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# Frequency Domain Channel Estimation for Cooperative Communication Networks

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***Abstract*—In this correspondence, we deal with the problem of channel estimation in amplify-and-forward (AF) wideband cooperative relay-based networks. Two types of frequency domain channel estimation techniques are proposed and analyzed. First, a training based technique is presented for which an optimal pilot placement and power allocation strategy is described. Second, hybrid techniques are introduced in which both training as well as channel output correlation information is utilized for channel estimation. A theoretical performance study of the proposed algorithms is presented and closed-form expressions for the mean squared channel estimation error are provided. The presented theoretical analysis is verified via extensive Monte Carlo simulations.**

***Index Terms*—Amplify-and-forward, channel estimation, cooperative networks, relays.**

## I. INTRODUCTION

Various time, frequency, and spatial diversity techniques, applied in multiantenna links over multiple-input multiple-output (MIMO) channels have been recently proposed to enhance transmission reliability and system performance [1]. The application of MIMO technology to mobile networks, however, often faces the practical implementation problem of packing many antennas on a small mobile terminal. In an attempt to overcome such a severe limitation, cooperative communication schemes, that exploit spatial diversity using relay nodes, have been proposed recently [2]–[5]. In most of the proposed cooperative transmission protocols, the relays either amplify and forward (AF) or decode, encode and forward (DF) the received signal. To enjoy all benefits offered by a relay network, accurate channel state information is required at the destination (for AF) or both the relay and destination (for DF). The channel estimation problem has been studied for AF networks in [6]–[9] and for DF schemes in [10] and [11]. Especially for AF relaying there exist two choices: either the source-relay $(S \rightarrow R)$ and relay-destination $(R \rightarrow D)$ channels are estimated separately or the overall source-relay-destination $(S \rightarrow R \rightarrow D)$ channel is estimated at the destination. In this work we adopt the second approach, which, as can be easily shown, offers higher spectral efficiency and lower implementation complexity.

In this paper, we present efficient channel estimation techniques for AF wideband relay networks. The new methods are implemented in the frequency domain (FD), to exploit the diagonal structure of the channel matrices, which is due to the block, single carrier, cyclic prefixed (CP) transmission. We adopt the time-division multple-access transmission protocol I, described in [4], which, as shown in [5], offers the optimal diversity/multiplexing tradeoff among the AF, single-relay,

Manuscript received December 16, 2009; accepted February 08, 2010. Date of publication March 11, 2010; date of current version May 14, 2010. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Ta-Sung Lee. This work was supported in part by the FP6 project Cooperative and Opportunistic Communications in Wireless Networks (COOPCOM, FP6- 033533) and the 03ED838 (PENED) research project funded by the Greek Secretariat of Research and Technology.

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Digital Object Identifier 10.1109/TSP.2010.2045413

half-duplex protocols. Initially, a training based (TB) technique is presented and issues such as: 1) the positioning of the pilot symbols; and 2) the power allocated in each phase of transmission are investigated. Then, more efficient channel estimation techniques, which are based not only on pilot transmission but also on system's output information (hybrid channel estimation), are developed. The proposed hybrid methods rely on a short training sequence transmitted in the $S \rightarrow D$ link only, while the $S \rightarrow R \rightarrow D$ link is estimated blindly in the FD up to multiple phase ambiguities. These phase ambiguities depend on the $S \rightarrow D$ channel frequency response (FR) and can thus be resolved based on the training assigned to the $S \rightarrow D$ link. A theoretical performance analysis in terms of the achievable MSE at high signal-to-noise ratio (SNR) conditions is also presented and verified through computer simulations.

The paper is organized as follows. In Section II, the adopted AF transmission model and protocol are presented. The proposed TB and hybrid channel estimation algorithms are described in Sections III and IV. A theoretical analysis of the proposed algorithms in terms of the MSE is presented in Section V. Simulation results are presented in Section VI and conclusions are summarized in Section VII.

*Notation:* We use $\mathbf{A}(k,l)$, $\mathbf{A}(k,:)$, $\mathbf{A}(:,l)$ to denote the $(k,l)$th element, the $k$th row and the $l$th column of matrix $\mathbf{A}$. With $\mathbf{A}(k)$ we denote either the $(k,k)$th element of matrix $\mathbf{A}$ or the $k$th element of vector $\mathbf{A}$. The expectation and trace operators are denoted by $E[\cdot]$ and $tr()$, respectively. $diag[\mathbf{x}]$ stands for a diagonal matrix that contains the elements of vector $\mathbf{x}$ on its diagonal. $\mathbf{F}$ represents the $M \times M$ FFT matrix whose $(k,l)$th element is given by $\mathbf{F}(k,l) = 1/\sqrt{M} e^{-j2\pi(k-1)(l-1)/M}$ with $1 \leq k, l \leq M$.

## II. Transmission Model

We consider a single relay network, where data are transmitted from the source terminal $S$ to the destination terminal $D$ with the assistance of the relay terminal $R$. All terminals are equipped with single antenna transmitters and receivers. In this paper, we propose and analyze efficient frequency domain techniques for estimating both the $S \rightarrow D$ and the overall $S \rightarrow R \rightarrow D$ channels. The $S \rightarrow D$, $S \rightarrow R$ and $R \rightarrow D$ channels are assumed to be frequency selective and static during the period of transmission. Their impulse responses (IRs) are defined as $\mathbf{h}_{SD} = [h_{SD}(0), \ldots, h_{SD}(L_{SD}-1)]^T$, $\mathbf{h}_{SR} = [h_{SR}(0), \ldots, h_{SR}(L_{SR}-1)]^T$ and $\mathbf{h}_{RD} = [h_{RD}(0), \ldots, h_{RD}(L_{RD}-1)]^T$, respectively. According to protocol I of [4], the transmission is accomplished in frames consisting of two time slots. During the first time slot the source terminal transmits to the relay and destination terminals. At the second time slot, both the relay and source terminals transmit to the destination terminal. Single carrier (SC) block transmission is considered, where at each time slot symbols are transmitted in blocks of length $M$. The blocks of symbols transmitted by the source at the two time slots are denoted by the $M \times 1$ vectors $\mathbf{x}_1, \mathbf{x}_2$. The average power per symbol of the signal transmitted from the source equals to $\sigma_x^2$.

