An Improved Threshold-Based Channel Selection Scheme for Wireless Communication Systems

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***Abstract*—In several wireless communication systems, low complexity channel selection schemes are required, as power efficient mechanisms to improve performance. In this context, the best selection scheme, in terms of performance, comes at the cost of increased complexity, since it necessitates a continuous monitoring of all available channels. In this paper, a new threshold-based channel selection strategy is proposed, which decreases the system complexity, without considerably affecting the system performance. Assuming independent but nonidentically distributed channel conditions, a generic analytical framework is first presented, based on the Markov chain theory. Then, the proposed selection scheme is applied to three specific communication scenarios, namely multichannel reception, transmit antenna selection with diversity reception, and cooperative relay selection. In all three cases, closed-form results are obtained and used to analyze the performance of the systems under consideration. It is shown that based on the proposed scheme, computational complexity is reduced and thus important energy savings can be achieved, without a significant loss in performance.**

***Index Terms*—Complexity and performance trade-off, Markov chain theory, relay selection, switch diversity, transmit antenna selection, threshold-based selection scheme.**

I. Introduction

BOTH multiple-input-multiple-output (MIMO) and cooperative diversity technologies, have promised to considerably improve spectral and power efficiency in contemporary wireless communication systems. This has motivated an increased research interest for these technologies, as well as their adoption in the 3rd Generation Partnership Project (3GPP) Long Term Evolution (LTE) and LTE-advanced (LTE-A) network architectures [1]. The main idea behind both of them is the reception of the same data signals over multiple independent channels, either in a single hop manner, e.g., in diversity reception [2], or under a cooperative scenario, e.g., [3]. However, in these systems there exists a trade-off between performance and complexity, since, depending on the employed channel selection scheme, a different amount of channel-state information may be required, entailing a different level of hardware complexity, while also leading to different performance [4].

Regarding low complexity diversity reception, the most well-known schemes include conventional selection diversity (SD), as well as channel selection strategies that are based on predefined thresholds, such as switch-and-stay (SS) and switch-and-examine (SE) [2]. However, both SS and SE do not take full advantage of the available channel estimates, and thus their performance is inferior compared to that offered by SD. Based on this observation, in [5], an alternative approach to SE was proposed, with improved performance and a relatively small increase in complexity. More specifically, the difference between classical SE and combining with postselection based SE (SECps), proposed in [5], is in their last step where SE keeps the last examined channel, while SECps selects the “strongest” channel. However, in [5], only independent and identically distributed (i.i.d.) statistics have been assumed, despite the fact that the assumption of i.i.d. fading is, in many cases, unrealistic for real radio propagation conditions. The same switching approach was also studied in [6], assuming dual-branch reception and correlated-but non identically distributed-Nakagami-m fading. Moreover, similar assumptions have been also made in a different communication scenario, where a transmit antenna selection (TAS) scheme was utilized [7]–[9]. In particular in [9], it was shown that the combination of TAS with SECps (TAS/SECps) is able to achieve similar average output signal to noise ratio (ASNR) and bit error rate (ABER) performance as compared with TAS/SD, when the predetermined SNR threshold is optimized, while it requires a significant less channel estimates.

In the context of cooperative networks employing multiple relays (or antennas), the clear benefits of the best relay selection (BRS) algorithm in terms of the diversity gain over other techniques have been highlighted in the past, e.g., [10], [11]. Since then, many efforts have been devoted to improve the performance of relay (or antenna) selection mechanism or reducing the induced complexity, e.g., [12]–[20]. In [14], a distributed SE scheme combined with threshold-based relay selection was proposed. The main idea in the scheme presented in [14] is that the relay satisfying the predefined threshold is chosen instead of

Manuscript received March 23, 2015; revised July 25, 2015; accepted October 2, 2015. Date of publication October 19, 2015; date of current version February 8, 2016. This work was supported by the European Union and Greek National Funds through the Operational Program Education and Lifelong Learning of the National Strategic Reference Framework (NSRF)—Research Funding Programs: Thales and Aristeia HSI-MARS. A preliminary version of part of this work was presented at IEEE VTC, Fall 2014. The work of G. K. Karagiannidis has been co-financed by EU and the Greek national funds through the Operational Program “Education and Lifelong Learning” of the NSRF—Research Funding Program: THALES-NTUA MIMOSA. The associate editor coordinating the review of this paper and approving it for publication was W. Su.

