 2, \ldots, M. \end{cases} \quad (4)$$

Manuscript received October 25, 2005; revised December 6, 2005. This work was supported in part by C.T.I.-R&D. V. Kekatos is a scholar of the Bodossaki Foundation. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Brian Sadler.

A. A. Rontogiannis is with the Institute of Space Applications and Remote Sensing, National Observatory of Athens, 15236 Athens, Greece. (e-mail: tronto@space.noa.gr).

V. Kekatos and K. Berberidis are with the Department of Computer Engineering and Informatics/C.T.I.-R&D, University of Patras, 26500 Rio, Greece. (e-mail: kekatos@ceid.upatras.gr; berberid@ceid.upatras.gr).

Digital Object Identifier 10.1109/LSP.2006.870373

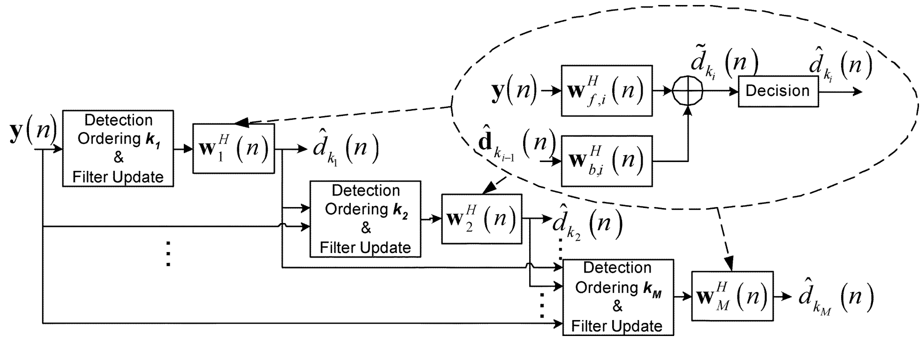

Fig. 1. Adaptive V-BLAST-type equalization architecture.

In (4), $\hat{\mathbf{d}}_{k_{i-1}}(n) = [\hat{d}_{k_1}(n), \hat{d}_{k_2}(n), \ldots, \hat{d}_{k_{i-1}}(n)]^T$ is the vector of already detected symbols, i.e., $\hat{d}_{k_i}(n) = f\{\tilde{d}_{k_i}(n)\}$, where $f\{\cdot\}$ is the decision device function, and $(\cdot)^T$ stands for simple transposition while $(\cdot)^H$ stands for complex conjugate transposition.

Under fast fading conditions, not only the equalizer filters but also the detection order need to be adapted at each time instant. An efficient recursive least squares (RLS) approach, which takes into consideration both requirements, has been introduced in [6]. More specifically, let us assume that $i-1$ symbols have been detected, i.e., symbols $\hat{d}_{k_1}(n), \hat{d}_{k_2}(n), \ldots, \hat{d}_{k_{i-1}}(n)$. In order to determine index k_i of the next symbol to be detected, as well as to update the corresponding DFE filters, a double minimization procedure is employed. First, each of the ith-order[1] LS cost functions

$$\mathcal{E}_{i,j}(n) = \sum_{l=1}^{n} \lambda^{n-l} |\hat{d}_j(l) - \mathbf{w}_{i,j}^H(n)\mathbf{y}_i(l)|^2 \tag{5}$$

is minimized with respect to $\mathbf{w}_{i,j}(n)$ for $j \in S_i = \{1, 2, \ldots, M\} \backslash \{k_1, k_2, \ldots, k_{i-1}\}$, where λ is the usual forgetting factor. Among the optimum filters $\mathbf{w}_{i,j}(n)$, the one that gives the lowest LS energy (i.e., $\mathcal{E}_i(n)$) is selected as the ith DFE at time n, denoted as $\mathbf{w}_i(n)$. The corresponding index j determines the order of the next symbol to be detected, i.e., k_i. It can easily be shown that the order selection criterion described above is equivalent to the criterion used in V-BLAST with the expectation operation replaced by exponentially weighted time averaging. By employing an alternative but equivalent parameterization of the equalizer filters, we describe in the next section a numerically robust, computationally efficient method for both detection order and DFE filters adaptation.