To eliminate interblock interference during the first time slot, a cyclic prefix (CP) of length $l$ is appended to the block of data, with $l > \max(L_{SD}, L_{SR})$. This CP is discarded at the destination and the relay. Based on the above transmission model, the received signal vectors at the destination and the relay at the first time-slot, are expressed as follows:

$$\mathbf{y}_1 = \mathbf{H}_{SD}\mathbf{x}_1 + \mathbf{w}_1^d, \quad \mathbf{r}_1 = \mathbf{H}_{SR}\mathbf{x}_1 + \mathbf{w}_1^r \tag{1}$$

where $\mathbf{H}_{SD}, \mathbf{H}_{SR}$ are $M \times M$ circulant matrices with entries $\mathbf{H}_{SD}(k,l) = \mathbf{h}_{SD}((k-l)\, mod M)$ and $\mathbf{H}_{SR}(k,l) = \mathbf{h}_{SR}((k-l)\, mod M)$ respectively and $\mathbf{w}_1^d, \mathbf{w}_1^r$ are the additive white Gaussian noise vectors at the source and the destination with zero mean and covariance matrices $\sigma_w^2 \mathbf{I}_M$ and $\sigma_r^2 \mathbf{I}_M$ respectively, with $\mathbf{I}_M$ being the $M \times M$ identity matrix. The relay terminal normalizes the received vector by a factor $a = \sqrt{E\left[\|\mathbf{r}_1\|^2\right]/M\sigma_x^2}$ (so that the average energy per symbol is $\sigma_x^2$). During the second time slot the relay adds a CP of length $l' > \max(L_{SD}, L_{RD})$ and retransmits the signal, while the source transmits $\mathbf{x}_2$ appended with a CP of length $l'$. The destination receives a superposition of the source and relay transmissions and discards the CP. The received signal can be expressed as follows:

$$\mathbf{y}_2 = \mathbf{H}_{SD}\mathbf{x}_2 + \mathbf{H}_R\mathbf{x}_1 + \tilde{\mathbf{w}}_2^d \tag{2}$$

where $\tilde{\mathbf{w}}_2^d = a\mathbf{H}_{RD}\mathbf{w}_1^r + \mathbf{w}_2^d$ and $\mathbf{H}_R \equiv a\mathbf{H}_{SR}\mathbf{H}_{RD} = a\mathbf{H}_{RD}\mathbf{H}_{SR}$. $\mathbf{H}_{RD}$ is an $M \times M$ circulant matrix defined similarly to $\mathbf{H}_{SD}, \mathbf{H}_{SR}$ and $\mathbf{w}_2^d$ is the additive white Gaussian noise at the destination during the second time slot. $\mathbf{H}_R$ is a circulant matrix having $\mathbf{h}_R$ as its first column, where $\mathbf{h}_R$ is the convolution of the zero-padded vectors $\mathbf{h}_{SR}$ and $\mathbf{h}_{RD}$. Due to their circulant structure, matrices $\mathbf{H}_{SD}$ and $\mathbf{H}_R$ can be diagonalized by applying the discrete Fourier transform (DFT) matrix operator $\mathbf{F}$. Premultiplying (1), (2), by $\mathbf{F}$ yields

$$\begin{aligned} \boldsymbol{\mathcal{Y}}_1 &= \boldsymbol{\Lambda}_{SD}\boldsymbol{\mathcal{X}}_1 + \boldsymbol{\mathcal{W}}_1^d, \\ \boldsymbol{\mathcal{Y}}_2 &= \boldsymbol{\Lambda}_{SD}\boldsymbol{\mathcal{X}}_2 + \boldsymbol{\Lambda}_R\boldsymbol{\mathcal{X}}_1 + \tilde{\boldsymbol{\mathcal{W}}}_2^d \end{aligned} \tag{3}$$

where $\boldsymbol{\mathcal{Y}}_j = \mathbf{F}\mathbf{y}_j$, $\boldsymbol{\mathcal{X}}_j = \mathbf{F}\mathbf{x}_j$, $j = 1, 2$, $\boldsymbol{\mathcal{W}}_1^d = \mathbf{F}\mathbf{w}_1^d$, $\tilde{\boldsymbol{\mathcal{W}}}_2^d = \mathbf{F}\tilde{\mathbf{w}}_2^d$, and $\boldsymbol{\Lambda}_{SD}, \boldsymbol{\Lambda}_R$ are diagonal matrices that contain the frequency response (FR) of the $S \rightarrow D$ and $S \rightarrow R \rightarrow D$ channels in $M$ frequency tones. It can be easily shown that $\tilde{\boldsymbol{\mathcal{W}}}_2^d$ in (3) has zero mean and its covariance matrix is expressed as

$$\begin{aligned} \mathbf{C} &= E\left[\tilde{\boldsymbol{\mathcal{W}}}_2^d \tilde{\boldsymbol{\mathcal{W}}}_2^{dH}\right] \\ &= |a|^2\sigma_r^2 diag\left[|\boldsymbol{\Lambda}_{RD}(1)|^2, |\boldsymbol{\Lambda}_{RD}(2)|^2 \cdots |\boldsymbol{\Lambda}_{RD}(M)|^2\right] + \sigma_w^2\mathbf{I}_M \end{aligned} \tag{4}$$

where $\boldsymbol{\Lambda}_{RD}(k)$ stands for the FR response of the $R \rightarrow D$ channel at the $k$th frequency tone. That is, $\mathbf{C}$ depends on the FR of the instantaneous $R \rightarrow D$ channel, which is unknown. To overcome this problem, an approximation of $\mathbf{C}$ can be defined by resorting to a stochastic assumption for the $R \rightarrow D$ channel IR. More specifically, by assuming that $\mathbf{h}_{RD}(j)$ is complex normal $(CN)$ with zero mean and variance $\sigma_j^2$, $j = 1, \ldots, L_{RD}-1$, it can be shown that $\boldsymbol{\Lambda}_{RD}(k) \sim CN\left(0, \sigma_{RD}^2\right)$ with $\sigma_{RD}^2 = \sum_{j=1}^{L_{RD}-1} \sigma_j^2$, $\forall k$. Under this assumption, $\mathbf{C}$ can be written as

$$E\left[\tilde{\boldsymbol{\mathcal{W}}}_2^d \tilde{\boldsymbol{\mathcal{W}}}_2^{dH}\right] \approx (|a|^2\sigma_r^2\sigma_{RD}^2 + \sigma_w^2)\mathbf{I}_M. \tag{5}$$