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Digital Object Identifier 10.1109/TWC.2015.2492538

that offering the best performance. In [18], the performance of an amplify-and-forward (AF) system with two low-complexity relay-selection schemes was investigated. These schemes are mainly based on the SE and SECps techniques, in which a single relay out of multiple relays is selected to forward the source message to the destination. A common observation in all study cases where threshold-based selection schemes are employed, i.e., SE or SECps, is that the performance gap between the best selection strategy and SE (or SECps) turns out to be large, especially for cases where the switching threshold is relatively low. Moreover, for SECps, an independent but non identically distributed (i.n.d.) analysis is not available, limiting thus its applicability to very specific communication scenarios.

Motivated by these observations, in this paper a new threshold-based channel selection strategy is proposed. In the proposed scheme, the system first examines whether the received value of the previously selected channel is larger than a predefined threshold. If this is the case, this channel is kept, otherwise the system switches to the channel that provides the highest value, similar to the best channel selection strategy. Besides the introduction of a new selection scheme, the contribution of this work includes, in summary, the following

- a generic analysis is presented, where i.n.d. channel conditions are taken into consideration for the first time in similar systems. First, it is shown that the proposed scheme can be represented by a finite-state Markov chain and can thus be analyzed by employing the well-established Markov chain theory. To this end, closed-form general expressions are derived for the transition and stationary probabilities of the Markov chain;
- based on these expressions, generic statistical characteristics of the new selection scheme's output are derived, also in closed form, such as the probability density function (PDF) and the cumulative distribution function (CDF);
- the proposed approach is then applied to three distinct communication scenarios, namely multichannel reception, TAS with diversity reception and cooperative relay selection;
- in all cases, the analytical tractability of the proposed approach leads to the derivation of closed-form expressions for important performance metrics of the considered systems, such as the outage probability (OP), ABER, channel capacity and ASNR;
- high SNR as well as complexity analyses are also provided, while diversity and coding gain issues are also addressed.

As a general remark it is noted that the performance of the proposed scheme approximates more closely the performance of best channel selection, its complexity is comparable to that of SECps, while, in addition, the assumption of i.n.d. channel conditions, adopted in this work, extends the applicability of threshold-based selections schemes.

The remainder of this paper is organized as follows. A general description and analysis of the new channel selection strategy is presented in Section II. Based on it, the proposed approach is applied in three different communication scenarios, namely diversity reception (in Section III), TAS with diversity reception (in Section IV) and relay selection (in Section V). Finally, concluding remarks are given in Section VI.

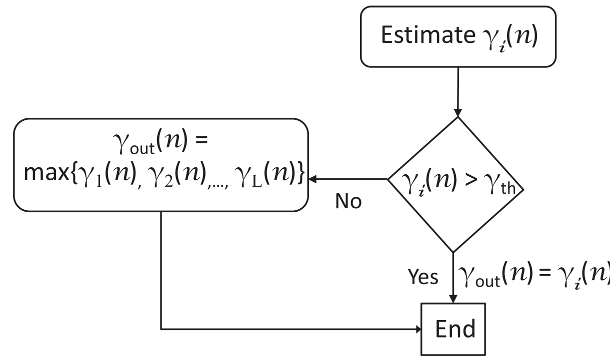

Fig. 1. The mode of operation of the proposed threshold-based selection scheme.

