Mobile Communication Systems in the Presence of Fading/Shadowing, Noise and Interference

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***Abstract*—In this paper, the effects of interference on composite fading environments, where multipath fading coexists with shadowing, are investigated. Based on some mathematical convenient expressions for the sum of squared $\mathcal{K}$-distributed random variables, which are derived for the first time, important statistical metrics for the signal to interference and noise ratio (SINR) are studied for various cases including non identical, identical and fully correlated statistics. Furthermore, our analysis is extended to multi-channel receivers and in particular to selection diversity (SD) receivers, investigating two distinct cases, namely, signal-to-noise ratio based and SINR-based SD. For all scenarios, simplified expressions are also provided for the interference limited case, where the influence of thermal noise is ignored. The derived expressions are used to analyze the performance, in terms of the average bit error probability (ABEP) and the outage probability (OP), of such systems operating over composite fading environments. A high SNR analysis is also presented for the OP and the ABEP, giving a clear physical insight of the system's performance in terms of the diversity and coding gains. The analysis is accompanied by numerical evaluated results, clearly demonstrating the usefulness of the proposed theoretical framework.**

***Index Terms*—Composite fading channels, minimum number of antennas, selection diversity, signal-to-interference and noise ratio, sum of squared $\mathcal{K}$-distributed RVs.**

I. Introduction

In many recent wireless communication systems, both licensed, e.g., long term evolution (LTE), or unlicensed, e.g., WiFi or bluetooth, a user quite often shares the same channels with other users and thus in the receiver side the signals need to be intelligently separated. This is imperative, since the aggressive frequency reuse that is frequently employed in cellular systems for increasing spectrum efficiency, causes co-channel as well as adjacent channel interference. The co-channel and adjacent channel interference depend on various physical factors, including interferers' spatial distribution, interfering channels fading, the power of the interferers and the wireless communication system considered. Depending upon the fading characteristics as well as the existence or not of multi-channel transmitters/receivers and/or relays, numerous contributions analyzing the effects of *fading* in conjunction with *interference* have been made, e.g., [2]–[7] and the references therein.

In terrestrial (indoor or outdoor) and satellite land-mobile systems, the link quality is also affected by slow variations of the mean signal level due to the *shadowing* from terrain, buildings and trees [8]. Under these circumstances, where multipath fading coexists with shadowing, the so-called *composite fading/shadowing environment* originates. In the past, this environment has been statistically modeled by using lognormal-based distributions such as Rayleigh-, Nakagami- and Rice-lognormal [8]–[10] and therefore rather mathematically complicated expressions have been derived for the performance analysis on such communication scenarios. To facilitate the communication systems performance evaluation in these environments, new families of distributions that accurately model composite fading conditions have been proposed, most notably as the $\mathcal{K}$ and the generalized-$\mathcal{K}$($\mathcal{K}_G$) distributions, e.g., [11], [12]. Based on the mathematical tractability of these new composite fading models, research efforts have been made recently for investigating the influence that interference has to the system's performance, e.g., [1], [13]–[15]. A common practice in all these works is that the research was restricted to interference-limited wireless communication systems and thus only the statistics of the signal-to-interference-ratio (SIR) was studied.

In this paper extending this approach, and in order to provide a more complete stochastic analysis framework of the composite fading environment, the effect of thermal noise is also taken into consideration. This is important since thermal noise may be the main source of system performance degradation especially in cases of weak interfering signals. Therefore, assuming such a complete model, our contribution in this paper can be summarized as follows:

- mathematical convenient expressions for the sum of squared $\mathcal{K}$-distributed random variables (RV)s are derived for the first time;
- based on these expressions, the signal to interference and noise ratio (SINR) statistics are studied for various scenarios including non identical, identical and fully correlated fading/shadowing effects;
- the analysis is extended to single-input multiple-output (SIMO) receivers and in particular to selection diversity (SD) receivers that operate in such environments;
- the derived expressions are used to investigate the system performance in terms of the average bit error probability (ABEP) and the outage probability (OP);

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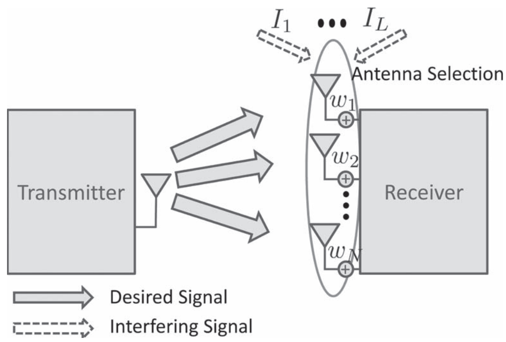

Fig. 1. System model.

- a high SNR analysis is also provided for the ABEP and the OP in the SIMO case, which is then employed to identify the coding and diversity gains of the system under study.

For all scenarios studied, simplified expressions are also provided for the interference limited cases, where the influence of thermal noise is ignored, whilst for SIMO diversity systems an investigation on the power efficiency is also presented.

The remainder of this paper is organized as follows. The system model is described in Section II. In Section III the statistics of the sum of squared $\mathcal{K}$-distributed RVs are investigated for the three scenarios under consideration, namely non identical, identical and fully correlated statistics. The statistics of the SINR for single-input single-output (SISO) and SIMO channels are derived in Sections IV and V, respectively. In Section VI the performance analysis of such systems is presented, while performance evaluation results are provided in Section VII. Finally, concluding remarks are given in Section VIII.

II. System Model

We consider a wireless-mobile communication system, with a single-antenna transmitter and (in general) a multiple-antennas receiver, where each receiver diversity branch is experiencing interference coming from L sources, as shown in Fig. 1. In a typical macrocellular mobile radio network, as it is the one considered in this study, the mobile users are usually surrounded by local scatterers so that the plane waves arrive from many directions without a direct line-of-sight (LoS) component [16]. For such a type of scattering environment the received envelope is Rayleigh distributed. This assumption holds for both the desired and interfering signals. However, in many wireless networks, due to the expected larger propagation distances, interfering signals are very likely to propagate over obstructed paths, and thus experience, in addition, severe shadowing conditions. This does not hold for the desired signals, since not many obstacles between the user and the tagged base station are expected to be present due to the (comparatively) small distance between them. It is noted that similar assumptions concerning the involved desired and interfering channel models in various settings have been made by many other researchers in the past, e.g., [16, pp. 140], [17]–[21]. It is also important to note that the adopted channel model offers also increased tractability, and thus give us the opportunity to analytically investigate the influence of the combination of fading, shadowing and multichannel reception to the system performance. We also assume that (in general) the level of interference at the receiver is such that the effect of thermal noise on system performance cannot be ignored [2]. Having defined our model, the complex baseband signal y_n received at the nth antenna, can be expressed as

$$y_n = h_{D_n} s_D + \sum_{i=1}^{L} h_{I_{n,i}} s_{I_i} + w_n \tag{1}$$

where h_{D_n} represents the complex channel gain between the transmitter and the nth receiver antenna (with its envelope following the Rayleigh distribution) and s_D is the desired transmitted complex symbol with energy $E_{s_D} = \mathbb{E}\langle |s_D|^2 \rangle$, and $\mathbb{E}\langle\cdot\rangle$ denoting statistical averaging. Furthermore, in (1), $h_{I_{n,i}}$ represents the complex channel gain (with its envelope following the $\mathcal{K}$-distribution) of the interfering signal s_{I_i} and w_n is the complex additive white Gaussian noise (AWGN) with zero mean and variance N_0. To proceed, we denote the instantaneous signal-to-noise-ratio (SNR) of the desired signal as $\gamma_{D_n} = |h_{D_n}|^2 E_{s_D}/N_0$, the corresponding average SNR as $\bar{\gamma}_{D_n} = \mathbb{E}\langle |h_{D_n}|^2 \rangle E_{s_D}/N_0$, whilst the instantaneous interference-to-noise-ratio (INR) of the ith interfering signal is defined as $\gamma_{I_{n,i}} = |h_{I_{n,i}}|^2 E_{s_{I_i}}/N_0$, with corresponding average INR equal to $\bar{\gamma}_{I_{n,i}} = \mathbb{E}\langle |h_{I_{n,i}}|^2 \rangle E_{s_{I_i}}/N_0$.

Since the desired signal is subject only to multipath fading, its instantaneous SNR γ_{D_n} at the nth antenna can be assumed to be exponentially distributed. Specifically, considering independent fading conditions, the probability density function (PDF) of γ_{D_n} is given by

$$f_{\gamma_{D_n}}(x) = \frac{1}{\bar{\gamma}_{D_n}} \exp\left(-\frac{x}{\bar{\gamma}_{D_n}}\right). \tag{2}$$

Here, the instantaneous INR, $\gamma_{I_{n,i}}$, of the interfering signals, which are subject to fading/shadowing effects, is assumed to follow a squared $\mathcal{K}$ distribution[1] with PDF given by [11]

$$f_{\gamma_{I_{n,i}}}(x) = 2\left(\frac{k_{n,i}}{\bar{\gamma}_{I_{n,i}}}\right)^{\frac{k_{n,i}+1}{2}} \frac{x^{\frac{k_{n,i}-1}{2}}}{\Gamma(k_{n,i})} K_{k_{n,i}-1}\left(2\sqrt{\frac{k_{n,i}x}{\bar{\gamma}_{I_{n,i}}}}\right). \tag{3}$$

In (3), $k_{n,i}$ denotes the shaping parameter of the distribution, $\Gamma(z) = \int_0^\infty \exp(-t)t^{z-1}dt$ [22, eq. (8.310/1)] and $K_v(\cdot)$ is the second kind modified Bessel function of vth order [22, eq. (8.407/1)]. In addition, $k_{n,i}$ is related to the severity of the shadowing, e.g., for small values for $k_{n,i}$, (3) models severe shadowing conditions, while as $k_{n,i} \to \infty$, it approximates the exponential distribution. The corresponding output SINR, at the nth receiver antenna, is expressed as [2]

$$\gamma_{\text{out}_n} = \frac{\gamma_{D_n}}{1+\gamma_{I_n}} \tag{4}$$

where γ_{I_n} denotes the total INR,[2] i.e., $\gamma_{I_n} = \sum_{i=1}^{L} \gamma_{I_{n,i}}$, and the PDF of $\gamma_{I_{n,i}}$ is given by (3). The PDF of γ_{out_n} can be evaluated as follows

$$f_{\gamma_{\text{out}_n}}(\gamma) = \int_0^\infty (1+x) f_{\gamma_{D_n}}\left[(1+x)\gamma\right] f_{\gamma_{I_n}}(x)dx. \tag{5}$$

[1]For simplification purposes and to avoid repetitions, from now on squared $\mathcal{K}$ distribution will be referred as $\mathcal{K}$.

[2]Without loosing the generality, it is assumed that all diversity branches are affected by the same interfering sources.

In the next section important statistical metrics of γ_{I_n} will be evaluated considering non identical, identical as well as fully correlated statistical parameters. The derived results (presented in Section III), will be used to obtain expressions for the instantaneous output SINR of both SISO and SIMO systems (presented in Sections IV and V, respectively).