Based on this approximation for the noise covariance matrix, an optimal least squares (LS) training based (TB) method is described in the next section. Two hybrid techniques[1] are also presented in Section IV. The proposed methods are developed completely in the frequency domain (FD) by exploiting the diagonal structure of the involved channel matrices.

## III. TB Channel Estimation

For TB channel estimation, a number of pilot tones must be suitably placed in the transmitted blocks. Here, we adopt the frequency expanding approach [12], in which pilot tones are inserted between blocks of information data in the FD. Such an approach offers improved channel estimation performance, at the cost of a reduced spectral efficiency. Let $L = \max(L_{SD}, L_{SR} + L_{RD} - 1)$. It should be noted that

[1]Hybrid channel estimation techniques are based on both training and channels' output information. We have chosen to use the term 'hybrid' instead of 'semi-blind' since in the latter case, the two estimation problems (i.e., training and blind) are combined into a single one, which is not the case here.

$L$ pilot tones would be sufficient for estimating each of the $S \longrightarrow D$ and $S \longrightarrow R \longrightarrow D$ channel IRs separately [13]. Assuming that $M/L$ is an integer and the covariance matrix of $\tilde{\boldsymbol{\mathcal{W}}}_2^d$ is given by (5), the following theorem can be proven.

*Theorem 1:* Under a certain constraint with respect to the power devoted to training at the source, $L$ pilot tones are sufficient for estimating both $\mathbf{h}_{SD}$ and $\mathbf{h}_R$. To minimize the mean squared channel estimation error (MSE) of the LS estimator the following assignment of data and pilots on available frequencies should be applied:

$$\mathbf{x}_1 = \mathbf{F}^H \mathbf{P} \begin{bmatrix} \boldsymbol{\mathcal{T}} \\ \boldsymbol{\mathcal{Z}}_1 \end{bmatrix}, \quad \mathbf{x}_2 = \mathbf{F}^H \mathbf{P} \begin{bmatrix} \mathbf{0}_{L\times 1} \\ \boldsymbol{\mathcal{Z}}_2 \end{bmatrix} \tag{6}$$

where $\boldsymbol{\mathcal{T}}$ is a $L \times 1$ vector containing equipowered pilot tones and the $(M-L) \times 1$ vectors $\boldsymbol{\mathcal{Z}}_1, \boldsymbol{\mathcal{Z}}_2$ correspond to the FD representation of the information data transmitted at first and second time slots. Finally, $\mathbf{P}$ is a permutation matrix that places the elements of $\boldsymbol{\mathcal{T}}$ in equispaced positions $\{k_1, \ldots, k_L\}$, where $k_l = k_{l-1} + M/L$, and the FD data in-between the pilots.

Estimation accuracy can be improved by placing pilots in more than one frames. Suppose that training has been placed in $Q$ consecutive frames and assume that all channels in the network do not change during this period. Furthermore, pilots in all frames are placed in the same equispaced positions $k_1, \ldots, k_L$. Let $\boldsymbol{\mathcal{T}}_Q$ be a $Q \times L$ matrix containing the overall training. Similarly, we define the $Q \times L$ matrices $\boldsymbol{\mathcal{Y}}_{1Q}, \boldsymbol{\mathcal{Y}}_{2Q}$ to denote the frequency domain outputs of the two slots in all frames. Their $(m,\ n)$th element is the output signal at the $k_m$ frequency position of the $n$th frame. Matrices $\boldsymbol{\mathcal{X}}_{1Q}, \boldsymbol{\mathcal{X}}_{2Q}, \boldsymbol{\mathcal{W}}_{1Q}^d, \tilde{\boldsymbol{\mathcal{W}}}_{2Q}^d$ are defined accordingly. Then from (3), we get

$$\begin{aligned} \boldsymbol{\mathcal{Y}}_{1Q}(:, k_i) &= \boldsymbol{\Lambda}_{SD}(k_i)\boldsymbol{\mathcal{T}}_Q(:, i) + \boldsymbol{\mathcal{W}}_{1Q}^d(:, k_i) \\ \boldsymbol{\mathcal{Y}}_{2Q}(:, k_i) &= \boldsymbol{\Lambda}_R(k_i)\boldsymbol{\mathcal{T}}_Q(:, i) + \tilde{\boldsymbol{\mathcal{W}}}_{2Q}^d(:, k_i) \end{aligned} \tag{7}$$

and the $S \longrightarrow D$, $S \longrightarrow R \longrightarrow D$ channels can be estimated using LS criterion as follows:

$$\begin{aligned} \hat{\boldsymbol{\Lambda}}_{SD}(k_i) &= \frac{\boldsymbol{\mathcal{T}}_Q^H(:, i)\boldsymbol{\mathcal{Y}}_{1Q}(:, k_i)}{\|\boldsymbol{\mathcal{T}}_Q(:, i)\|^2} \\ &= \boldsymbol{\Lambda}_{SD}(k_i) + \frac{\boldsymbol{\mathcal{T}}_Q^H(:, i)\boldsymbol{\mathcal{W}}_{1Q}^d(:, k_i)}{\|\boldsymbol{\mathcal{T}}_Q(:, i)\|^2} \end{aligned} \tag{8}$$

$$\begin{aligned} \hat{\boldsymbol{\Lambda}}_R(k_i) &= \frac{\boldsymbol{\mathcal{T}}_Q^H(:, i)\boldsymbol{\mathcal{Y}}_{2Q}(:, k_i)}{\|\boldsymbol{\mathcal{T}}_Q(:, i)\|^2} \\ &= \boldsymbol{\Lambda}_R(k_i) + \frac{\boldsymbol{\mathcal{T}}_Q^H(:, i)\tilde{\boldsymbol{\mathcal{W}}}_{2Q}^d(:, k_i)}{\|\boldsymbol{\mathcal{T}}_Q(:, i)\|^2}. \end{aligned} \tag{9}$$