II. Threshold Based Selection Strategy

Let n be a discrete time index and $\gamma_i(n), i = 1, 2, \ldots, L$, represent L independent random sequences (RS)s, each consisting of i.i.d. (continuous) positive random variables (RV)s. In this work, we propose a new threshold-based selection scheme, which is shown in Fig. 1. According to the proposed selection policy, at each discrete time instant n, it is examined whether the previously (at time $n-1$) selected RS exceeds a predefined threshold γ_{th}. If this is the case, the algorithm keeps the selected RS, otherwise it switches to the RS having the highest value at time n. It is readily verified that the proposed selection scheme defines a L-state ergodic and regular Markov chain, whose i-th state corresponds to the event that RS i is selected at time n [21]. These events define a new RS $\gamma_{\text{out}}(n)$, which corresponds to the output of the proposed scheme and is expressed as

$$\gamma_{\text{out}}(n) = \gamma_i(n) \text{ iff } \begin{cases} \gamma_{\text{out}}(n-1) = \gamma_i(n-1) \text{ and } \gamma_i(n) \geq \gamma_{\text{th}} \text{ or} \\ \gamma_{\text{out}}(n-1) = \gamma_k(n-1) \text{ and } \gamma_k(n) < \gamma_{\text{th}} \\ \text{ and } \gamma_i(n) = \max\{\gamma_1(n), \ldots, \gamma_L(n)\}. \end{cases} \quad (1)$$

Since the previously mentioned events are mutually exclusive, the CDF of the RS $\gamma_{\text{out}}(n)$ at time n, $F_{\gamma_{\text{out}}(n)}(\gamma)$, is given by

$$F_{\gamma_{\text{out}}(n)}(\gamma) = \Pr[\gamma_{\text{out}}(n) \leq \gamma] = \sum_{i=1}^{L} \Pr[\gamma_{\text{out}}(n) = \gamma_i(n), \, \gamma_i(n) \leq \gamma]. \quad (2)$$

In the following, in order to simplify notations, the time index n is omitted and we further define the set $\mathbb{G} = \{\gamma_1, \gamma_2, \ldots, \gamma_L\}$. Then, if π_i's stand for the stationary probabilities of the aforementioned Markov chain, i.e., π_i is the limiting probability that the ith RS is selected, the CDF in (2) is expressed as

$$F_{\gamma_{\text{out}}}(\gamma) = \begin{cases} \sum_{i=1}^{L} \pi_i \{\Pr[\gamma_{\text{th}} \leq \gamma_i \leq \gamma] + \Pr[\gamma_i < \gamma_{\text{th}}] \\ \times \Pr[\max\{\mathbb{G} - \{\gamma_i\}\} \leq \gamma]\}, \gamma \geq \gamma_{\text{th}} \\ \Pr[\max\{\mathbb{G}\} \leq \gamma], \gamma < \gamma_{\text{th}}. \end{cases} \quad (3)$$

Assuming, in general, i.n.d. RSs, $F_{\gamma_{\text{out}}}(\gamma)$ can be written as

$$F_{\gamma_{\text{out}}}(\gamma)=\begin{cases}\sum_{i=1}^{L}\pi_i\left\{F_{\gamma_i}(\gamma)-F_{\gamma_i}(\gamma_{\text{th}})\right.\\ \left.+F_{\gamma_i}(\gamma_{\text{th}})\prod_{j=1,\neq i}^{L}F_{\gamma_j}(\gamma)\right\},\ \gamma\geq\gamma_{\text{th}}\\ \prod_{j=1}^{L}F_{\gamma_j}(\gamma),\quad \gamma<\gamma_{\text{th}}\end{cases}\tag{4}$$