III. On the Sum of $\mathcal{K}$-Distributed RVs

In this section the statistics of the sum of $\mathcal{K}$-distributed RVs will be investigated for three different scenarios, namely for $\mathcal{K}$-distributed RVs which, i) are independent but non identically distributed (i.n.d.), ii) are independent and identically distributed (i.i.d.) and iii) have fully correlated mean values. It should be noted here that in the past, several efforts have been devoted to accurately approximating the $\mathcal{K}$-distribution, e.g., [23]. However, to the best of the authors knowledge, none of them led to exact expressions for the sum of $\mathcal{K}$ RVs, which are obtained for the first time in the following analysis. Since the analytical results provided in this and the next sections refer to the arbitrary nth antenna, the antenna index n will be omitted and will be reestablished in Section V.

A. Independent But Not Identically Distributed RVs

Theorem 1: Let γ_I denote a RV defined as

$$\gamma_I \triangleq \sum_{i=1}^{L} \gamma_{I_i} \tag{6}$$

where the PDF of γ_{I_i} is given by (3). Considering *non identical (interference) statistics*, the PDF of γ_I can be expressed as

$$f_{\gamma_I}(\gamma) = \mathcal{G}_1 \left[\left(\frac{\gamma}{\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}} \right)^{\frac{\mathcal{S}^L_{k_i,1}-1}{2}} I_{\mathcal{S}^L_{k_i,1}-1}\left(2\sqrt{\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}\gamma}\right) \right.$$

$$\left. + \underbrace{\sum_{\substack{i,i\\ \lambda_1,\ldots,\lambda_i}}^{L} \sum_{h=0}^{\infty} \left(\frac{\gamma}{\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}} \right)^{\frac{\mathcal{G}_{2_i}-1}{2}} \mathcal{G}_3 I_{\mathcal{G}_{2_i}-1}\left(2\sqrt{\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}\gamma}\right)}_{IS} \right]. \tag{7}$$

In (7), $\mathcal{S}^z_{x_q,y_q} = \sum_{q=1}^{z} \frac{x_q}{y_q}$, $\mathcal{G}_1 = \left[\prod_{i=1}^{L}\left(\frac{k_i}{\bar{\gamma}_{I_i}}\right)^{k_i}\Gamma(1-k_i)\right]$, $\mathcal{G}_{2_j} = \sum_{i=1}^{L} k_i + j - \sum_{i=1}^{j} k_{\lambda_i} + h$, $\mathcal{G}_3 = t_{h,\lambda_1}$, if $i=1$, $\mathcal{G}_3 = \tilde{t}_{1,h} = \sum_{p=0}^{h} t_{p,\lambda_1} t_{h-p,\lambda_2}$, if $i=2$, $\mathcal{G}_3 = \tilde{t}_{i-1,h} = \sum_{p=0}^{h} \tilde{t}_{i-2,p} t_{h-p,\lambda_i}$, if $i>2$, with $t_{h,\lambda_1} = \frac{(-1)^h (k_{\lambda_1}/\bar{\gamma}_{I_{\lambda_1}})^{1-k_{\lambda_1}+h}}{h!\Gamma(1-k_{\lambda_1})(1-k_{\lambda_1}+h)}$ and $I_v(\cdot)$ denoting the first kind modified Bessel function of vth order [22, eq. (8.406/1)].

TABLE I
Minimum Number of Terms H of (7) Required for Obtaining Accuracy Better Than $\pm 10^{-5}$

γ	5dB $k=1.5$	5dB $k=3$	15dB $k=1.5$	15dB $k=3$
1	4	5	1	2
5	9	16	4	7
10	13	20	4	8
15	16	23	5	9
20	18	25	7	11

Proof: See Appendix A. ■

It can be shown that the infinite series in *IS* converges everywhere.[3] In addition the rate of convergence of $f_{\gamma_I}(\gamma)$ given in (7), was investigated experimentally. This is important because in practice, to evaluate $f_{\gamma_I}(\gamma)$, the infinite series in *IS* is truncated and a number of series terms H is retained. In Table I the minimum values of H, required for accuracy better than $\pm 10^{-5}$ are presented for various values of the average INR $\bar{\gamma}_{I_i}$ and distribution's parameters. These results have been obtained by assuming an exponentially decaying profile, i.e., $\bar{\gamma}_{I_i} = 10^{\mathrm{dB}/10}\exp[-d(i-1)]$, $k_i = 3-0.3i$ (with $i=1,2,\ldots,L$), $d=0.1$ and $L=3$. It is clearly depicted from this table that a relatively small number of terms is sufficient to achieve a high accuracy and as a consequence the PDF in (7) converges fast. Note that H mainly depends on the value of the variable of the Bessel function $I_v(\cdot)$ in (7), and increases as this variable increases. As a result, H increases as $\bar{\gamma}_{I_i}$ decreases and/or k_i, γ increase. Moreover, comparing these results with other employing infinite series, e.g., [25], [26], the required number of terms is significantly smaller for a similar target accuracy.

Lemma 1: For i.n.d. statistics, the CDF of γ_I can be obtained as

$$F_{\gamma_I}(\gamma) = \mathcal{G}_1 \left[\left(\frac{\gamma}{\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}} \right)^{\frac{\mathcal{S}^L_{k_i,2}}{2}} I_{\mathcal{S}^L_{k_i,1}}\left(2\sqrt{\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}\gamma}\right) \right.$$

$$\left. + \sum_{\substack{i,i\\ \lambda_1,\ldots,\lambda_i}}^{L} \sum_{h=0}^{\infty} \left(\frac{\gamma}{\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}} \right)^{\frac{\mathcal{G}_{2_i}}{2}} \mathcal{G}_3 I_{\mathcal{G}_{2_i}}\left(2\sqrt{\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}\gamma}\right) \right]. \tag{8}$$

Proof: Substituting (7) in the definition of the CDF, $F_{\gamma_I}(\gamma) = \int_0^{\gamma} f_{\gamma_I}(x)dx$, making changes of variables of the form $x^{1/2} = y$ and $y = \sqrt{\gamma}z$ and using [22, eq. (6.561/7)], i.e., $\int_0^1 x^{v+1} I_v(ax)dx = a^{-1} I_{v+1}(a)$, (8) is obtained. ■

In Fig. 2(a), several PDFs, obtained by using (7), are plotted for various values of the number of interferers L. These distributions have been evaluated by also assuming exponential decaying average INR, i.e., $\bar{\gamma}_{I_i} = 10^{\mathrm{dB}/10}\exp[-d(i-1)]$, $k_i = k - 0.3i$ (with $i = 1,2,\ldots,L$), and dB = 15, $d = 0.1$. Additionally, simulated results are also included in this figure, verifying in all cases the agreement between the analytical and the simulated PDFs.

[3] Detailed analysis is available in the technical report [24], which has been also prepared.

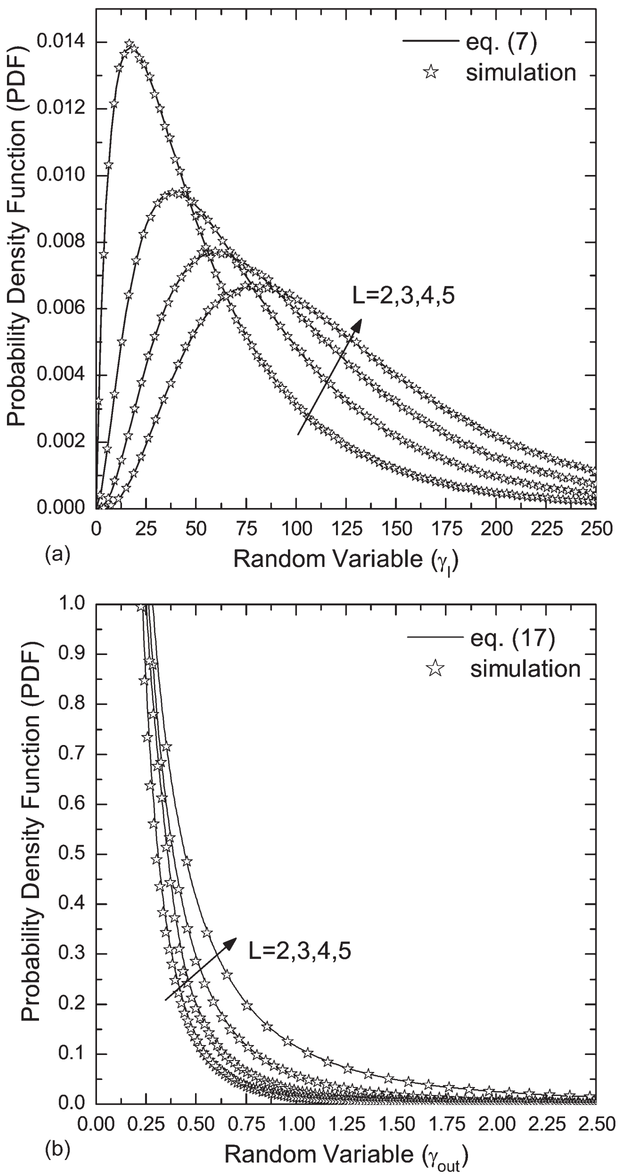

Fig. 2. (a) The PDF of γ_I given in (7). (b) The PDF of γ_{out} given in (17).

B. Independent and Identically Distributed RVs

Theorem 2: For the same RV defined in (6), considering *identical (interference) statistics*, i.e., $\overline{\gamma}_{I_i} = \overline{\gamma}_I$ and $k_i = k$, the PDF of γ_I can be expressed as

$$f_{\gamma_I}(\gamma) = L\sum_{i=0}^{L}\binom{L}{i}\Gamma(1-k)^{L-i}(-1)^i \times \left[\sum_{h=0}^{\infty} c_h \left(\frac{k}{\overline{\gamma}_I L}\right)^{\frac{1+\mathcal{G}_4}{2}} \gamma^{\frac{\mathcal{G}_4-1}{2}} I_{\mathcal{G}_4-1}\left(2\sqrt{\frac{kL}{\overline{\gamma}_I}\gamma^{\frac{1}{2}}}\right)\right] \quad (9)$$

where $c_0 = a_0^i$, $c_h = \frac{1}{ha_0}\left[\sum_{t=1}^{h}(ti-h+t)a_t c_{h-t}\right]$ for $h \geq 1 a_0 = \frac{1}{1-k}$, $a_t = \frac{(-1)^t}{t!(1-k+t)}$,with $\mathcal{G}_4 = Lk + h + (1-k)i$.

Proof: See Appendix B. ■

Lemma 2: For i.i.d. statistics, the CDF of γ_I can be obtained as

$$F_{\gamma_I}(\gamma) = \sum_{i=0}^{L}\binom{L}{i}\Gamma(1-k)^{L-i}(-1)^i \times \left[\sum_{h=0}^{\infty} c_h \left(\frac{k}{\overline{\gamma}_I L}\right)^{\frac{\mathcal{G}_4}{2}} \gamma^{\frac{\mathcal{G}_4}{2}} I_{\mathcal{G}_4}\left(2\sqrt{\frac{kL}{\overline{\gamma}_I}\gamma^{\frac{1}{2}}}\right)\right]. \quad (10)$$

Proof: Substituting (9) in the definition of the CDF, $F_{\gamma_I}(\gamma) = \int_0^{\gamma} f_{\gamma_I}(x)dx$, making changes of variables of the form $x^{1/2} = y$ and $y = \sqrt{\gamma}z$ and using again [22, eq. (6.561/7)], (10) is obtained. ■

C. Fully Correlated Shadowing

When the L different interfering paths exhibit identical shadowing effects, the mean values of the corresponding $\mathcal{K}$-distributed RVs are fully correlated. This is the so-called *fully (or totally) correlated shadowing* communication scenario, which has gained considerable interest lately, e.g., [27]–[30], and will be investigated in this section. This type of shadowing arises in situations where the interferers have approximately the same distance from the receiver, and thus the same obstacles shadow in a quite similar way the various interfering signals. As a result, the local mean powers of the interfering signals become correlated [14], [31], a situation that quite often arises in indoor communication systems [32].