Having the FR of the $S \longrightarrow D$ and $S \longrightarrow R \longrightarrow D$ channels in $L$ frequency tones, their IRs can be obtained as

$$\hat{\mathbf{h}}_{SD} = \frac{1}{\sqrt{M}} \boldsymbol{F}_L^{-1} \hat{\boldsymbol{\Lambda}}_{SD}^L, \quad \hat{\mathbf{h}}_R = \frac{1}{\sqrt{M}} \boldsymbol{F}_L^{-1} \hat{\boldsymbol{\Lambda}}_R^L \tag{10}$$

where $\boldsymbol{F}_L$ is the $L \times L$ submatrix of $\boldsymbol{F}$ formed by the rows corresponding to the indices $k_i, i = 1, 2, \ldots, L$ and its first $L$ columns and the vectors $\hat{\boldsymbol{\Lambda}}_{SD}^L$ and $\hat{\boldsymbol{\Lambda}}_R^L$ contain the FR of the $S \longrightarrow D$ and $S \longrightarrow R \longrightarrow D$ channels at the positions $\{k_1, \ldots, k_L\}$.

## IV. Hybrid Channel Estimation Techniques

In this section, it is shown that by making use of pilots exclusively on the $S \longrightarrow D$ link we can estimate the $S \longrightarrow R \longrightarrow D$ channel IR blindly, by exploiting the special structure of the received block autocorrelation matrix. From (1), (2), it can be easily shown that the received signals at the $k$th frequency carrier may be expressed as follows:

$$\underbrace{\begin{bmatrix} \boldsymbol{\mathcal{Y}}_1(k) \\ \boldsymbol{\mathcal{Y}}_2(k) \end{bmatrix}}_{\boldsymbol{Y}(k)} = \underbrace{\begin{bmatrix} \boldsymbol{\Lambda}_{SD}(k) & 0 \\ \boldsymbol{\Lambda}_R(k) & \boldsymbol{\Lambda}_{SD}(k) \end{bmatrix}}_{\boldsymbol{\Lambda}(k)} \underbrace{\begin{bmatrix} \boldsymbol{\mathcal{X}}_1(k) \\ \boldsymbol{\mathcal{X}}_2(k) \end{bmatrix}}_{\boldsymbol{X}(k)} + \underbrace{\begin{bmatrix} \boldsymbol{\mathcal{W}}_1^d(k) \\ \tilde{\boldsymbol{\mathcal{W}}}_2^d(k) \end{bmatrix}}_{\boldsymbol{W}^d(k)}. \tag{11}$$

Notice that the $2 \times 2$ matrix $\boldsymbol{\Lambda}(k)$, which is to be estimated, possesses a lower triangular structure. Assuming complex, zero-mean, and uncorrelated input and noise signals, the autocorrelation matrix of $\boldsymbol{Y}(k)$ is expressed as

$$\boldsymbol{\Phi}(k) = E\left[\boldsymbol{Y}(k)\boldsymbol{Y}^H(k)\right] = \sigma_x^2 \boldsymbol{\Lambda}(k)\boldsymbol{\Lambda}^H(k) + \begin{bmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_{\tilde{w}}^2(k) \end{bmatrix} \tag{12}$$

where $\sigma_{\tilde{w}}^2(k) = \mathbf{C}(k) = |a|^2 |\boldsymbol{\Lambda}_{RD}(k)|^2 \sigma_r^2 + \sigma_w^2$. Ignoring the noise terms and defining $\boldsymbol{\mathcal{R}}_{1,1}(k) = E\left[|\boldsymbol{\mathcal{Y}}_1(k)|^2\right]$ and $\boldsymbol{\mathcal{R}}_{2,1}(k) = E\left[\boldsymbol{\mathcal{Y}}_2(k)\boldsymbol{\mathcal{Y}}_1^*(k)\right]$, it is straightforward to show using (12), that the FRs of the channels at the $k$th tone can be estimated as

$$\hat{\boldsymbol{\Lambda}}_{SD}(k) = \frac{e^{j\phi_{SD}(k)}}{\sigma_x} \boldsymbol{\mathcal{R}}_{1,1}^{1/2}(k) \tag{13}$$

$$\begin{aligned} \hat{\boldsymbol{\Lambda}}_R(k) &= \frac{e^{j\phi_{SD}(k)}}{\sigma_x} \boldsymbol{\mathcal{R}}_{1,1}^{-1/2}(k)\boldsymbol{\mathcal{R}}_{2,1}(k) \\ &= \frac{\boldsymbol{\mathcal{R}}_{2,1}(k)}{\sigma_x^2 \hat{\boldsymbol{\Lambda}}_{SD}^*(k)} \end{aligned} \tag{14}$$

where $\phi_{SD}(k)$ is the phase of the $k$th frequency tone of the $S \longrightarrow D$ FR. We observe from the last equations that the FR of the $S \longrightarrow R \longrightarrow D$ channel can be estimated based on the system output autocorrelation matrix and the knowledge of the $S \longrightarrow D$ channel in the frequency domain. Notice further that, the phase information of the $S \longrightarrow D$ channel in the frequency domain would be sufficient for estimating all channels in the network, even in case that more relays are engaged sequentially.