III. Derivation of the New Algorithm

It is well known that minimization of $\mathcal{E}_{i,j}(n)$ in (5) with respect to $\mathbf{w}_{i,j}(n)$ leads to the following solution:

$$\mathbf{w}_{i,j}(n) = \mathbf{\Phi}_i^{-1}(n)\mathbf{z}_{i,j}(n) \tag{6}$$

[1]The ith-order problem is defined as the one that corresponds to the $(N + i - 1) \times 1$ input data vector $\mathbf{y}_i(n)$.

where $\mathbf{\Phi}_i(n)$ stands for the $(N + i - 1) \times (N + i - 1)$ time-averaged input data autocorrelation matrix, and $\mathbf{z}_{i,j}(n)$ is the $(N + i - 1) \times 1$ time-averaged input-output cross-correlation vector given by [8]

$$\mathbf{z}_{i,j}(n) = \sum_{l=1}^{n} \lambda^{n-l} \mathbf{y}_i(l)\hat{d}_j^*(l) \tag{7}$$

where $(\cdot)^*$ stands for complex conjugation. Let $\mathbf{R}_i(n)$ denote the upper triangular Cholesky factor of $\mathbf{\Phi}_i(n)$, i.e., $\mathbf{\Phi}_i(n) = \mathbf{R}_i^H(n)\mathbf{R}_i(n)$. Then the LS solution given in (6) can alternatively be expressed as

$$\mathbf{w}_{i,j}(n) = \mathbf{R}_i^{-1}(n)\mathbf{p}_{i,j}(n) \tag{8}$$

and $\mathbf{p}_{i,j}(n)$ is defined as

$$\mathbf{p}_{i,j}(n) = \mathbf{R}_i^{-H}(n)\mathbf{z}_{i,j}(n). \tag{9}$$

By substituting (6) in (5), we obtain the following expression of the minimum LS error energy for symbol stream j and order i:

$$\mathcal{E}_{i,j}(n) = \alpha_j(n) - \mathbf{w}_{i,j}^H(n)\mathbf{z}_{i,j}(n) \tag{10}$$

where $\alpha_j(n) = \sum_{l=1}^{n} \lambda^{n-l} |\hat{d}_j(l)|^2$. Moreover, from (8) and (9), (10) is rewritten as follows:

$$\mathcal{E}_{i,j}(n) = \alpha_j(n) - |\mathbf{p}_{i,j}(n)|^2. \tag{11}$$

Let us now define the matrix

$$\mathbf{Q}(n) = \sum_{l=1}^{n} \lambda^{n-l} \hat{\mathbf{d}}(l)\hat{\mathbf{d}}^H(l) = \lambda\mathbf{Q}(n-1) + \hat{\mathbf{d}}(n)\hat{\mathbf{d}}^H(n) \tag{12}$$

where $\hat{\mathbf{d}}(n) = [\hat{d}_1(n), \hat{d}_2(n), \ldots, \hat{d}_M(n)]^T$. It is straightforward to show that

$$\mathcal{E}_{i,j}(n) = q_{j,j}(n) - |\mathbf{p}_{i,j}(n)|^2 \tag{13}$$

where $q_{j,j}(n)$ stands for the (j, j)th entry of $\mathbf{Q}(n)$. Furthermore, from (7) and (12) it is easily verified that

$$\mathbf{z}_{i,j}(n) = \begin{bmatrix} \mathbf{z}_{i-1,j}^T(n) & q_{k_{i-1},j}(n) \end{bmatrix}^T. \tag{14}$$

In order to compute the minimum LS error energies from (13), vector $\mathbf{p}_{i,j}(n)$ must be obtained first. In the following, we show that $\mathbf{p}_{i,j}(n)$ can be order updated very efficiently, resulting in the significant computational savings of the proposed scheme.