where $F_{\gamma_i}(\cdot)$ represents the CDF of γ_i. It is well known from the Markov chain theory (e.g., [21]) that since the previously defined Markov chain is ergodic, then it will be characterized by a unique stable vector of stationary probabilities $\boldsymbol{\pi}=[\pi_1,\pi_2,\ldots,\pi_L]$ that can be obtained by solving the system of equations $\boldsymbol{\pi}=\boldsymbol{\pi}\mathbf{P}$ under the constraint $\sum_{i=1}^{L}\pi_i=1$. The $L\times L$ matrix $\mathbf{P}=[P_{i,j}]$ is the so-termed transition probability matrix of the Markov chain. Based on the proposed selection scheme, the transition probabilities of the corresponding Markov chain are given as follows

$$P_{i,j}=\begin{cases}\Pr\left[\gamma_i\geq\gamma_{\text{th}}\right]+\Pr\left[\gamma_i<\gamma_{\text{th}},\gamma_i=\max\{\mathbb{G}\}\right], & i=j\\ \Pr\left[\gamma_i<\gamma_{\text{th}},\gamma_j=\max\{\mathbb{G}\}\right], & i\neq j.\end{cases}\tag{5}$$

From (5), by defining a new RV $\gamma_{\max}=\max\{\mathbb{G}-\{\gamma_i\}\}$ and since $\gamma_{\max}\leq\gamma_i$, when $\gamma_i<\gamma_{\text{th}}$, it is straightforward that

$$P_{i,i}=1-F_{\gamma_i}(\gamma_{\text{th}})+\int_0^{\gamma_{\text{th}}}f_{\gamma_i}(x)F_{\gamma_{\max}}(x)dx.\tag{6}$$

Moreover as shown in Appendix A, for $i\neq j$,

$$P_{i,j}=\int_0^{\gamma_{\text{th}}}\left[\int_x^{\infty}f_{\gamma_j}(y)dy\right]F_{\gamma_{\mathbb{L}\max}}(x)f_{\gamma_i}(x)dx+\sum_{b=1}^{L-2}\sum_{n_{1:b}=1}^{L}\int_0^{\gamma_{\text{th}}}\left[\int_x^{\infty}\int_{x_1}^{\infty}\cdots\int_{x_{b-1}}^{\infty}\int_{x_b}^{\infty}f_{\gamma_j}(y)f_{\gamma_{n_b}}(x_b)f_{\gamma_{n_{b-1}}}(x_{b-1})\cdots\times f_{\gamma_{n_1}}(x_1)dydx_bdx_{b-1}\cdots dx_1\right]\left[\int_0^{x}f_{\gamma_{\mathbb{L}\max}}(z)dz\right]f_{\gamma_i}(x)dx\tag{7}$$

where $\gamma_{\mathbb{L}\max}=\max\{\mathbb{L}\}$. The set $\mathbb{L}$ and $\sum_{n_{1:b}=1}^{L}$ appearing in (7), are defined in Appendix A. Differentiating (4) with respect to γ, yields the corresponding expression for the PDF of γ_{out} as

$$f_{\gamma_{\text{out}}}(\gamma)=\begin{cases}\sum_{i=1}^{L}\pi_i\left\{f_{\gamma_i}(\gamma)+F_{\gamma_i}(\gamma_{\text{th}})\sum_{k=1,\neq i}^{L}f_{\gamma_k}(\gamma)\right.\\ \left.\times\prod_{j=1,\neq k,i}^{L}F_{\gamma_j}(\gamma)\right\},\ \gamma\geq\gamma_{\text{th}}\\ \sum_{j=1}^{L}f_{\gamma_j}(\gamma)\prod_{k=1,\neq j}^{L}F_{\gamma_k}(\gamma),\ \gamma<\gamma_{\text{th}}.\end{cases}\tag{8}$$