Using a number of $L$ pilots, the FR of the $S \longrightarrow D$ channel in $L$ equispaced positions can be estimated and the unknown phases may be resolved. Note however, that the amplitudes of the $S \longrightarrow D$ FR may also be computed directly from the elements of the correlation matrix $\boldsymbol{\Phi}(k)$ in (12). Thus, a proper combination of these techniques may be preferable in some cases. A more detailed description of the two alternatives for estimating the $S \longrightarrow D$ FR is given here.

### A. Direct Estimation of the $S \longrightarrow D$ FR

Under a certain power constraint for training at the source, it can be shown from [13] that $L$ equispaced and equipowered pilots can be used to optimally estimate in the LS sense the $S \longrightarrow D$ channel. These pilots can be transmitted as follows:

$$\mathbf{x}_1 = \mathbf{F}^H \mathbf{P} \begin{bmatrix} \boldsymbol{\mathcal{T}} \\ \boldsymbol{\mathcal{Z}}_1 \end{bmatrix}, \quad \mathbf{x}_2 = \mathbf{F}^H \boldsymbol{\mathcal{X}}_2 \tag{15}$$

where $\boldsymbol{\mathcal{T}}$ is the $L \times 1$ vector of equipowered pilot tones transmitted at the first time slot, $\boldsymbol{\mathcal{Z}}_1$ is an $(M-L) \times 1$ information data vector in the FD and $\mathbf{P}$ a permutation matrix that places the pilots in equispaced positions in the FD. Alternatively, the same pilots can be placed in equispaced positions in the second time slot. However, to be able to achieve the same MSE, the relay should transmit nulls at the frequency positions in which the source transmits pilots, in order to avoid interference. Since such an approach decreases spectral efficiency, we adopt the scheme described in (15). Note that for $Q$ training frames, $\boldsymbol{\Lambda}_{SD}(k_i), \quad i = 1, 2, \ldots, L$ can be estimated similarly to (8).

### B. Hybrid Estimation of the $S \longrightarrow D$ Frequency Response

As an alternative to TB estimation of Section IV-A, a hybrid method can be followed that uses both information from the pilots and the estimated correlation quantities, according to (13). In practice, with $K$ output blocks available, the correlation term $\mathcal{R}_{1,1}(k_i)$ is estimated based on the sample average

$$\hat{\mathcal{R}}_{1,1}(k_i) = \frac{1}{K}\sum_{n=1}^{K}|\mathcal{Y}_{1K}(n,k_i)|^2 \tag{16}$$

which can be computed recursively in time. Matrix $\mathcal{Y}_{1K}$ is defined similarly to $\mathcal{Y}_{1Q}$.

As aforementioned, in cases where $Q$ is small and $K$ large, it may be preferable to impose the amplitude computed as the square root of $\hat{\mathcal{R}}_{1,1}(k_i)$, given in (16), to the training based estimate of (8). This amplitude refinement may be mathematically expressed as

$$\hat{\hat{\mathbf{\Lambda}}}_{SD}(k_i) = \frac{\hat{\mathbf{\Lambda}}_{SD}(k_i)}{\sigma_x\left|\hat{\mathbf{\Lambda}}_{SD}(k_i)\right|}\sqrt{\hat{\mathcal{R}}_{1,1}(k_i)}. \tag{17}$$

Having estimated the $S \longrightarrow D$ FR in $L$ positions, $\mathbf{h}_{SD}$ can be obtained from (10). Then, the FR of the $S \longrightarrow R \longrightarrow D$ channel at any position can be computed from (14). In these expressions the following sample average can be used to estimate $\mathcal{R}_{2,1}(k_i)$

$$\hat{\mathcal{R}}_{2,1}(k_i) = \frac{1}{K}\sum_{n=1}^{K}\mathcal{Y}_{2K}(n,k_i)\mathcal{Y}_{1K}^*(n,k_i). \tag{18}$$

Finally, it is straightforward from (10) to estimate $\mathbf{h}_R$.

To summarize, it has been shown that the transmission of $L$ pilots in the $S \longrightarrow D$ link, is sufficient for estimating all channel IRs of the relay network.

## V. Performance Study

In this section the performance of the proposed channel estimation schemes is studied at high SNR conditions and in terms of the attained MSE between the actual and estimated channel IRs. Before proceeding to the analysis, it should be noted that the time-domain MSE can be expressed as follows[2]:

$$E\left[\left\|\mathbf{h}_{SD}-\hat{\mathbf{h}}_{SD}\right\|^2\right] = \frac{1}{L}\sum_{i=1}^{L}E\left[\left|\mathbf{\Lambda}_{SD}(k_i)-\hat{\mathbf{\Lambda}}_{SD}(k_i)\right|^2\right] \tag{19}$$

$$E\left[\left\|\mathbf{h}_{R}-\hat{\mathbf{h}}_{R}\right\|^2\right] = \frac{1}{L}\sum_{i=1}^{L}E\left[\left|\mathbf{\Lambda}_{R}(k_i)-\hat{\mathbf{\Lambda}}_{R}(k_i)\right|^2\right] \tag{20}$$

### A. Theoretical MSE of the Training Based Scheme

For training based channel estimation with $Q$ training blocks, the MSEs of the FR estimates at frequency $k_i$ are given from (8), (4), (10) by

$$E\left[\left|\mathbf{\Lambda}_{SD}(k_i)-\hat{\mathbf{\Lambda}}_{SD}(k_i)\right|^2\right] = \frac{\sigma_w^2}{Q\sigma_x^2} \tag{21}$$

$$E\left[\left|\mathbf{\Lambda}_{R}(k_i)-\hat{\mathbf{\Lambda}}_{R}(k_i)\right|^2\right] = \frac{|a|^2\sigma_r^2\left|\mathbf{\Lambda}_{RD}(k_i)\right|^2+\sigma_w^2}{Q\sigma_x^2} \tag{22}$$

[2]Note that for $M/L$ integer $\boldsymbol{F}_L^H\boldsymbol{F}_L = \boldsymbol{F}_L\boldsymbol{F}_L^H = L/M\mathbf{I}_L$.