As it will be shown below, the fully correlated shadowing communication scenario is mathematically tractable compared to the more general models described above, in the sense that easy-to-compute closed-form expressions can be easily derived, which provide useful insights to the system performance. In such a communication scenario, where the multipath Rayleigh components of the interferers are independent but all of them experience a common local average power, $\overline{\gamma}_I$, it has been shown that the moments generating function (MGF) of γ_I can be expressed as [11, eq. (9)]

$$\mathcal{M}_{\gamma_I}(s) = \left(\frac{k}{\overline{\gamma}_I s}\right)^k U\left(k, k-L+1, \frac{k}{\overline{\gamma}_I s}\right) \quad (11)$$

where $U(\cdot)$ denotes the confluent hypergeometric function defined as $U\left(k,k-L+1,\frac{k}{\overline{\gamma}_I s}\right) = \frac{\Gamma(L-k)}{\Gamma(L)}\,{}_1F_1\left(k;k-L+1;\frac{k}{\overline{\gamma}_I s}\right) + \frac{\Gamma(k-L)}{\Gamma(k)}\left(\frac{k}{\overline{\gamma}_I s}\right)^{L-k}{}_1F_1\left(L;1-k+L;\frac{k}{\overline{\gamma}_I s}\right)$ [22, eq. (9.210/2)], with ${}_1F_1(\alpha,\gamma,z) = \sum_{k=0}^{\infty}\frac{(\alpha)_k z^k}{(\gamma)_k k!}$ representing the confluent hypergeometric function [22, eq. (9.210/1)] and $(\cdot)_v$ denoting the pochhammer symbol [22, pp. xliii]. Based on this expression and the definition of ${}_1F_1(\cdot)$, (11) yields

$$\mathcal{M}_{\gamma_I}(s) = \frac{\Gamma(L-k)}{\Gamma(L)}\left(\frac{k}{\overline{\gamma}_I s}\right)^k \sum_{i=0}^{\infty}\frac{(k)_i\left[k/(\overline{\gamma}_I s)\right]^i}{i!(k-L+1)_i} + \frac{\Gamma(k-L)}{\Gamma(k)}\left(\frac{k}{\overline{\gamma}_I s}\right)^L \sum_{i=0}^{\infty}\frac{(L)_i\left[k/(\overline{\gamma}_I s)\right]^i}{i!(1-k+L)_i}. \quad (12)$$

Applying the inverse Laplace transform in (12), yields the following expression for the PDF of γ_I

$$f_{\gamma_I}(\gamma)=\frac{\Gamma(L-k)}{\Gamma(k)\Gamma(L)}\left(\frac{k\gamma}{\bar{\gamma}_I}\right)^k\frac{1}{\gamma}\sum_{i=0}^{\infty}\frac{(k\gamma/\bar{\gamma}_I)^i}{i!(k-L+1)_i}$$
$$+\frac{\Gamma(k-L)}{\Gamma(k)\Gamma(L)}\left(\frac{k\gamma}{\bar{\gamma}_I}\right)^L\frac{1}{\gamma}\sum_{i=0}^{\infty}\frac{(k\gamma/\bar{\gamma}_I)^i}{i!(1-k+L)_i}. \quad (13)$$

Furthermore, based on the definition of the generalized hypergeometric function, i.e., ${}_0F_1(b;z)=\sum_{i=0}^{\infty}z^i/(i!(b)_i)$ [33, eq. (07.17.02.0001.01)], and by employing [33, eq. (03.04.27.0005.01)], a simpler closed-form expression for the PDF of γ_I, when fully correlated shadowing effects are present, can be extracted as

$$f_{\gamma_I}(\gamma)=2\frac{(k/\bar{\gamma}_I)^{\frac{L+k}{2}}\gamma^{\frac{L+k}{2}-1}}{\Gamma(L)\Gamma(k)}K_{L-k}\left(2\sqrt{\frac{k}{\bar{\gamma}_I\gamma}}\right). \quad (14)$$

Note that in a different research topic, i.e., free space optical communications, a similar PDF expression, as the one given in (14), has been independently derived in [34]. Substituting now (14) in the definition of the CDF [35, eq. (4.17)], using the Meijer-G function representation for the $K_{L-k}(\cdot)$ [33, eq. (03.04.26.0006.01)], i.e., $K_v(\sqrt{z^2})=\frac{1}{2}\mathcal{G}_{2,0}^{0,2}\left(\frac{z^2}{4}\,\middle|\,\frac{v}{2},-\frac{v}{2}\right)$, and then [36, eq. (26)], the CDF of γ_I can be expressed in a simple closed form as

$$F_{\gamma_I}(\gamma)=\frac{(k/\bar{\gamma}_I)^{\frac{L+k}{2}}}{\Gamma(L)\Gamma(k)}\gamma^{\frac{L+k}{2}}\mathcal{G}_{1,3}^{2,1}\left(\frac{k\gamma}{\bar{\gamma}_I}\,\middle|\,\begin{matrix}1-\frac{L+k}{2}\\ \frac{L-k}{2},\frac{k-L}{2},-\frac{L+k}{2}\end{matrix}\right) \quad (15)$$

where $\mathcal{G}_{p,q}^{m,n}[\cdot|\cdot]$ is the Meijer's G-function [22, eq. (9.301)]. Obviously, the derived expressions for the fully correlated case of shadowing, can be directly employed for further analytical purposes. Moreover, as it will be shown in Section VII, the system performance results, which are obtained for the fully correlated case, are quite close to those obtained for the i.i.d. case. Finally, it should be noted that the previously derived expressions for the sum of $\mathcal{K}$-distributed RVs can also be applied to analyze the performance of various receiver structures, including a maximal ratio combiner output SNR study.

IV. SINR Statistics for SISO Systems

In this section, based on the previously derived expressions for the sum of $\mathcal{K}$-distributed RVs, a statistical analysis of the instantaneous output SINR γ_{out} is presented for the three communication scenarios under consideration, namely i.n.d., i.i.d. and fully correlated interference conditions. In all cases simplified expressions for the SIR are also obtained.

A. Non Identical Interference Statistics

Substituting the PDF for the SNR given by (2) and the PDF for the INR given by (7) in (5), it can be shown that integrals of the following form appear

$$I_1=\int_0^{\infty}\Phi\left(q_1,\sum_{i=1}^{L}\frac{k_i}{\bar{\gamma}_{I_i}}\right)dx \quad (16)$$

where $\Phi(a,b)=x^{\frac{a-1}{2}}(1+x)\exp\left[-\frac{(1+x)\gamma}{\bar{\gamma}_D}\right]I_{a-1}(2\sqrt{bx})$ with $q_1=\sum_{i=1}^{L}k_i$ or $q_1=\mathcal{G}_{2_h}$. This type of integrals can be solved in closed form using [22, eq. (6.643/2)]. After some straightforward mathematical manipulations the PDF of γ_{out} under i.n.d. interference conditions can be expressed as

$$f_{\gamma_{\text{out}}}(\gamma)=\frac{\exp(-\gamma/\bar{\gamma}_D)}{\bar{\gamma}_D}\mathcal{G}_1\exp\left(\frac{\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}}{2\gamma/\bar{\gamma}_D}\right)\left\{\left(\frac{\bar{\gamma}_D}{\gamma\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}}\right)^{\mathcal{S}^L_{k_i,2}}\right.$$
$$\times\left[\mathcal{S}^L_{k_i,1}\frac{\bar{\gamma}_D}{\gamma}M_{-\mathcal{S}^L_{k_i,2}-1,\frac{\mathcal{S}^L_{k_i,1}-1}{2}}\left(\frac{\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}}{\gamma/\bar{\gamma}_D}\right)\right.$$
$$\left.+M_{-\mathcal{S}^L_{k_i,2},\frac{\mathcal{S}^L_{k_i,1}-1}{2}}\left(\frac{\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}}{\gamma\bar{\gamma}_D}\right)\right]+\sum_{\substack{i,i\\ \lambda_1,\ldots,\lambda_i}}^{L}\sum_{h=0}^{\infty}\mathcal{G}_3$$
$$\times\left(\frac{\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}\gamma}{\bar{\gamma}_D}\right)^{-\frac{\mathcal{G}_{2_i}}{2}}\left[\frac{\mathcal{G}_{2_i}\bar{\gamma}_D}{\gamma}M_{-\frac{\mathcal{G}_{2_i}}{2}-1,\frac{\mathcal{G}_{2_i}-1}{2}}\left(\frac{\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}}{\gamma/\bar{\gamma}_D}\right)\right.$$
$$\left.\left.+M_{-\frac{\mathcal{G}_{2_i}}{2},\frac{\mathcal{G}_{2_i}-1}{2}}\left(\frac{\mathcal{S}^L_{k_i,\bar{\gamma}_{I_i}}}{\gamma/\bar{\gamma}_D}\right)\right]\right\} \quad (17)$$

where $M_{\lambda,\mu}(\cdot)$ is the Whittaker function [22, eq. (9.220/2)], which is a built-in function in many mathematical software packages. Here, it should be mentioned that (17) represents a valid PDF, since it is a nonnegative function, and using [33, eqs. (07.44.26.0007.01) and (01.03.26.0004.01)] and [36, eq. (21)], it can be verified that $\int_0^{\infty}f_{\gamma_{\text{out}}}(\gamma)d\gamma=1$ [37]. In Fig. 2(b), several PDFs, obtained by using (17), are plotted for the same parameters as the ones used in Fig. 2(a) and various values for the number of interferers L. The simulated results also included in this figure depict in all cases the tight agreement between analytical and simulated PDFs. Furthermore, for the infinite series appearing in (17), a similar rate of convergence has been observed with that of the series appearing in (7). By definition, the CDF of the instantaneous output SINR is expressed as

$$F_{\gamma_{\text{out}}}(\gamma)=\int_0^{\infty}F_{\gamma_D}\left[(1+x)\gamma\right]f_{\gamma_I}(x)dx. \quad (18)$$

Substituting the exponential CDF, i.e.,

$$F_{\gamma_D}(x)=1-\exp\left(-\frac{x}{\bar{\gamma}_D}\right) \quad (19)$$

and (7) in (18), following a similar procedure as the one used for deriving (17), yields the following closed-form expression