If we now assume, i.i.d. RSs, (4) simplifies to[1]

$$F_{\gamma_{\text{out}}}(\gamma)=\begin{cases}F_{\gamma_i}(\gamma)-F_{\gamma_i}(\gamma_{\text{th}})+F_{\gamma_i}(\gamma_{\text{th}})F_{\gamma_i}(\gamma)^{L-1},\ \gamma\geq\gamma_{\text{th}}\\ F_{\gamma_i}(\gamma)^L,\quad \gamma<\gamma_{\text{th}}\end{cases}\tag{9}$$

and the corresponding expression for the PDF to

$$f_{\gamma_{\text{out}}}(\gamma)=\begin{cases}f_{\gamma_i}(\gamma)+(L-1)F_{\gamma_i}(\gamma_{\text{th}})f_{\gamma_i}(\gamma)F_{\gamma_i}(\gamma)^{L-2},\ \gamma\geq\gamma_{\text{th}}\\ Lf_{\gamma_i}(\gamma)F_{\gamma_i}(\gamma)^{L-1},\quad \gamma<\gamma_{\text{th}}.\end{cases}\tag{10}$$

A. Exponentially Distributed RVs

The previous generic statistical framework may apply to any distribution model. Here, we will focus on the exponential one. In this case, the γ_i's are characterized by the following PDF

$$f_{\gamma_i}(\gamma)=\frac{1}{\overline{\gamma}_i}\exp\left(-\frac{\gamma}{\overline{\gamma}_i}\right)\tag{11}$$

where $\overline{\gamma}_i$ denotes the distribution's mean value. The corresponding expression for the CDF is

$$F_{\gamma_i}(\gamma)=1-\exp\left(-\frac{\gamma}{\overline{\gamma}_i}\right).\tag{12}$$

In general, the CDF of the maximum of L i.n.d. RVs, e.g., $\gamma_{\max}=\max\{\gamma_1,\gamma_2,\ldots,\gamma_L\}$, is given by $F_{\gamma_{\max}}(\gamma)=\prod_{j=1}^{L}F_{\gamma_j}(\gamma)$. Using (12) on this product, the following simplified expression can be derived

$$\prod_{j=1}^{L}\left[1-\exp\left(-\frac{\gamma}{\overline{\gamma}_j}\right)\right]=1+\sum_{j,-}^{L}\exp\left[-\left(\sum_{p=1}^{j}\frac{\gamma}{\overline{\gamma}_{\lambda_p}}\right)\right]\tag{13}$$

where $\sum_{x,y}^{z}=\sum_{\substack{x=1\\x\neq y}}^{z}(-1)^x\sum_{\substack{\lambda_1=1\\\lambda_1\neq y}}^{z-x+1}\sum_{\substack{\lambda_2=\lambda_1+1\\\lambda_2\neq\lambda_1}}^{z-x+2}\cdots\sum_{\substack{\lambda_{z-1}=\lambda_{z-2}+1\\\lambda_{z-1}\neq\lambda_{z-2}}}^{z}$. For example, $\sum_{x,2}^{3}e^{-\sum_{i=1}^{x}\alpha_i}=-\sum_{\substack{\lambda_1=1\\\lambda_1\neq 2}}^{3}e^{-\sum_{i=1}^{1}\alpha_{\lambda_i}}+\sum_{\substack{\lambda_1=1\\\lambda_1\neq 2}}^{2}\sum_{\substack{\lambda_2=\lambda_1+1\\\lambda_1\neq\lambda_2}}^{3}e^{-\sum_{i=1}^{2}\alpha_{\lambda_i}}$. Using (13) in (4) and after some mathematical manipulations, yields

$$F_{\gamma_{\text{out}}}(\gamma)=\begin{cases}\sum_{i=1}^{L}\pi_i\left\{F_{\gamma_i}(\gamma)-F_{\gamma_i}(\gamma_{\text{th}})+F_{\gamma_i}(\gamma_{\text{th}})\right.\\ \left.\times\left[1+\sum_{j,i}^{L}\exp\left[-\left(\sum_{p=1}^{j}\frac{\gamma}{\overline{\gamma}_{\lambda_p}}\right)\right]\right]\right\},\ \gamma\geq\gamma_{\text{th}}\\ 1+\sum_{j,-}^{L}\exp\left[-\left(\sum_{p=1}^{j}\frac{\gamma}{\overline{\gamma}_{\lambda_p}}\right)\right],\ \gamma<\gamma_{\text{th}}.\end{cases}\tag{14}$$

[1] A preliminary version of the i.i.d. analysis has been presented in [22].