where $\sigma_x^2$ is the average power assigned to each pilot tone. By inspecting (19)–(22) we get

$$E\left[\left\|\mathbf{h}_{SD}-\hat{\mathbf{h}}_{SD}\right\|^2\right] = \frac{\sigma_w^2}{Q\sigma_x^2} \tag{23}$$

$$E\left[\left\|\mathbf{h}_{R}-\hat{\mathbf{h}}_{R}\right\|^2\right] = \sum_{i=1}^{L}\frac{|a|^2\sigma_r^2\left|\mathbf{\Lambda}_{RD}(k_i)\right|^2+\sigma_w^2}{Q\sigma_x^2} \tag{24}$$

### B. Theoretical MSE of the Hybrid $S \longrightarrow D$ IR Estimate

In the rest of this section, we assume that we have already estimated the $S \longrightarrow D$ channel via training, i.e., Section IV-A. To simplify derivations, the analysis is carried out at high SNR conditions at the destination (i.e., $\sigma_w^2 \longrightarrow 0$). By inspecting (21), it is reasonable to assume that a TB technique would lead to perfect channel knowledge of the $S \longrightarrow D$ link. Thus, imposing the amplitude estimated from the correlation matrix (17) would result in a performance degradation in terms of the MSE. More specifically, the MSE between the actual and estimated $S \longrightarrow D$ channel IR will be given by

$$\begin{aligned} &E\left[\left|\mathbf{\Lambda}_{SD}(k_i)-\hat{\mathbf{\Lambda}}_{SD}(k_i)\right|^2\right] \\ &= E\left[\left|\mathbf{\Lambda}_{SD}(k_i)-\frac{\mathbf{\Lambda}_{SD}(k_i)}{\sigma_x\left|\mathbf{\Lambda}_{SD}(k_i)\right|}\hat{\mathcal{R}}_{1,1}^{1/2}(k_i)\right|^2\right] \\ &= \frac{1}{\sigma_x^2}E\left[\left|\sigma_x\left|\mathbf{\Lambda}_{SD}(k_i)\right|-\hat{\mathcal{R}}_{1,1}^{1/2}(k_i)\right|^2\right]. \end{aligned} \tag{25}$$

Under high SNR conditions at the destination ($\sigma_w^2 \longrightarrow 0$), we get

$$E\left[\left|\mathbf{\Lambda}_{SD}(k_i)-\hat{\mathbf{\Lambda}}_{SD}(k_i)\right|^2\right] = \frac{1}{\sigma_x^2}E\left[\left|\mathcal{R}_{1,1}^{1/2}(k_i)-\hat{\mathcal{R}}_{1,1}^{1/2}(k_i)\right|^2\right]. \tag{26}$$

For a large DFT size $M$, the input sequence can be considered as a complex Gaussian random variable (RV). Based on this assumption, it can be shown that $\hat{\mathcal{R}}_{1,1}^{1/2}(k_i)$ follows (with a very good approximation) a normal distribution with $E\left[\hat{\mathcal{R}}_{1,1}^{1/2}(k_i)\right] \approx \mathcal{R}_{1,1}^{1/2}(k_i)$ and

$$E\left[\left|\mathcal{R}_{1,1}^{1/2}(k_i)-\hat{\mathcal{R}}_{1,1}^{1/2}(k_i)\right|^2\right] = \frac{\sigma_x^2\left|\mathbf{\Lambda}_{SD}(k_i)\right|^2}{4K}+\frac{\sigma_w^4}{4K}. \tag{27}$$

Combining (26), (27) and (19) we get for high SNR

$$E\left[\left\|\mathbf{h}_{SD}-\hat{\mathbf{h}}_{SD}\right\|^2\right] \approx \frac{1}{L}\sum_{i=1}^{L}\frac{\left|\mathbf{\Lambda}_{SD}(k_i)\right|^2}{4K}. \tag{28}$$

Thus, imposing the amplitudes estimated from the output autocorrelation to the TB estimate of the $S \longrightarrow D$ channel IR results in a performance degradation at high SNR conditions. More specifically, we see from (28) that the MSE exhibits an error floor for high SNR, which depends on the number of output blocks $K$. However, this approach may be superior compared to the purely TB method at low SNRs and for a small number of training blocks.

### C. Theoretical MSE of the $S \longrightarrow R \longrightarrow D$ IR Estimate

To evaluate the performance of estimating the $S \longrightarrow R \longrightarrow D$ IR we assume again that the $S \longrightarrow D$ channel has been perfectly estimated via training. Then, from (14) the MSE for a frequency carrier $k_i$ will be expressed as follows:

$$\begin{aligned} E\left[\left|\mathbf{\Lambda}_R(k_i)-\hat{\mathbf{\Lambda}}_R(k_i)\right|^2\right] &= E\left[\left|\mathbf{\Lambda}_R(k_i)-\frac{\hat{\boldsymbol{\mathcal{R}}}_{2,1}(k_i)}{\sigma_x^2\mathbf{\Lambda}_{SD}^*(k_i)}\right|^2\right] \\ &= \frac{E\left[\left|\boldsymbol{\mathcal{R}}_{2,1}(k_i)-\hat{\boldsymbol{\mathcal{R}}}_{2,1}(k_i)\right|^2\right]}{\sigma_x^4\left|\mathbf{\Lambda}_{SD}(k_i)\right|^2}. \end{aligned} \tag{29}$$

By assuming that the input sequences are complex Gaussian distributed and using results for quadratic forms of normal RVs from [14], it can be shown that $E[\hat{\boldsymbol{\mathcal{R}}}_{2,1}(k_i)]=\boldsymbol{\mathcal{R}}_{2,1}(k_i)$ and

$$\begin{aligned} &E\left[\left|\boldsymbol{\mathcal{R}}_{2,1}(k_i)-\hat{\boldsymbol{\mathcal{R}}}_{2,1}(k_i)\right|^2\right]\approx \\ &\left(\sigma_x^2\left|\mathbf{\Lambda}_{SD}(k_i)\right|^2+\sigma_w^2\right)\left(\sigma_x^2\left|\mathbf{\Lambda}_{SD}(k_i)\right|^2+\sigma_x^2\left|\mathbf{\Lambda}_R(k_i)\right|^2+\sigma_{\tilde{w}}^2(k_i)\right) \end{aligned} \tag{30}$$