TABLE II
SISO SIR STATISTICS

Non Identical Interference	$f_{\gamma_{\text{out}}}(\gamma) = \mathcal{G}_1 \exp\left(\frac{\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}}{2\gamma/\bar\gamma_D}\right)\frac{1}{\gamma}\left\{\left(\frac{\bar\gamma_D}{\gamma\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}}\right)^{\mathcal{S}^L_{k_i,2}}\mathcal{S}^L_{k_i,1} M_{-\mathcal{S}^L_{k_i,2}-1,\frac{\mathcal{S}^L_{k_i,1}-1}{2}}\left(\frac{\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}}{\gamma/\bar\gamma_D}\right) + \sum^L{}_{\substack{i,i\\\lambda_1,\ldots,\lambda_i}}\sum_{h=0}^{\infty}\mathcal{G}_3\mathcal{G}_{2_i}\left(\frac{\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}\gamma}{\bar\gamma_D}\right)^{-\frac{\mathcal{G}_{2_i}}{2}} M_{-\frac{\mathcal{G}_{2_i}}{2}-1,\frac{\mathcal{G}_{2_i}-1}{2}}\left(\frac{\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}}{\gamma/\bar\gamma_D}\right)\right\}$
	$F_{\gamma_{\text{out}}}(\gamma) = 1-\mathcal{G}_1 \exp\left(\frac{\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}}{2\gamma/\bar\gamma_D}\right)\left\{\left(\frac{\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}\gamma}{\bar\gamma_D}\right)^{-\mathcal{S}^L_{k_i,2}} M_{-\mathcal{S}^L_{k_i,2},\frac{\mathcal{S}^L_{k_i,1}-1}{2}}\left(\frac{\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}}{\gamma\bar\gamma_D}\right) + \sum^L{}_{\substack{i,i\\\lambda_1,\ldots,\lambda_i}}\sum_{h=0}^{\infty}\mathcal{G}_3\left(\frac{\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}\gamma}{\bar\gamma_D}\right)^{-\frac{\mathcal{G}_{2_i}}{2}} M_{-\frac{\mathcal{G}_{2_i}}{2},\frac{\mathcal{G}_{2_i}-1}{2}}\left(\frac{\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}}{\gamma/\bar\gamma_D}\right)\right\}$
Identical Interference	$f_{\gamma_{\text{out}}}(\gamma) = \sum_{i=0}^{L}\binom{L}{i}\Gamma(1-k)^{L-i}(-1)^i\sum_{n=0}^{\infty}c_n\frac{\mathcal{G}_4}{\gamma}\left(\frac{k\bar\gamma_D}{\bar\gamma_I L\gamma}\right)^{\frac{\mathcal{G}_4}{2}}\exp\left(\frac{kL\bar\gamma_D}{2\bar\gamma_I\gamma}\right)M_{-\frac{\mathcal{G}_4+2}{2},\frac{\mathcal{G}_4-1}{2}}\left(\frac{kL\bar\gamma_D}{\bar\gamma_I\gamma}\right)$
	$F_{\gamma_{\text{out}}}(\gamma) = 1-\sum_{i=0}^{L}\binom{L}{i}\Gamma(1-k)^{L-i}(-1)^i\sum_{n=0}^{\infty}c_n\left(\frac{k\bar\gamma_D}{\gamma L\bar\gamma_I}\right)^{\frac{\mathcal{G}_4}{2}}\exp\left(\frac{kL\bar\gamma_D}{2\gamma\bar\gamma_I}\right)M_{-\frac{\mathcal{G}_4}{2},\frac{\mathcal{G}_4-1}{2}}\left(\frac{kL\bar\gamma_D}{\gamma\bar\gamma_I}\right)$
Fully Correlated Interference	$f_{\gamma_{\text{out}}}(\gamma) = Lk\left(\frac{k\bar\gamma_D}{\bar\gamma_I\gamma}\right)^{\frac{L+k-1}{2}}\exp\left(\frac{k\bar\gamma_D}{2\bar\gamma_I\gamma}\right)\gamma^{-1}W_{-\frac{L+k+1}{2},\frac{L-k}{2}}\left(\frac{k\bar\gamma_D}{\bar\gamma_I\gamma}\right)$
	$F_{\gamma_{\text{out}}}(\gamma) = 1-\left(\frac{k\bar\gamma_D}{\bar\gamma_I\gamma}\right)^{\frac{L+k-1}{2}}\exp\left(\frac{k\bar\gamma_D}{2\bar\gamma_I\gamma}\right)W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\bar\gamma_D}{\bar\gamma_I\gamma}\right)$
	$M_{\gamma_{\text{out}}}(s) = \frac{1}{\Gamma(k)\Gamma(L)}\left(\frac{k\bar\gamma_D}{\bar\gamma_I}\right)^{\frac{L+k-1}{2}} s^{\frac{L+k-1}{2}}\mathcal{G}^{1,3}_{3,1}\left(\frac{\bar\gamma_I}{k\bar\gamma_D s}\middle\vert\begin{matrix}\frac{L+k+1}{2},1-\frac{L-k+1}{2},1+\frac{L-k-1}{2}\\1+\frac{L+k-1}{2}\end{matrix}\right)$

for the CDF of γ_{out}[4]

$$F_{\gamma_{\text{out}}}(\gamma) = 1-\exp\left(-\frac{\gamma}{\bar\gamma_D}\right)\mathcal{G}_1\exp\left(\frac{\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}}{2\gamma/\bar\gamma_D}\right) \times\left\{\left(\frac{\gamma\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}}{\bar\gamma_D}\right)^{-\mathcal{S}^L_{k_i,2}} M_{-\mathcal{S}^L_{k_i,2},\frac{\mathcal{S}^L_{k_i,1}-1}{2}}\left(\frac{\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}}{\gamma\bar\gamma_D}\right) + \sum_{\substack{i,i\\\lambda_1,\ldots,\lambda_i}}^{L}\sum_{h=0}^{\infty}\mathcal{G}_3\left(\frac{\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}\gamma}{\bar\gamma_D}\right)^{-\frac{\mathcal{G}_{2_i}}{2}} M_{-\frac{\mathcal{G}_{2_i}}{2},\frac{\mathcal{G}_{2_i}-1}{2}}\left(\frac{\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}}{\gamma/\bar\gamma_D}\right)\right\}. \quad (20)$$

For the special case of $L=1$ $\mathcal{K}$-distributed interferer, (20) simplifies to

$$F_{\gamma_{\text{out}}}(\gamma) = 1-\exp\left(-\frac{\gamma}{\bar\gamma_D}\right)\left(\frac{k\bar\gamma_D}{\bar\gamma_I\gamma}\right)^{\frac{k}{2}} \times\exp\left(\frac{k\bar\gamma_D}{2\bar\gamma_I\gamma}\right)W_{-\frac{k}{2},\frac{1-k}{2}}\left(\frac{k\bar\gamma_D}{\bar\gamma_I\gamma}\right). \quad (21)$$

In a similar communication scenario consisting of a Nakagami-m desired signal and a Nakagami-lognormal interfering signal, an integral expression for the outage performance was derived in [16, eq. (3.59)]. Setting in that expression $m=1$, i.e., considering Rayleigh/Rayleigh-lognormal fading/shadowing conditions, and comparing the resulting formula with (21), the mathematical simplification offered by the latter is obvious.

[4]A simpler expression for this CDF, can be obtained using the MGF-based approach that is proposed in [3].

Simplified Expressions for the SIR: In many cases, the mobile communication systems tend to be interference limited rather than noise limited, since the thermal and man-made noise effects are often insignificant compared to the signal levels of cochannel users [38]. This case will be also studied here, where considering an interference limited environment, i.e., ignoring the AWGN at the user terminal, the received SIR is given by

$$\gamma_{\text{out}} = \frac{\gamma_D}{\gamma_I} \quad (22)$$

whilst its PDF and CDF expressions are

$$f_{\gamma_{\text{out}}}(\gamma) = \int_0^\infty x f_{\gamma_D}(x\gamma)f_{\gamma_I}(x)dx$$
$$F_{\gamma_{\text{out}}}(\gamma) = \int_0^\infty F_{\gamma_D}(x\gamma)f_{\gamma_I}(x)dx \quad (23)$$

respectively. Substituting (2) and (7) (or (19) and (7)) in (23), and following a similar procedure as the one for deriving (17), yields the PDF and CDF expressions, respectively, for the i.n.d. case given in Table II.

B. Identical Interference Statistics

Substituting (2) and (9) in (5) and after some mathematical procedure yields

$$f_{\gamma_{\text{out}}}(\gamma) = \frac{L}{\bar\gamma_D}\sum_{i=0}^{L}\binom{L}{i}\Gamma(1-k)^{L-i}(-1)^i\sum_{n=0}^{\infty}c_n \times\left(\frac{k}{\bar\gamma_I L}\right)^{\frac{\mathcal{G}_4}{2}}\left[\int_0^\infty\Phi\left(\mathcal{G}_4,\frac{kL}{\bar\gamma_I}\right)dx\right]. \quad (24)$$

The integral in (24) can be solved in closed form using [22, eq. (6.643/2)] and thus the PDF of γ_{out} under i.i.d. interference conditions can be expressed as

$$f_{\gamma_{\text{out}}}(\gamma)=\frac{\exp(-\gamma/\bar{\gamma}_D)}{\bar{\gamma}_D}\sum_{i=0}^{L}\binom{L}{i}\Gamma(1-k)^{L-i}(-1)^i$$
$$\times\sum_{n=0}^{\infty}c_n\left(\frac{k\bar{\gamma}_D}{\gamma L\bar{\gamma}_I}\right)^{\frac{\mathcal{G}_4}{2}}\exp\left(\frac{kL\bar{\gamma}_D}{2\gamma\bar{\gamma}_I}\right)\left[M_{-\frac{\mathcal{G}_4}{2},\frac{\mathcal{G}_4-1}{2}}\left(\frac{kL\bar{\gamma}_D}{\gamma\bar{\gamma}_I}\right)\right.$$
$$\left.+\frac{\mathcal{G}_4\bar{\gamma}_D}{\gamma}M_{-\frac{\mathcal{G}_4+2}{2},\frac{\mathcal{G}_4-1}{2}}\left(\frac{kL\bar{\gamma}_D}{\gamma\bar{\gamma}_I}\right)\right]. \tag{25}$$

The corresponding expression for the CDF is given by

$$F_{\gamma_{\text{out}}}(\gamma)=1-\exp\left(-\frac{\gamma}{\bar{\gamma}_D}\right)\sum_{i=0}^{L}\binom{L}{i}\Gamma(1-k)^{L-i}(-1)^i$$
$$\times\sum_{n=0}^{\infty}c_n\left(\frac{k\bar{\gamma}_D}{\gamma L\bar{\gamma}_I}\right)^{\frac{\mathcal{G}_4}{2}}\exp\left(\frac{kL\bar{\gamma}_D}{2\gamma\bar{\gamma}_I}\right)M_{-\frac{\mathcal{G}_4}{2},\frac{\mathcal{G}_4-1}{2}}\left(\frac{kL\bar{\gamma}_D}{\gamma\bar{\gamma}_I}\right). \tag{26}$$

For the SIR case simplified expression for the PDF and CDF are given in Table II.