A. Order Update of $\mathbf{p}_{i,j}(n)$

Let $\mathbf{p}_{i-1}(n)$ be the optimum vector of order $i-1$, which is related to $\mathbf{w}_{i-1}(n)$ via an expression similar to (8). Then it can be shown that the upper triangular factor $\mathbf{R}_i(n)$ is given by the following order update expression [9]:

$$\mathbf{R}_i(n) = \begin{bmatrix} \mathbf{R}_{i-1}(n) & \mathbf{p}_{i-1}(n) \\ \mathbf{0}^T & \sqrt{\mathcal{E}_{i-1}(n)} \end{bmatrix} \tag{15}$$

where $\mathcal{E}_{i-1}(n)$ is the minimum LS error energy of order $i-1$. The last expression is the result of the particular characteristics of the LS problems defined in the previous section. More specifically, it is easily verified from (4) and (5) that the first $N+i-2$ elements of the input data vector of the ith-order problem are identical to the input data vector of the $(i-1)$th-order problem, while its last element coincides with $\hat{d}_{k_{i-1}}(n)$. Since $\mathbf{R}_i(n)$ is the upper triangular factor in a QR decomposition of the ith-order input data matrix, the expression given in (15) is easily derived. From (9), (14), and (15), we get

$$\mathbf{p}_{i,j}(n) = \begin{bmatrix} \mathbf{R}_{i-1}^{-H}(n) & \mathbf{0} \\ -\frac{1}{\sqrt{\mathcal{E}_{i-1}(n)}}\mathbf{p}_{i-1}^H(n)\mathbf{R}_{i-1}^{-H}(n) & \frac{1}{\sqrt{\mathcal{E}_{i-1}(n)}} \end{bmatrix} \begin{bmatrix} \mathbf{z}_{i-1,j}(n) \\ q_{k_{i-1},j}(n) \end{bmatrix}$$

or

$$\mathbf{p}_{i,j}(n) = \begin{bmatrix} \mathbf{p}_{i-1,j}(n) \\ \frac{q_{k_{i-1},j}(n) - \mathbf{p}_{i-1}^H(n)\mathbf{p}_{i-1,j}(n)}{\sqrt{\mathcal{E}_{i-1}(n)}} \end{bmatrix}. \tag{16}$$

Having computed matrix $\mathbf{Q}(n)$ from (12) and $\mathbf{p}_{i,j}(n)$ from (16) for all $j \in S_i$, the LS error energies $\mathcal{E}_{i,j}(n)$, $j \in S_i$, given in (13), can be very efficiently obtained from

$$\mathcal{E}_{i,j}(n) = \mathcal{E}_{i-1,j}(n) - |[\mathbf{p}_{i,j}(n)]_{N+i-1}|^2 \tag{17}$$

where $[\mathbf{p}_{i,j}(n)]_{N+i-1}$ is the last element of $\mathbf{p}_{i,j}(n)$. The minimum of these energies is denoted as $\mathcal{E}_i(n)$ and the corresponding vector as $\mathbf{p}_i(n)$.

B. Computation of $\hat{d}_{k_i}(n)$

So far in our analysis, we have assumed that the detected symbols at time n are available. However, we see from (2) that this requires knowledge of the optimum filter at time n. To overcome this problem, we assume, as in [6], that the detected symbols at time n are computed using the optimum vector and detection order at time $n-1$, i.e.,

$$\bar{d}_{k_i}(n) = \mathbf{w}_i^H(n-1)\mathbf{y}_i(n), \quad \hat{d}_{k_i}(n) = f\{\bar{d}_{k_i}(n)\} \tag{18}$$

for $i = 1, \dots, M$, where k_i refers to the detection order at time $n-1$. Using the proposed alternative parameterization and (2) and (8), the detected symbol is expressed as

$$\hat{d}_{k_i}(n) = f\{\bar{d}_{k_i}(n)\} = f\{\mathbf{p}_i^H(n-1)\mathbf{g}_i(n)\} \tag{19}$$

where $\mathbf{g}_i(n)$ is defined as follows:

$$\mathbf{g}_i(n) = \mathbf{R}_i^{-H}(n-1)\mathbf{y}_i(n). \tag{20}$$

Substituting the inverse Cholesky factor in the last equation, as in (16), and using the relation $\mathbf{y}_i(n) = [\mathbf{y}_{i-1}^T(n), \hat{d}_{k_{i-1}}(n)]^T$, it is easily shown that

$$\mathbf{g}_i(n) = \begin{bmatrix} \mathbf{g}_{i-1}(n) \\ \frac{\hat{d}_{k_{i-1}}(n) - \mathbf{p}_{i-1}^H(n-1)\mathbf{g}_{i-1}(n)}{\sqrt{\mathcal{E}_{i-1}(n-1)}} \end{bmatrix} \tag{21}$$

or

$$\mathbf{g}_i(n) = \begin{bmatrix} \mathbf{g}_{i-1}(n) \\ \frac{\hat{d}_{k_{i-1}}(n) - \bar{d}_{k_{i-1}}(n)}{\sqrt{\mathcal{E}_{i-1}(n-1)}} \end{bmatrix}. \tag{22}$$

Thus, $\mathbf{g}_i(n)$ can be very efficiently order updated, provided that $\mathbf{g}_1(n)$ is available.

C. Initial Time-Update Recursions

To complete the proposed algorithm, the involved quantities of first order vectors, namely, $\mathbf{p}_{1,j}(n)$ for $j = 1, 2, \dots, M$, must be computed. This is accomplished through the time-update recursions described below. Let us assume that $\mathbf{R}_1^{-1}(n-1)$ has been calculated in the previous time instant. Then

$$\mathbf{g}_1(n) = \mathbf{R}_1^{-H}(n-1)\mathbf{y}(n). \tag{23}$$

Next we produce a sequence of N elementary complex Givens rotation matrices [8], whose product is denoted by $\mathbf{T}(n)$, according to the expression

$$\mathbf{T}(n)\begin{bmatrix} -\frac{\mathbf{g}_1(n)}{\sqrt{\lambda}} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \star \end{bmatrix} \tag{24}$$

where $\star$ denotes a "do not care" element. The kth elementary matrix $k = 1, 2, \dots, N$ annihilates the kth element of $-\mathbf{g}_1(n)/\sqrt{\lambda}$ with respect to the last element of the whole vector, which initially equals 1. It can be shown [9] that the same rotation matrices can be used for time updating the inverse Cholesky factor as[2]

$$\mathbf{T}(n)\begin{bmatrix} \lambda^{-1/2}\mathbf{R}_1^{-H}(n-1) \\ \mathbf{0}^T \end{bmatrix} = \begin{bmatrix} \mathbf{R}_1^{-H}(n) \\ \star \end{bmatrix}. \tag{25}$$

Moreover, and most importantly, matrix $\mathbf{T}(n)$ also updates in time $\mathbf{p}_{1,j}(n)$ for $j = 1, 2, \dots, M$, i.e., [9]

$$\mathbf{T}(n)\begin{bmatrix} \lambda^{1/2}\mathbf{p}_{1,j}(n-1) \\ \hat{d}_j^*(n) \end{bmatrix} = \begin{bmatrix} \mathbf{p}_{1,j}(n) \\ \star \end{bmatrix}. \tag{26}$$

Note that it is not necessary to compute matrix $\mathbf{T}(n)$ explicitly. Only the rotation parameters are calculated from (24) and are then used to update the respective quantities in (25) and (26).

D. Algorithm

The basic steps of the proposed equalization algorithm are summarized in Table I. In the initial training phase, step 2 of the algorithm is not executed, and the respective decisions are replaced by a known training sequence. After convergence, the algorithm switches to the decision-directed mode, in which the decisions are computed as described in step 2. The new algorithm is mathematically equivalent to the algorithm proposed in [6], but it is expected to have enhanced numerical robustness, since it stems from the Cholesky factorization of the input data correlation matrix.