The corresponding PDF expression is then given by

$$f_{\gamma_{\text{out}}}(\gamma)=\begin{cases}\sum_{i=1}^{L}\pi_i\left\{f_{\gamma_i}(\gamma)+F_{\gamma_i}(\gamma_{\text{th}})\left[\sum_{j,i}^{L}\left(-\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)\right.\right.\\ \left.\left.\times\exp\left[-\left(\sum_{p=1}^{j}\frac{\gamma}{\overline{\gamma}_{\lambda_p}}\right)\right]\right]\right\},\ \gamma\geq\gamma_{\text{th}}\\ \sum_{j,-}^{L}\left(-\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)\exp\left(-\sum_{p=1}^{j}\frac{\gamma}{\overline{\gamma}_{\lambda_p}}\right),\ \gamma<\gamma_{\text{th}}.\end{cases}\tag{15}$$

Based on (11)–(13) and after repeatedly evaluating integrals of the form $I=\int_x^{\infty}\exp(-\alpha x)dx$ with $\alpha\in\mathbb{R}$, the transition probabilities appearing in (6) and (7) can be expressed as

$$P_{i,i}=1+\frac{1}{\overline{\gamma}_i}\sum_{k,i}^{L}\left(\frac{1-\exp\left[-\gamma_{\text{th}}\left(\sum_{p=1}^{k}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)_{\overline{\gamma}_i^{-1}}\right]}{\left(\sum_{p=1}^{k}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)_{\overline{\gamma}_i^{-1}}}\right).\tag{16}$$

and as in (17), shown at the bottom of the page. In these equation, $\overline{\gamma}_{i,j}=\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}$ and $(X)_y=y+X$. From (16) and (17) it is evident that for practical communications scenarios, where $\overline{\gamma}_i>0, \forall i\in[1,...,L]$, the transition probabilities will be positive numbers, i.e., it is possible to get from any state of the corresponding Markov chain to any other state with some probability, and thus the ergodicity of the Markov chain is justified. For the case of i.i.d. RSs, simplified expressions for the CDF and PDF of γ_{out} are given in Table I. In the following sections, the proposed threshold-based selection approach will be used in three different applications, namely i) as a diversity reception scheme, ii) for receiver channel selection in a TAS scheme and iii) for relay selection in a cooperative communication system.

III. Threshold-Based Selection Diversity

We consider a diversity receiver, termed threshold-based selection diversity (t-SD) receiver, equipped with L antenna branches that employs the selection strategy presented in the previous section and operates over slow faded Rayleigh channels. Its mode of operation is as follows. In each time slot, the system examines if the received instantaneous SNR of the branch that was selected in the previous time slot (denoted by γ_i with $i=1,2,\ldots,L$) exceeds a predefined threshold γ_{th}. If that branch exceeds the threshold, it is kept, otherwise the system switches to the branch with the highest SNR. Note that (i) a single (branch) receiver is used when the SNR of the tagged antenna branch is above the threshold (i.e., $\gamma_i\geq\gamma_{\text{th}}$) and (ii) L-branch SD is employed when $\gamma_i<\gamma_{\text{th}}$.