C. Fully Correlated Interference Statistics

Substituting (2) and (14) in (5) yields the following expression for the SINR of $f_{\gamma_{\text{out}}}(\gamma)$

$$f_{\gamma_{\text{out}}}(\gamma)=\frac{2}{\bar{\gamma}_D}\frac{(k/\bar{\gamma}_I)^{\frac{L+k}{2}}}{\Gamma(L)\Gamma(k)}\underbrace{\int_0^{\infty}x^{\frac{L+k}{2}-1}(1+x)}_{I_2}$$
$$\underbrace{\times\exp\left[-\frac{(1+x)\gamma}{\bar{\gamma}_D}\right]K_{L-k}\left(2\sqrt{\frac{k}{\bar{\gamma}_I}}x^{1/2}\right)dx}_{I_2}. \tag{27}$$

After performing some straightforward mathematical manipulations and using [22, eq. (6.643/3)] a closed-form expression for the PDF of γ_{out} can be derived as

$$f_{\gamma_{\text{out}}}(\gamma)=\frac{1}{\bar{\gamma}_D}\left(\frac{k\bar{\gamma}_D}{\gamma\bar{\gamma}_I}\right)^{\frac{L+k-1}{2}}\exp\left(-\frac{\gamma}{\bar{\gamma}_D}\right)\exp\left(\frac{k\bar{\gamma}_D}{2\gamma\bar{\gamma}_I}\right)$$
$$\times\left[W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\bar{\gamma}_D}{\gamma\bar{\gamma}_I}\right)+\frac{Lk\bar{\gamma}_D}{\gamma}W_{-\frac{L+k+1}{2},\frac{L-k}{2}}\left(\frac{k\bar{\gamma}_D}{\gamma\bar{\gamma}_I}\right)\right] \tag{28}$$

where $W_{\lambda,\mu}(\cdot)$ is the Whittaker function [22, eq. (9.220/4)]. Substituting the exponential CDF and (14) in (18) and using [22, eq. (6.561/16)], i.e., $\int_0^{\infty}x^{\mu}K_v(\alpha x)dx=\frac{2^{\mu-1}}{\alpha^{\mu+1}}\Gamma\left(\frac{1+\mu+v}{2}\right)\Gamma\left(\frac{1+\mu-v}{2}\right)$ and [22, eq. (6.631/3)], yields the following closed-form expression for the CDF of γ_{out}

$$F_{\gamma_{\text{out}}}(\gamma)=1-\exp\left(-\frac{\gamma}{\bar{\gamma}_D}\right)\left(\frac{k\bar{\gamma}_D}{\bar{\gamma}_I\gamma}\right)^{\frac{L+k-1}{2}}$$
$$\times\exp\left(\frac{k\bar{\gamma}_D}{2\bar{\gamma}_I\gamma}\right)W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\bar{\gamma}_D}{\bar{\gamma}_I\gamma}\right). \tag{29}$$

Simplified Expressions for the SIR: The closed-form expression for the PDF and the CDF of γ_{out} are given in Table II.

V. SINR Statistics for SIMO: The Fully Correlated Case

In this section, focusing on the fully correlated shadowing case for the interfering signals, we consider a SD receiver and investigate two distinct selection techniques, namely the SNR-based and the SINR-based. We also assume that the receive antennas are sufficiently spaced, so that the L interfering signals received in any of them are totally independent from the ones received by any other antenna. In this context, new closed-form expressions are derived for important statistical metrics of the instantaneous output SINR of the two techniques under consideration. It is noted that the analytical framework presented here can also be applied to the i.n.d. as well as to i.i.d. interference scenarios. However, due to space limitations these results are not presented here. In [1], some preliminary results limited to the SNR-based SD scenario were presented.

A. SNR-Based Criterion

For the SNR-based SD criterion in the presence of AWGN and multiple interfering signals, the diversity receiver monitors the available diversity branches continuously and selects the branch with the largest instantaneous SNR for data detection. This SD technique requires the separation of the desired signal from the interfering signals, which can be practically achieved by using different pilot signals for each of them [4]. Mathematically speaking, the instantaneous system output SINR of this SIMO system can be expressed as

$$\gamma_{\text{SD}_{\text{out}}}=\frac{\gamma_{\text{SD}}}{1+\gamma_I} \tag{30}$$

where $\gamma_{\text{SD}}=\max\{\gamma_{D_1},\gamma_{D_2},\ldots,\gamma_{D_N}\}$ represents the instantaneous output SNR of the SD receiver, with γ_{D_n} denoting the SNR of the nth branch, following the PDF given by (2). The CDF of γ_{SD} for i.n.d. fading conditions,[5] is given by

$$F_{\gamma_{\text{SD}}}(x)=\prod_{n=1}^{N}F_{\gamma_{D_n}}(x). \tag{31}$$

For i.i.d. fading conditions, (31) simplifies to $F_{\gamma_{\text{SD}}}(x)=[F_{\gamma_D}(x)]^N$, with $F_{\gamma_D}(x)$ given in (19). Based on (A.4) and after some straightforward mathematical manipulations, $F_{\gamma_{\text{SD}}}(x)$ can be expressed as

$$F_{\gamma_{\text{SD}}}(x)=1+\sum_{\substack{n,n\\ \lambda_1,\ldots,\lambda_n}}^{N}\exp\left(-\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}x\right) \tag{32}$$

which for the case of i.i.d. fading conditions simplifies to

$$F_{\gamma_{\text{SD}}}(x)=\sum_{n=0}^{N}\binom{N}{n}(-1)^n\exp\left(-\frac{n}{\bar{\gamma}_D}x\right). \tag{33}$$

[5]For SNR-based SIMO, the i.n.d. and i.i.d. conditions refer to the fading statistics of the instantaneous SNRs of the desired signal at the branches of the receiver. For the interferers, fully correlated shadowing has been assumed.

TABLE III
SIMO SIR STATISTICS

Non Identical Fading	$f_{\gamma_{\mathrm{SD_{out}}}}(\gamma)=\sum_{\lambda_1,\ldots,\lambda_n}^{N}{}_{n,n}\ \frac{Lk}{\gamma}\left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right)^{\frac{L+k-1}{2}}\exp\left(\frac{k/\overline{\gamma}_I}{2\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right)W_{-\frac{L+k+1}{2},\frac{L-k}{2}}\left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right)$
	$F_{\gamma_{\mathrm{SD_{out}}}}(\gamma)=1+\sum_{\lambda_1,\ldots,\lambda_n}^{N}{}_{n,n}\left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right)^{\frac{L+k-1}{2}}\exp\left(\frac{k/\overline{\gamma}_I}{2\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right)W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right)$
	$M_{\gamma_{\mathrm{SD_{out}}}}(s)=\sum_{\lambda_1,\ldots,\lambda_n}^{N}{}_{n,n+1}\left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}}\right)^{\frac{L+k-1}{2}}\frac{s^{\frac{L+k-3}{2}}}{\Gamma(k)\Gamma(L)}\mathcal{G}^{1,3}_{3,1}\left(\frac{\overline{\gamma}_I\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}}{ks}\,\middle\vert\,\begin{matrix}\frac{L+k+1}{2},\frac{1-L+k}{2},\frac{1+L-k}{2}\\ \frac{L+k+1}{2}\end{matrix}\right)$
Identical Fading	$f_{\gamma_{\mathrm{SD_{out}}}}(\gamma)=Lk\sum_{n=1}^{N}\binom{N}{n}(-1)^{n+1}\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I n\gamma}\right)^{\frac{L+k-1}{2}}\exp\left(\frac{k\overline{\gamma}_D}{2\overline{\gamma}_I n\gamma}\right)\gamma^{-1}W_{\frac{-1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I n\gamma}\right)$
	$F_{\gamma_{\mathrm{SD_{out}}}}(\gamma)=1+\sum_{n=1}^{N}\binom{N}{n}(-1)^{n}\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I n\gamma}\right)^{\frac{L+k-1}{2}}\exp\left(\frac{k\overline{\gamma}_D}{2\overline{\gamma}_I n\gamma}\right)W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I n\gamma}\right)$
	$M_{\gamma_{\mathrm{SD_{out}}}}(s)=\sum_{n=1}^{N}\binom{N}{n}(-1)^{n+1}\left(\frac{k\overline{\gamma}_D}{n\overline{\gamma}_I}\right)^{\frac{L+k-1}{2}}\frac{s^{\frac{L+k-3}{2}}}{\Gamma(k)\Gamma(L)}\mathcal{G}^{1,3}_{3,1}\left(\frac{n\overline{\gamma}_I}{k\overline{\gamma}_D s}\,\middle\vert\,\begin{matrix}\frac{L+k+1}{2},\frac{k-L+1}{2},\frac{1+L-k}{2}\\ \frac{L+k+1}{2}\end{matrix}\right)$

Starting from (30) and substituting (32) and (14) in (18), integrals of the form I_2 appearing in (27), are obtained. Therefore following the procedure proposed in Section IV-C, the CDF of $\gamma_{\mathrm{SD_{out}}}$, can be obtained in closed form as

$$F_{\gamma_{\mathrm{SD}_{out}}}(\gamma)=1+\sum_{\substack{n,n\\ \lambda_1,\ldots,\lambda_n}}^{N}\left(\frac{k/\bar{\gamma}_I}{\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\gamma}\right)^{\frac{L+k-1}{2}}\exp\left(-\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\gamma\right)\times\exp\left(\frac{k/\bar{\gamma}_I}{2\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\gamma}\right)W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k/\bar{\gamma}_I}{\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\gamma}\right).\quad(34)$$

Its corresponding PDF is given by

$$f_{\gamma_{\mathrm{SD}_{out}}}(\gamma)=\sum_{\substack{n,n\\ \lambda_1,\ldots,\lambda_n}}^{N}\left(-\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\right)\exp\left(-\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\gamma\right)\times\left(\frac{k/\bar{\gamma}_I}{\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\gamma}\right)^{\frac{L+k-1}{2}}\exp\left(\frac{k/\bar{\gamma}_I}{2\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\gamma}\right)\times\left[W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k/\bar{\gamma}_I}{\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\gamma}\right)+\frac{Lk}{\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\gamma}W_{-\frac{L+k+1}{2},\frac{L-k}{2}}\left(\frac{k/\bar{\gamma}_I}{\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\gamma}\right)\right].\quad(35)$$

Considering i.i.d. fading conditions (34) simplifies to

$$F_{\gamma_{\mathrm{SD}_{out}}}(\gamma)=1-\sum_{n=1}^{N}\binom{N}{n}(-1)^{n+1}\exp\left(-\frac{n}{\bar{\gamma}_D}\gamma\right)\times\left(\frac{k\bar{\gamma}_D}{\bar{\gamma}_I n\gamma}\right)^{\frac{L+k-1}{2}}\exp\left(\frac{k\bar{\gamma}_D}{2\bar{\gamma}_I n\gamma}\right)W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\bar{\gamma}_D}{\bar{\gamma}_I n\gamma}\right)\quad(36)$$

whilst (35) simplifies to

$$f_{\gamma_{\mathrm{SD}_{out}}}(\gamma)=\sum_{n=1}^{N}\binom{N}{n}(-1)^{n}\left(-\frac{n}{\bar{\gamma}_D}\right)\exp\left(-\frac{n}{\bar{\gamma}_D}\gamma\right)\times\left(\frac{k\bar{\gamma}_D}{\bar{\gamma}_I n\gamma}\right)^{\frac{L+k-1}{2}}\exp\left(\frac{k\bar{\gamma}_D}{2\bar{\gamma}_I n\gamma}\right)\left\{W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\bar{\gamma}_D}{\bar{\gamma}_I n\gamma}\right)+Lk\left(\frac{n}{\bar{\gamma}_D}\gamma\right)^{-1}W_{\frac{-1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\bar{\gamma}_D}{\bar{\gamma}_I n\gamma}\right)\right\}.\quad(37)$$

Simplified Expressions for the SIR: For the SIR case, the instantaneous system output SIR can be expressed as

$$\gamma_{\mathrm{SD_{out}}}=\frac{\gamma_{\mathrm{SD}}}{\gamma_I}.\quad(38)$$

The simplified PDF and CDF expressions for both i.n.d. and i.i.d. cases are given in Table III.