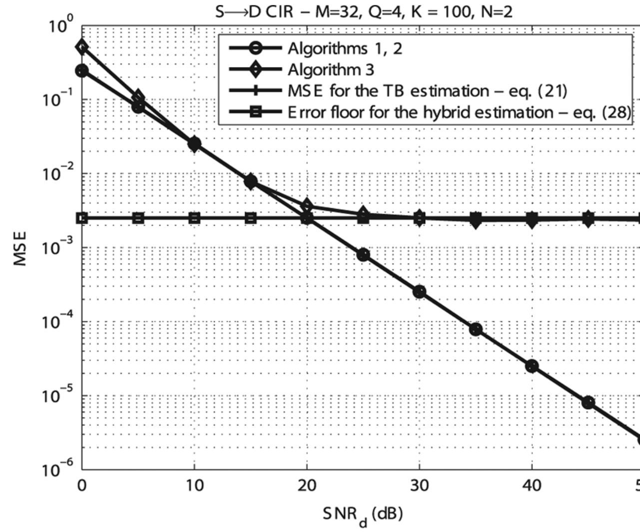
where $\sigma_{\tilde{w}}^2(k_i)=|a|^2\left|\mathbf{\Lambda}_{RD}(k_i)\right|^2\sigma_r^2+\sigma_w^2$. From (20), (29), and (30) we get for $\sigma_w^2\to 0$

$$\begin{aligned} &E\left[\left\|\mathbf{h}_R-\hat{\mathbf{h}}_R\right\|^2\right]\approx \\ &\qquad \frac{1}{L}\sum_{i=1}^{L}\frac{\left|\mathbf{\Lambda}_{SD}(k_i)\right|^2+\left|\mathbf{\Lambda}_R(k_i)\right|^2+|a|^2\left|\mathbf{\Lambda}_{RD}(k_i)\right|^2\sigma_r^2}{K} \end{aligned} \tag{31}$$

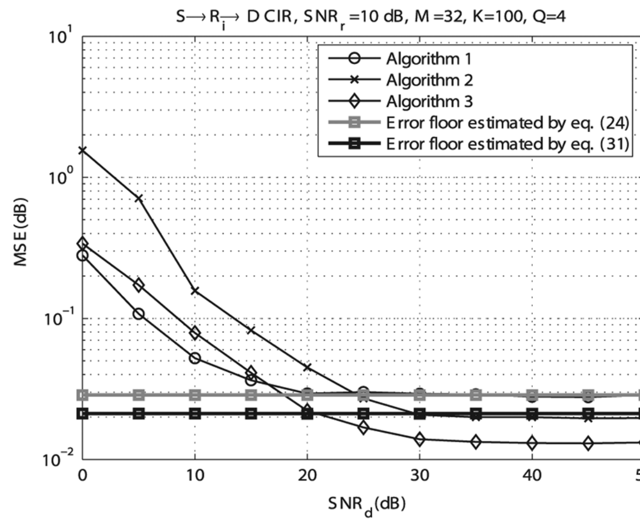
## VI. Simulation Results

The techniques described in the previous sections can be extended to networks with more than one relays $R_i$, $i=1,\ldots,N$, in which the transmission protocol presented in [5] is used. According to this protocol the source transmits to all the relays in a sequential manner. Communication with each relay (and the destination) is done based on protocol I. Note that for the hybrid techniques described in Section IV, transmission of $L$ pilots exclusively in the $S\longrightarrow D$ link is sufficient for estimating all channels ($S\longrightarrow R_i\longrightarrow D$) in the network.

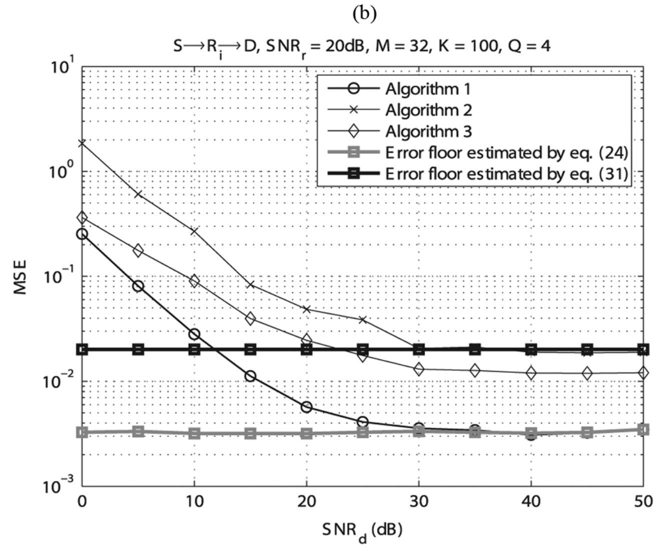
The performance of the presented techniques was evaluated through computer simulations. We consider 2 relays cooperating with the source according to the protocol aforedescribed. The $S\longrightarrow D$, $S\longrightarrow R_i$, $R_i\longrightarrow D$, ($i=1,2$) links are modeled as frequency selective channels with memory lengths $L_{SD}=L_{SR_i}=L_{R_iD}=4$, $i=1,2$. All channel taps are assumed to be Rayleigh fading and the transmission is done in blocks of $M=32$ QPSK symbols. The power profile has been considered to be uniform and thus each channel tap is modeled as a circular symmetric Gaussian RV of zero mean and variance equal to $1/4$. The average transmitted energy per symbol $\sigma_x^2$ has been selected to be equal to one. The SNR at each relay and the destination was defined as $SNR_r=\sigma_x^2/\sigma_r^2$ and $SNR_d=\sigma_x^2/\sigma_w^2$, respectively. The proposed algorithms, i.e., algorithm 1 (8)–(10), algorithm 2 (8), (10), (14), and (10), and algorithm 3 (8), (17), (10), (14), and (10), were evaluated in terms of the MSE between actual and estimated channel IRs, as shown in Fig. 1. In Fig. 1(a) the MSE between the actual and estimated $S\longrightarrow D$ channel IR along with the theoretical expression given in (21) are plotted for $Q=4$ training blocks and for $SNR_r=10$ dB. Eight equispaced pilots have been placed within each training block. Note that algorithms 1 and 2 are identical in terms of $S\longrightarrow D$ channel IR estimation. The theoretical result for training based estimation (21) coincides with the experimental one for algorithm 1, even for a small number of training blocks. Furthermore, the hybrid scheme exhibits a floor, as $SNR_d$ increases,