Furthermore, the algorithm offers significant computational savings compared to the algorithm described in [6], which, to the best of our knowledge, is the fastest V-BLAST-type equalization algorithm. The required operations for each step of the new algorithm are provided in brackets in Table I. The overall computational complexity of the two equalization schemes in terms of the number of complex multiplications and additions

[2]Even though the analysis in [9] is done for real signals, it can be extended to complex signals in a rather straightforward manner.

TABLE I
SQUARE-ROOT ADAPTIVE V-BLAST ALGORITHM

1) Compute $\mathbf{g}_1(n)$, $\mathbf{T}(n)$ and $\mathbf{R}_1^{-1}(n)$ from (23), (24), and (25), respectively. ($3M^2,\ 3M^2/2$)
2) Compute decisions $\hat{d}_{k_i}(n)$, for $i=1,2,\ldots,M$ from (19) and (22). ($3M^2/2,\ 3M^2/2$)
3) Time update $\mathbf{Q}(n)$ from (12). ($2M^2,\ 0$)
4) For $i=1,2,\ldots,M$
   a) Compute $\mathbf{p}_{i,j}(n),\ \forall j\in S_i$ from (26) for i=1 and from (16) for i>1. ($2M^3/3+9M^2/2,\ 2M^3/3+M^2$)
   b) Evaluate $\mathcal{E}_{i,j}(n),\ \forall j\in S_i$ from (13) for i=1 and from (17) for i>1. Set $\mathcal{E}_i(n)$ the minimum and $\mathbf{p}_i(n)$ the corresponding $\mathbf{p}_{i,j}(n)$ vector. ($3M^2/2,\ 3M^2/2$)
5) Go to step 1.

Initialization: k_i=i $\forall i$, $\mathbf{R}^{-1}(0)$=$\delta^{-1}\mathbf{I}$, $\mathbf{Q}(0)$=$\mathbf{0}$, $\mathbf{p}_{1,j}(0)$=$\mathbf{1}$ for j=1,2,...,M, $\mathbf{p}_i(0)=\delta\mathbf{1}$ and $\mathcal{E}_i(0)$=0 for i=1,...,M. δ is a small positive constant.

TABLE II
COMPARISON OF COMPLEXITIES

	Complex Multiplications	Complex Additions
Algorithm of [6]	$2M^3+\frac{19}{2}M^2+O(M)$	$\frac{4}{3}M^3+4M^2+O(M)$
Proposed Algorithm	$\frac{2}{3}M^3+\frac{25}{2}M^2+O(M)$	$\frac{2}{3}M^3+\frac{11}{2}M^2+O(M)$

TABLE III
PERCENTAGE REDUCTION IN COMPUTATIONAL COMPLEXITY

	Complex Multiplications	Complex Additions
$M=4$	13.3%	12.5%
$M=8$	30%	26.1%
$M=12$	39%	32.5%

is shown in Table II for the case $M = N$.[3] The reduction in computational complexity in favor of the proposed scheme is increased as the number of antennas increases. This reduction is clearly shown in Table III for different values of M. Finally, it should be noted that the new algorithm requires the computation of M square roots in each iteration.

IV. SIMULATIONS

The numerical robustness of the proposed scheme is verified in Fig. 2. The curves of Fig. 2 represent the instantaneous squared error at the output of the equalizers, averaged over all DFEs. A MIMO system with $M = N = 5$ is used in our simulation, whose input independent streams are taken from a QPSK sequence. A number of 30 symbol periods is used for training, and the SNR is set to 20 dB. Each channel coefficient is varied according to Jakes model with a normalized Doppler frequency $f_d T_s = 5 \cdot 10^{-4}$, where f_d stands for the Doppler frequency, and T_s is the symbol period. To track system variations, a forgetting factor $\lambda = 0.98$ was employed.