B. SINR-Based Criterion

Under a SINR-based criterion, the diversity receiver selects the branch with the highest instantaneous SINR for coherent detection. This technique is more complex to be implemented, than the SNR-based one, since computationally demanding processing operations are required at the receiving end [4]. These include signal separation, SINR calculation per branch, statistical ordering of the resulting SINRs and SINR-based selection via a maximum selection criterion. For the SINR-based scenario the instantaneous SINR of the system output can be expressed as $\gamma_{\mathrm{SD_{out}}}=\max(\gamma_{\mathrm{out}_1},\gamma_{\mathrm{out}_2},\ldots,\gamma_{\mathrm{out}_N})$, where γ_{out_n} represents the instantaneous SINR of the nth branch, with PDF and CDF given by (28) and (29), respectively. Therefore, considering i.n.d. fading conditions[6] the CDF of the output SINR can be expressed as

$$F_{\gamma_{\mathrm{SD}_{out}}}(\gamma) = \left[\prod_{n=1}^{N} F_{\gamma_{\mathrm{out}_n}}(\gamma)\right]. \tag{39}$$

For the i.i.d. fading case, (39) simplifies to

$$F_{\gamma_{\mathrm{SD}_{out}}}(\gamma) = \left[F_{\gamma_{\mathrm{out}}}(\gamma)\right]^N. \tag{40}$$

The corresponding expressions for the PDFs, considering i.n.d. fading conditions, are

$$f_{\gamma_{\mathrm{SD}_{out}}}(\gamma) = \sum_{n=1}^{N} f_{\gamma_{\mathrm{out}_n}}(\gamma) \prod_{\substack{m=1\\ m\neq n}}^{N} F_{\gamma_{\mathrm{out}_m}}(\gamma). \tag{41}$$

For i.i.d. fading conditions (41) simplifies to

$$f_{\gamma_{\mathrm{SD}_{out}}}(\gamma) = N f_{\gamma_{\mathrm{out}}}(\gamma)\left[F_{\gamma_{\mathrm{out}}}(\gamma)\right]^{N-1}. \tag{42}$$

Simplified Expressions for the SIR: The CDF and PDF for the i.n.d. fading case can be obtained by substituting the PDF and the CDF expressions of the fully correlated case presented in Table III in (39) and (41), respectively. For i.i.d. fading, the same expressions should be substituted in (40) and in (42).

VI. Performance Analysis: The Fully Correlated Case

In this section, focusing also on the fully correlated case and using the previously derived expressions for the instantaneous output SINR of the SD receivers under consideration, important performance quality indicators are studied. In particular, the performance of both SNR-based and SINR-based SD receivers is evaluated using the OP and the ABEP criteria. In addition, in all cases an asymptotic analysis is included for the high SNR regime.

A. Outage Probability (OP)

The OP is defined as the probability that the SINR falls below a predetermined threshold γ_{th} and is given by $P_{\mathrm{out}} = F_{\gamma_{\mathrm{out}}}(\gamma_{\mathrm{th}})$.

1) SNR-Based Criterion: Considering the SNR-based criterion, the OP can be obtained by using (34) (for i.n.d. fading) or (36) (for i.i.d. fading). The corresponding CDF expression for the SIR case can be found in Table III.

High SNR Approximation: The exact results presented in the previous sections do not provide a clear physical insight of the system's performance. Therefore, here, we focus on the high SNR regime to obtain important system-design parameters such as the diversity gain (G_d) and the coding gain (G_c). Additionally, these analytic expressions help to quantify the amount of performance variations, which are due to the interfering effects as well as to the receiver's architecture. At high SNR the exponential CDF can be closely approximated by $F_{\gamma_D}(x) \approx \frac{x}{\bar{\gamma}_D}$ [39]. Based on this approximated expression, and assuming i.i.d. fading conditions (the i.n.d. case can be similarly analyzed) the CDF of γ_{SD} can be expressed as $F_{\gamma_{\mathrm{SD}}}(x) = [F_{\gamma_{\mathrm{D}}}(x)]^N$. Therefore following the procedure proposed in Section IV-C, and employing [22, eq. (6.561/16)], the OP of $\gamma_{\mathrm{SD}_{\mathrm{out}}}$, for high SNR, can be expressed as

$$F_{\gamma_{\mathrm{SD}_{out}}}(\gamma_{\mathrm{th}}) \approx \underbrace{\left[\sum_{n=0}^{N}\binom{N}{n}\frac{\Gamma(L+n)\Gamma(k+n)}{(k/\bar{\gamma}_I)^n\,\Gamma(L)\Gamma(k)}\right]\gamma_{\mathrm{th}}^N}_{\mathcal{D}_1}\bar{\gamma}_D^{-N}. \tag{43}$$

It is obvious that (43) is of the form $(G_c\bar{\gamma}_D)^{-G_d}$, where G_d represents the diversity gain, which as expected equals N, and $G_c = \mathcal{D}_1^{-1/N}$ is the coding gain [40], [41]. Therefore, the coding gain of the system is affected by the number of branches N, the average INR $\bar{\gamma}_I$, the outage threshold γ_{th}, the severity of the shadowing effects on the interfering channels, described by k, and the number of interferers L, with the first three parameters having the highest impact. In particular, the coding gain increases as $\bar{\gamma}_I$ and/or L and/or N and/or γ_{th} increase or k decreases. As far as N and γ_{th} are concerned, it is obvious that their increase will result to a higher coding gain. Regarding the shadowing conditions on the interfering signals, it can be shown from (43) that as they get worst, i.e., k decreases or $\bar{\gamma}_I$ increases, the coding gain increases. This is a reasonable result since worst channel conditions on the interfering signals lead to an increased SINR and thus to an improved coding gain.

2) SINR-Based Criterion: Considering the SINR-based criterion, the OP can be obtained by substituting (29) in (39) (for i.n.d. fading) or (40) (for i.i.d. fading). For the SIR case, the corresponding CDF expression, which should be substituted (39) (for i.n.d. fading) or (40) (for i.i.d. fading), can be found in Table II.

High SNR Approximation: For high values of the average SNR, by following a similar procedure as the one used for the SNR-based case, the OP can be closely approximated as

$$F_{\gamma_{\mathrm{SD}_{\mathrm{out}}}}(\gamma_{\mathrm{th}}) \approx \underbrace{(1+L\bar{\gamma}_I\gamma_{\mathrm{th}})^N}_{\mathcal{D}_2}\bar{\gamma}_D^{-N} \tag{44}$$

where $G_d = N$, $G_c = \mathcal{D}_2^{-1/N}$, while similar conclusions with the SNR-based case can be drawn for the diversity and coding gains. On the other hand, as it is also noted in [42], G_c is independent of the interfering signals parameter k. This does not apply to the SNR-based case due to its different mode of operation.

B. Average Bit Error Probability (ABEP)

The ABEP will be evaluated by using the MGF and CDF based approaches as described next.

1) SNR-Based Criterion: Considering i.n.d. fading conditions, substituting (35) in the definition of the MGF, using

[6]For the SINR-based criterion the i.n.d. and i.i.d. conditions refer to the fading statistics of the instantaneous SINRs γ_{out_n}.

[33, eq. (07.45.26.0005.01)], and employing [22, eq. (7.813/1)], yields the following expression for the MGF of $\gamma_{\mathrm{SD_{out}}}$

$$M_{\gamma_{\mathrm{SD}_{out}}}(s)=\sum_{\substack{n,n\\ \lambda_1,\ldots,\lambda_n}}^{N}\frac{\left(-\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\right)}{\Gamma(k)\Gamma(L)}\left[\frac{k\left(\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}+s\right)}{\bar{\gamma}_I\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}}\right]^{\frac{L+k-1}{2}}$$
$$\times\left\{\left(\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}+s\right)^{-1}\mathcal{G}^{1,3}_{3,1}\times\left(\frac{\frac{\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\bar{\gamma}_I}{k}}{\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}+s}\,\middle|\,\begin{matrix}\frac{L+k-1}{2},\frac{1-L+k}{2},\frac{1+L-k}{2}\\ \frac{L+k-1}{2}\end{matrix}\right)\right.$$
$$\left.+\left(\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\right)^{-1}\times\mathcal{G}^{1,3}_{3,1}\left(\frac{\frac{\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}\bar{\gamma}_I}{k}}{\mathcal{S}^n_{1,\bar{\gamma}_{D_{\lambda_m}}}+s}\,\middle|\,\begin{matrix}\frac{L+k+1}{2},\frac{1-L+k}{2},\frac{1+L-k}{2}\\ \frac{L+k+1}{2}\end{matrix}\right)\right\}. \tag{45}$$

For the i.i.d. fading case, based on (37), (45) simplifies to

$$M_{\gamma_{\mathrm{SD_{out}}}}(s)=\sum_{n=1}^{N}\binom{N}{n}(-1)^{n+1}\left[\frac{(n/\bar{\gamma}_D+s)k\bar{\gamma}_D}{n\bar{\gamma}_I}\right]^{\frac{L+k-1}{2}}$$
$$\times\frac{\left(1+\frac{s\bar{\gamma}_D}{n}\right)^{-1}}{\Gamma(k)\Gamma(L)}\left\{\mathcal{G}^{1,3}_{3,1}\left(\frac{n\bar{\gamma}_I/k}{n+s\bar{\gamma}_D}\,\middle|\,\begin{matrix}\frac{L+k-1}{2},\frac{k-L+1}{2},\frac{1+L-k}{2}\\ \frac{L+k-1}{2}\end{matrix}\right)\right.$$
$$\left.+\left(1+\frac{s\bar{\gamma}_D}{n}\right)\mathcal{G}^{1,3}_{3,1}\left(\frac{n\bar{\gamma}_I/k}{n+s\bar{\gamma}_D}\,\middle|\,\begin{matrix}\frac{L+k+1}{2},\frac{k-L+1}{2},\frac{1+L-k}{2}\\ \frac{L+k+1}{2}\end{matrix}\right)\right\} \tag{46}$$

which for the SISO system becomes

$$M_{\gamma_{\mathrm{out}}}(s)=\frac{(k\bar{\gamma}_D/\bar{\gamma}_I)^{\frac{L+k}{2}}}{\Gamma(L)\Gamma(k)\bar{\gamma}_D}\left\{\left(\frac{1}{\bar{\gamma}_D}+s\right)^{\frac{L+k}{2}-1}\right.$$
$$\times\mathcal{G}^{1,3}_{3,1}\left(\frac{\bar{\gamma}_I/(k\bar{\gamma}_D)}{1/\bar{\gamma}_D+s}\,\middle|\,\begin{matrix}\frac{L+k}{2},1-\frac{L-k}{2},1+\frac{L-k}{2}\\ \frac{L+k}{2}\end{matrix}\right)+\left(\frac{1}{\bar{\gamma}_D}+s\right)^{\frac{L+k}{2}}$$
$$\left.\times\bar{\gamma}_D\,\mathcal{G}^{1,3}_{3,1}\left(\frac{\bar{\gamma}_I/(k\bar{\gamma}_D)}{1/\bar{\gamma}_D+s}\,\middle|\,\begin{matrix}\frac{L+k}{2}+1,1-\frac{L-k}{2},1+\frac{L-k}{2}\\ \frac{L+k}{2}+1\end{matrix}\right)\right\}. \tag{47}$$

For the SIR and for both i.n.d. and i.i.d. fading conditions, MGF expressions for $\gamma_{\mathrm{SD_{out}}}$ are included in Table III (or Table II for the SISO system). Using the previously derived MGF expressions and following the MGF-based approach, the ABEP can be readily evaluated for a variety of modulation schemes [8]. More specifically, the ABEP can be calculated: *i*) directly for non-coherent differential binary phase shift keying (DBPSK), that is $P_{\mathrm{b}}^{DBPSK}=0.5M_{\gamma_{\mathrm{SD_{out}}}}(1)$; and *ii*) via numerical integration for Gray encoded M-PSK, that is $P_{\mathrm{b}}^{M-PSK}=\frac{1}{\pi\log_2 M}\int_0^{\pi-\pi/M}M_{\gamma_{\mathrm{SD_{out}}}}\left[\frac{\log_2 M\sin^2(\pi/M)}{\sin^2\phi}\right]d\phi$.