(a)




(b)




(c)

Fig. 1. (a) MSE of the $S\longrightarrow D$ IR. (b) MSE of the $S\longrightarrow R_i\longrightarrow D$ IR, versus SNR at the destination. (c) MSE of the $S\longrightarrow R_i\longrightarrow D$ channel IR versus SNR at the destination.

which is a function of the number of relays in the network and the number of the blocks used for estimating the correlation quantities.

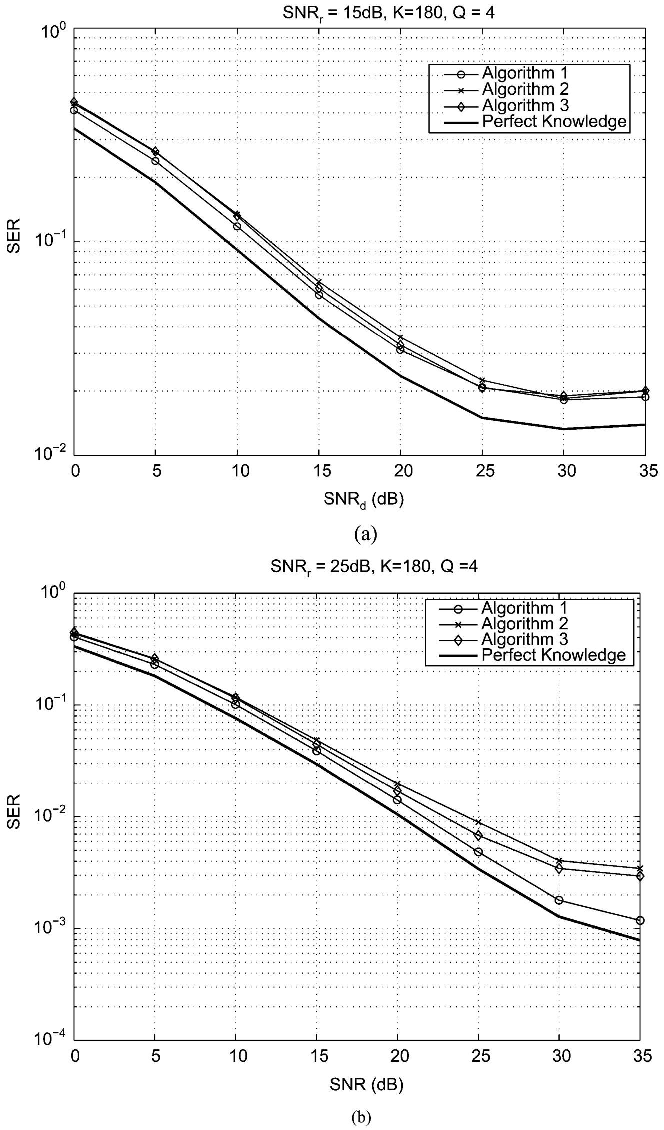



Fig. 2. (a) SER versus SNR at the destination $(SNR_r = 15 \text{ dB})$. (b) SER versus SNR at the destination $(SNR_r = 25 \text{ dB})$.

Again, theoretical (28) and experimental curves coincide. The MSE between the actual and estimated $S \rightarrow R \rightarrow D$ channel IRs averaged over all relays, along with the theoretical expressions given in (24) and (31) are plotted in Fig. 1(b) and (c) for two different $SNR_r$s. By inspecting (24) we observe that the TB estimation of the $S \rightarrow R_i \rightarrow D$ channel exhibits a floor as $SNR_d$ increases, which depends on the noise variance at the relay. This is also confirmed by simulations. Moreover, according to the theoretical analysis, the MSE of algorithms 2,3 is expected to be the same at high SNR conditions and unaffected from the $SNR_r$. In practice, there is a slight difference between these two schemes. This is due to the fact that the assumption of perfect $S \rightarrow D$ channel knowledge is not adequately satisfied for algorithm 3, since the hybrid estimation of $S \rightarrow D$ channel exhibits a floor in MSE as $SNR_d$ increases.

The performance of the proposed methods in terms of the symbol error rate (SER) is shown in Fig. 2 for two different SNRs at the relay. A linear MMSE equalizer has been used at the destination followed by maximum likelihood detection. The performance of perfect channel knowledge is also provided as a benchmark. From the figure we see that the proposed methods have similar SER performance, which is very close to that of a scheme having perfect channel knowledge.

## VII. Conclusion

Efficient channel estimation techniques for wideband cooperative systems operating in AF mode have been presented. We initially studied pilot-based channel estimation for relay-assisted cyclic prefixed block transmissions. The minimum MSE pilot placement scheme under a certain power constraint at the source has been derived. In addition, it has been shown that all channels from the source to the destination can be obtained from the elements of the channel output autocorrelation matrix in the frequency domain. Thus, by using a few pilot symbols in the $S \rightarrow D$ link all channels of the network $(S \rightarrow R_i \rightarrow D)$ can be efficiently estimated. The proposed methods have been analyzed theoretically at high SNR conditions and their performance has been verified via simulations.

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