We observe from Fig. 2 that both algorithms converge very fast and have identical performance for a number of iterations. However, after about 800 iterations, the algorithm of [6] starts diverging due to accumulation of numerical errors. On the

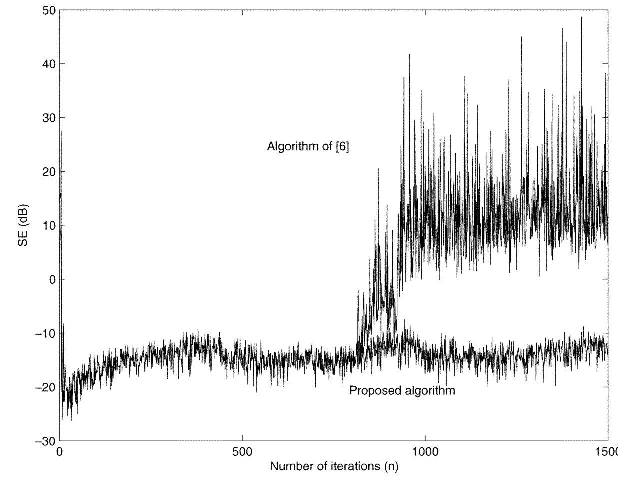

Fig. 2. Squared error curves.

contrary, the proposed algorithm keeps tracking the underlying system and retains a numerically robust performance for the whole simulation period. Notice that a lower steady-state squared error could be achieved after convergence of both algorithms by reducing parameter λ. In such a case, however, divergence of the algorithm of [6] would occur earlier.

V. CONCLUSION

A new square-root adaptive V-BLAST receiver for flat and fast-fading MIMO channels has been proposed. The new adaptive algorithm stems from the Cholesky factorization of the equalizer's input autocorrelation matrix and is mathematically equivalent to a recently proposed adaptive V-BLAST scheme [6]. To the best of our knowledge, these two algorithms are the only V-BLAST schemes that update efficiently in time both the equalizer taps and symbol detection order. However, the proposed receiver offers not only reduced computational complexity but also enhanced numerical robustness.

[3]In [6, Table I (a)], the number of complex multiplications and additions is calculated as $(4/3)M^3+7M^2$ and $(4/3)M^3+5M^2$, respectively. We believe, however, that careful counting leads to the figures in Table II.

REFERENCES

[1] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolniansky, "Simplified processing for high spectral efficiency wireless communications employing multi-element arrays," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 9, pp. 1841–1852, Nov. 1999.
[2] B. Hassibi, "An efficient square-root algorithm for BLAST," in *Proc. IEEE Int. Conf. Acoustics, Speech, Signal Processing*, Istanbul, Turkey, Jun. 2000, pp. 737–740.
[3] W. Zha and S. Blostein, "Modified decorrelating decision-feedback detection of BLAST space-time system," in *Proc. IEEE Int. Conf. Communications*, Helsinki, Finland, Jun. 2002, pp. 737–740.
[4] H. Zhu, Z. Lei, and F. Chin, "An improved square-root algorithm for BLAST," *IEEE Signal Process. Lett.*, vol. 11, no. 9, pp. 772–775, Sep. 2004.
[5] G. Ginis and J. M. Cioffi, "On the relation between V-BLAST and the GDFE," *IEEE Commun. Lett.*, vol. 5, no. 9, pp. 364–366, Sep. 2001.
[6] J. Choi, H. Yu, and Y. H. Lee, "Adaptive MIMO decision feedback equalization for receivers in time-varying channels," in *Proc. IEEE Vehicular Technology Conf.*, Korea, 2003, pp. 1851–1856.
[7] ——, "Adaptive MIMO decision feedback equalization for receivers with time-varying channels," *IEEE Trans. Signal Process.*, vol. 53, no. 11, pp. 4295–4303, Nov. 2005.
[8] S. Haykin, *Adaptive Filter Theory*. Englewood Cliffs, NJ: Prentice-Hall, 2002.
[9] A. A. Rontogiannis and S. Theodoridis, "New fast QR decomposition least squares adaptive algorithms," *IEEE Trans. Signal Process.*, vol. 46, no. 8, pp. 2113–2121, Aug. 1998.