High SNR Approximation: To evaluate the ABEP performance at the high SNR regime, the CDF-based approach will be employed. Specifically, the ABEP can be evaluated as [43]

$$P_{\mathrm{b}}=\int_0^{\infty}-P_e'(\gamma)F_{\gamma_{\mathrm{SD_{out}}}}(\gamma)d\gamma. \tag{48}$$

where $-P_e'(\gamma)$ denotes the negative derivative of the conditional error probability. For example assuming DBPSK modulation $-P_e'(\gamma)=\alpha\beta\exp(-\beta\gamma)$, where $\alpha=1/2,\beta=1$ [43]. Substituting (43) in (48) and using [22, eq. (3.351/3)], i.e., $\int_0^{\infty}x^n\exp(-\mu x)dx=\frac{n!}{\mu^{n+1}}$, yields the following approximation for the ABEP

$$P_{\mathrm{b}}\approx\underbrace{\left[\sum_{n=0}^{N}\binom{N}{n}\frac{\alpha\Gamma(L+n)\Gamma(k+n)\Gamma(N+1)}{(k/\bar{\gamma}_I)^n\Gamma(L)\Gamma(k)\beta^N}\right]}_{\mathcal{D}_3}\bar{\gamma}_D^{-N} \tag{49}$$

and $G_d=N,G_c=\mathcal{D}_3^{-1/N}$. It should be noted that similar observations as the ones obtained for (43) can be also made for (49).

2) SINR-Based Criterion: To evaluate the ABEP performance for the SINR-based criterion the CDF-based approach is used. For the i.n.d. fading case, (39) is substituted in (48), whilst for i.i.d. fading (40) is employed. For both cases numerical integration techniques must be applied, using any of the well-known mathematical software packages, since a direct derivation in terms of closed forms is not possible. Similar to the SINR analysis, the ABEP for the SIR can be evaluated using the CDF expressions for the fully correlated case provided in Table III. Based on these expressions and substituting (39) (for the i.n.d. fading) as well as (40) (for the i.i.d. fading case) in (48) and employing numerical integration the ABEP can be readily evaluated.

High SNR Approximation: Substituting (44) in (48), the following closed-form approximated expression can be derived for the ABEP in the high-SNR regime

$$P_{\mathrm{b}}\approx\underbrace{\alpha\left(\frac{1+L\bar{\gamma}_I}{\beta}\right)^N\Gamma(N+1)}_{\mathcal{D}_4}\bar{\gamma}_D^{-N}. \tag{50}$$

From the last expression we get $G_d=N,G_c=\mathcal{D}_4^{-1/N}$.

VII. Numerical Results and Discussion

In this section various numerical performance evaluation results, which have been obtained using the previous analysis, will be presented. In particular, results related to SISO as well as SIMO systems, SNR- or SINR-based techniques and different $\mathcal{K}$-distributed i.i.d. fading and fully correlated shadowing conditions will be presented and discussed.

For obtaining Figs. 3 and 4, we have assumed $L=5,k=1.6,\bar{\gamma}_I=5$ dB, while we have used (36) and (40), respectively. In both these figures, the number of branches required for achieving a predefined target OP is plotted, for different values of the normalized outage threshold, $\gamma_{\mathrm{th}}/\bar{\gamma}_D$. In Fig. 3, considering SNR-based SD, it is shown that the number of branches increases as $\gamma_{\mathrm{th}}/\bar{\gamma}_D$ increases and/or the target OP decreases. Moreover, it is easily verified that for relatively low values of $\gamma_{\mathrm{th}}/\bar{\gamma}_D$ and/or high target OP, it is not necessary to employ SD, since even with SISO the target OP is achieved. This means that the overall power consumption of the receiver side

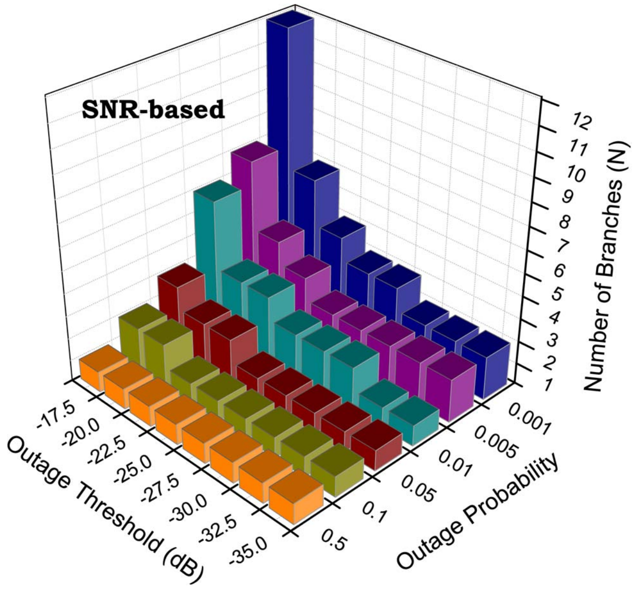

Fig. 3. The number of branches as a function of the OP and the normalized outage threshold for SNR-based SD reception.

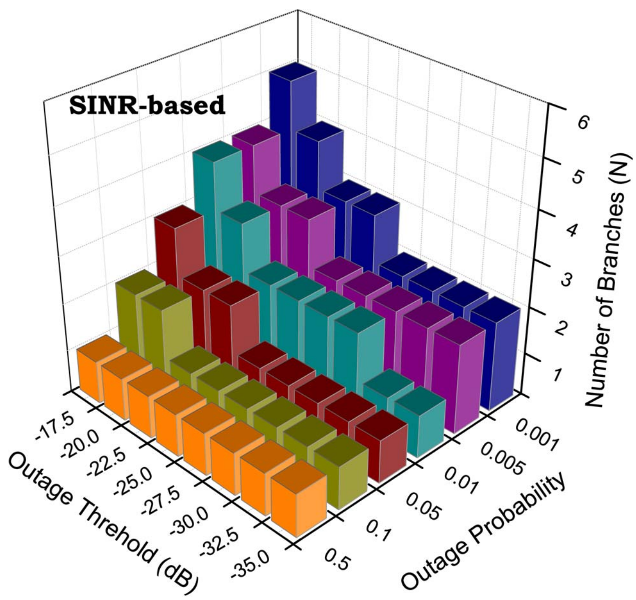

Fig. 4. The number of branches as a function of the OP and the normalized outage threshold for SINR-based SD reception.

can be reduced by avoiding unnecessary circuity and channel estimations. The same observations hold also for the SINR-based SD scenario, which is depicted in Fig. 4. By comparing Figs. 3 and 4, we observe that the SINR-based receiver requires considerably less reception branches than the SNR-based one for obtaining the same target OP. This gain, however, comes at the cost of much higher signal processing requirements for the SINR-based approach. Moreover, from these two figures it can be also verified that in most cases the predefined OP target is achieved by employing at most 2 or 3 branches. Therefore, both SNR-and SINR-based SD schemes represent excellent compromises as far as the performance and complexity (or energy efficiency) tradeoff is concerned.

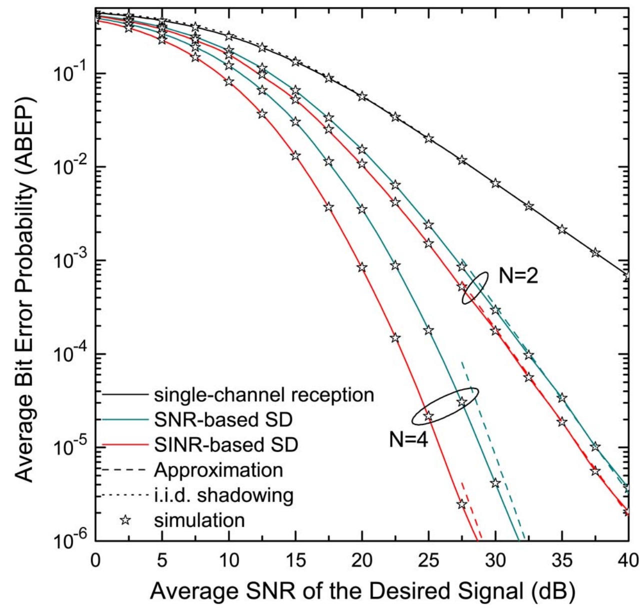

Fig. 5. The ABEP for SISO and SIMO SD receivers with SNR-based and SINR-based techniques, assuming DBPSK modulation.

Using (47), (46) and (48), the ABEP of DBPSK is plotted in Fig. 5, as a function of the average SNR of the desired signal, $\bar{\gamma}_D$, for SISO, SNR-based SD and SINR-based SD, respectively. In this figure the parameters are taken as $k = 2$, $\bar{\gamma}_I = 5$ dB and $L = 4$. As expected, the best performance is provided by the SINR-based receiver, while the performance gap between SINR and SNR-based SD increases as the number of diversity branches employed also increases. Additionally, in the same figure and for the higher SNR values, i.e., > 25 dB, the corresponding curves for the SNR-and-SINR-based cases are plotted, using (49) and (50), respectively. In both cases the asymptotic expressions for the ABEP are quite close to the exact ones, especially for higher values of average input SNR and/or small number of branches. Finally, assuming i.i.d. (and not fully correlated) shadowing conditions for the different interfering paths, the corresponding performance of a SISO system is also plotted. It is shown that the ABEP of the i.i.d. case is always slightly worst, verifying thus that the fully correlated shadowing represents the worst case of shadowing.

In Fig. 6, assuming $\bar{\gamma}_I = 5$ dB and $L = 4$, the OP is plotted as a function of the normalized outage threshold under two communication scenarios, namely SISO system and SINR-based SD, for various values of N. We observe that the performance improves as the number of branches increases, with a decreased rate of improvement though. A worth mentioning observation that comes out of this figure is that as the interfering signals shadowing parameter k increases, the OP decreases. This is a reasonable result since severe shadowing conditions in the interfering signals result to a lower INR and thus to a higher SINR. Additionally, to better understand how interference affects system's performance, an interference limited communication scenario has been considered by entirely neglecting noise effects. Therefore, in the same figure, the corresponding OP performance for the SIR case has been depicted, using the CDF expression given in Table III. In all cases, when

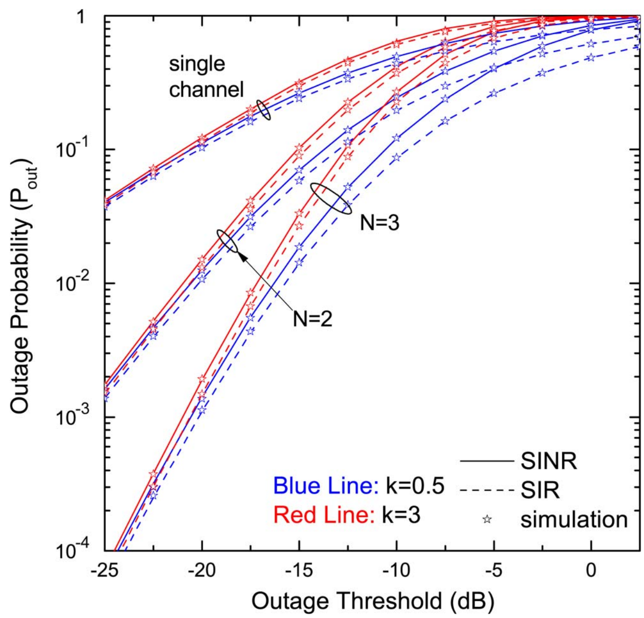

Fig. 6. The OP for SISO as well as SINR-based SD SIMO system.

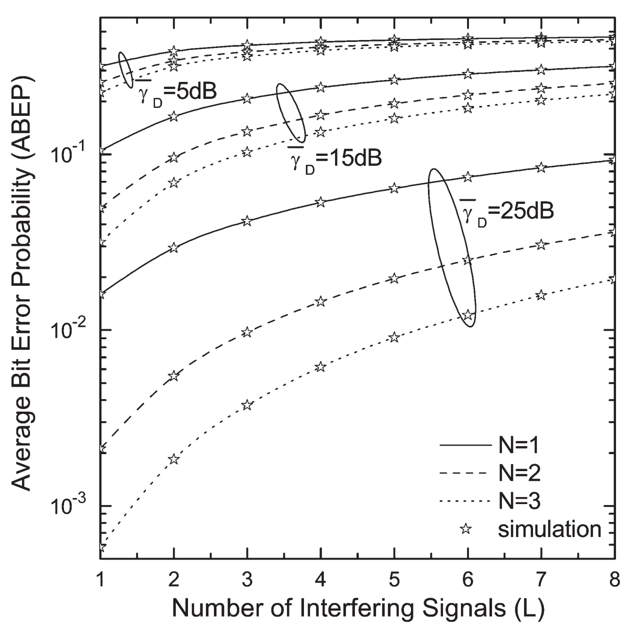

Fig. 7. The ABEP for SNR-based SD reception versus the number of interfering signals, assuming DBPSK modulation.

noise is not present (SIR case) the performance shows an improvement, as expected. An interesting observation though is that the performance gap between the SINR and SIR scenarios, mainly depends on the number of diversity branches employed. Specifically, the noise effects seem to play a more important role when SD reception is used with an increased number of antennas.

In Fig. 7, considering SNR-based SD and assuming $\bar\gamma_I = 10$ dB, $k = 2$, the ABEP is plotted as a function of the number of interfering signals L, for various values of the number of branches N and different average SNRs of the desired signal $\bar\gamma_D$. We observe that as $\bar\gamma_D$ and/or N increase the ABEP decreases, whilst in all cases the performance worsens with the increase of L. It is interesting to note that for higher values of $\bar\gamma_D$, the line gap of the performances obtained using different values of N increases. For comparison purposes, computer simulation performance results are also included in Figs. 5–7, verifying in all cases the validity of the proposed theoretical approach.

VIII. Conclusion

In this paper, an analytical framework for evaluating important statistical metrics of the instantaneous output SINR of SISO as well as SIMO diversity receivers operating over composite fading channels has been presented. The proposed analysis is based on convenient expressions that have been extracted for the PDF and CDF of the sum of $\mathcal{K}$-distributed RVs, assuming identical, non-identical as well as fully-correlated distributed parameters. Focusing on the latter case, various statistical characteristics of SISO, SNR-based SD and SINR-based SD receivers are derived in closed form, which are then used to study system performance in terms of ABEP and OP. An asymptotic high SNR analysis has been also presented based on which the diversity and coding gain expressions are studied. The obtained results indicate that the combination of fading/shadowing and interference disrupts seriously the performance of the system. Furthermore, it is shown that a power efficient solution that can considerably improve this situation is the employment of SD reception with a relatively small number of diversity branches.

Appendix A
Proof of Theorem 1

The moments generating function (MGF) of $\gamma_I \triangleq \sum_{i=1}^{L}\gamma_{I_i}$ can be expressed as [11]

$$\mathcal{M}_{\gamma_{I_i}}(s)=\left(\frac{k_i}{\bar\gamma_{I_i}s}\right)^{k_i}\exp\left(\frac{k_i}{\bar\gamma_{I_i}s}\right)\Gamma\left(1-k_i,\frac{k_i}{\bar\gamma_{I_i}s}\right) \tag{A.1}$$

where $\Gamma(\alpha,x)=\int_x^{\infty}\exp(-t)t^{\alpha-1}dt$ is the upper incomplete Gamma function [22, eq. (8.350/2)]. The MGF of γ_I, for i.n.d. interference conditions, is given by

$$\mathcal{M}_{\gamma_I}(s)=\prod_{i=1}^{L}\mathcal{M}_{\gamma_{I_i}}(s). \tag{A.2}$$

Therefore, assuming non integer values for k_i, substituting (A.1) in (A.2) and using the infinite series representation for the incomplete gamma function, i.e., [22, eq. (8.354/2)], yields

$$\mathcal{M}_{\gamma_I}(s)=\mathcal{G}_1\left(\frac{1}{s}\right)^{\mathcal{S}^L_{k_i,1}}\exp\left[\left(\mathcal{S}^L_{k_i,\bar\gamma_{I_i}}\right)\frac{1}{s}\right]\left[\prod_{i=1}^{L}(1-\hat t_{h,i})\right] \tag{A.3}$$

where $\hat t_{h,i}=\sum\limits_{h=0}^{\infty}t_{h,i}$. Furthermore, since

$$\prod_{i=1}^{L}(1-\hat t_{h,i})=1+\sum_{\substack{i,i\\ \lambda_1,\ldots,\lambda_i}}^{L}\prod_{n=1}^{i}\hat t_{h,\lambda_n} \tag{A.4}$$

where $\sum_{\substack{x,y \\ \lambda_1,\ldots,\lambda_x}}^{z} = \sum_{x=1}^{z} (-1)^y \sum_{\lambda_1=1}^{z-x+1} \sum_{\lambda_2=\lambda_1+1}^{z-x+2} \cdots \sum_{\lambda_x=\lambda_{x-1}+1}^{z}$, (A.3) can be rewritten as

$$\mathcal{M}_{\gamma_I}(s) = \left[\prod_{i=1}^{L} \left(\frac{k_i}{\bar{\gamma}_{I_i}}\right)^{k_i} \Gamma(1-k_i)\right] \left(\frac{1}{s}\right)^{S^L_{k_i,1}} \times \exp\left(\frac{S^L_{k_i,\bar{\gamma}_{I_i}}}{s}\right) \left(1 + \sum_{\substack{i,i \\ \lambda_1,\ldots,\lambda_i}}^{L} \prod_{n=1}^{i} \hat{t}_{h,\lambda_n}\right). \quad \text{(A.5)}$$

Capitalizing on the power series identity $\sum_{k=0}^{\infty} \alpha_k x^k \sum_{k=0}^{\infty} \beta_k x^k = \sum_{k=0}^{\infty} c_k x^k$, where $c_h = \sum_{k=0}^{h} \alpha_k \beta_{h-k}$ given in [22, eq. (0.316)], (A.3) can be simplified as

$$\mathcal{M}_{\gamma_I}(s) = \left[\prod_{i=1}^{L} \left(\frac{k_i}{\bar{\gamma}_{I_i}}\right)^{k_i} \Gamma(1-k_i)\right] \left(\frac{1}{s}\right)^{S^L_{k_i,1}} \times \exp\left(\frac{S^L_{k_i,\bar{\gamma}_{I_i}}}{s}\right) \left(1 + \sum_{\substack{i,i \\ \lambda_1,\ldots,\lambda_i}}^{L} \sum_{h=0}^{\infty} \mathcal{G}_3\right). \quad \text{(A.6)}$$

The MGF expression presented in (A.6) is in a convenient form for applying the inverse Laplace transform given in [44, eq. (29.3.81)], leading to (7), which completes the proof.

Appendix B
Proof of Theorem 2

Considering i.i.d. interference conditions the MGF function of γ_I can be expressed as

$$\mathcal{M}_{\gamma_I}(s) = \left[\mathcal{M}_{\gamma_{I_i}}(s)\right]^L. \quad \text{(B.1)}$$

Assuming non integer values for k, substituting (A.1) in this equation, with $\bar{\gamma}_{I_i} = \bar{\gamma}_I$, using also the infinite series representation for the incomplete gamma function, i.e., [22, eq. (8.354/2)] and employing the binomial identity, (B.1) can be written as

$$\mathcal{M}_{\gamma_I}(s) = \left(\frac{k}{\bar{\gamma}_I s}\right)^{Lk} \exp\left(\frac{Lk}{\bar{\gamma}_I s}\right) \sum_{i=0}^{L} \binom{L}{i} (-1)^i \times \Gamma(1-k)^{L-i} \left[\left(\frac{k}{\bar{\gamma}_I s}\right)^{1-k} \sum_{h=0}^{\infty} \frac{(-1)^h}{h!(1-k+h)} \left(\frac{k}{\bar{\gamma}_I s}\right)^h\right]^i. \quad \text{(B.2)}$$

Using the useful power series identity provided in [22, eq. (0.314)], i.e., $\left(\sum_{q=0}^{\infty} \alpha_q x^q\right)^h = \sum_{q=0}^{\infty} c_q x^q$, with $c_0 = \alpha_0^h$, $c_m = \frac{1}{m\alpha_0} \sum_{q=1}^{m} (qh - m + q) \alpha_q c_{m-q}$ for $m \geq 1$, a simplified expression for (B.2) can be derived as

$$\mathcal{M}_{\gamma_I}(s) = \exp\left(\frac{Lk}{\bar{\gamma}_I s}\right) \sum_{i=0}^{L} \binom{L}{i} \Gamma(1-k)^{L-i} (-1)^i \times \left[\sum_{h=0}^{\infty} c_h \left(\frac{k}{\bar{\gamma}_I s}\right)^{\mathcal{G}_4}\right]. \quad \text{(B.3)}$$

Based on the convenient expression derived in (B.3), the inverse Laplace transform given in [44, eq. (29.3.81)] can be applied, and thus (9) is derived, which completes the proof.

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