A. Statistical Characteristics

It is obvious from the previous discussion that the CDF of the output SNR for the scheme under consideration is given by (14) and its PDF by (15). Moreover:

1) Moments Generating Function (MGF): Substituting (15) in the definition of the MGF, i.e., $M_{\gamma_{\text{out}}}(s)=\mathbb{E}\langle\exp(-s\gamma_{\text{out}})\rangle$, with $\mathbb{E}\langle\cdot\rangle$ denoting expectation, and using [23, eq. (3.310)], yields the following expression for the MGF of γ_{out}

$$M_{\gamma_{\text{out}}}(s)=\sum_{i=1}^{L}\pi_i\left\{\frac{\exp\left[-\left(1/\overline{\gamma}_i\right)_s\gamma_{\text{th}}\right]}{\overline{\gamma}_i\left(1/\overline{\gamma}_i\right)_s}+F_{\gamma_i}(\gamma_{\text{th}})\right.$$
$$\left.\times\left[\sum_{j,i}^{L}\left(-\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)\frac{\exp\left[-\left(\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)_s\gamma_{\text{th}}\right]}{\left(\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)_s}\right]\right\}$$
$$+\sum_{j,-}^{L}\left(-\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)\frac{\exp\left[-\left(\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)_s\gamma_{\text{th}}\right]}{\left(\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)_s}.\tag{18}$$

The corresponding expression for i.i.d. fading conditions is given in Table I.

2) Moments: The mth order moment of γ_{out} can be evaluated by substituting (15) in the definition of the moments, i.e., $\mu_{\gamma_{\text{out}}}(m)=\mathbb{E}\langle\gamma_{\text{out}}^m\rangle$. Then, using [23, eqs. (8.350/1 and 8.350/2)], the following closed-form expression is obtained

$$\mu_{\gamma_{\text{out}}}(m)=\sum_{i=1}^{L}\pi_i\left\{\frac{1}{\overline{\gamma}_i}\frac{\Gamma\left(m+1,\gamma_{\text{th}}/\overline{\gamma}_i\right)}{\left(1/\overline{\gamma}_i\right)^{m+1}}+F_{\gamma_i}(\gamma_{\text{th}})\right.$$
$$\left.\times\left[\sum_{j,i}^{L}\left(-\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)\frac{\Gamma\left(m+1,\sum_{p=1}^{j}\gamma_{\text{th}}/\overline{\gamma}_{\lambda_p}\right)}{\left(\sum_{p=1}^{j}1/\overline{\gamma}_{\lambda_p}\right)^{m+1}}\right]\right\}$$

$$P_{i,j}=\frac{1}{\overline{\gamma}_i}\left\{\left[\left[\frac{1-\exp\left(-\overline{\gamma}_{i,j}\gamma_{\text{th}}\right)}{\overline{\gamma}_{i,j}}+\sum_{k,[i,y]}^{L}\left(\frac{1-\exp\left[-\gamma_{\text{th}}\left(\sum_{p=1}^{k}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)_{\overline{\gamma}_{i,j}}\right]}{\left(\sum_{p=1}^{k}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)_{\overline{\gamma}_{i,j}}}\right)\right]+\sum_{b=1}^{L-2}\left[\sum_{n_{1:b}=1}^{b}\left(\prod_{t=1}^{b}\frac{1/\overline{\gamma}_{n_t}}{\left(\sum_{q=1}^{t}\frac{1}{\overline{\gamma}_{n_q}}\right)_{\overline{\gamma}_j^{-1}}}\right)\right.\right.\right.$$
$$\left.\left.\left.\times\frac{1-\exp\left[-\gamma_{\text{th}}\left(\sum_{t=1}^{b}\frac{1}{\overline{\gamma}_{n_t}}\right)_{\overline{\gamma}_{i,j}}\right]}{\overline{\gamma}_{i,j}+\sum_{t=1}^{b}\frac{1}{\overline{\gamma}_{n_t}}}+\sum_{n_{1:b}=1}^{b}\sum_{k',[i,j,n_b]}^{L}\left(\prod_{t=1}^{b}\frac{1/\overline{\gamma}_{n_t}}{\left(\sum_{q=1}^{t}\frac{1}{\overline{\gamma}_{n_q}}\right)_{\overline{\gamma}_j^{-1}}}\right)\frac{1-\exp\left[-\left(\left(\sum_{t=1}^{b}\frac{1}{\overline{\gamma}_{n_t}}\right)_{\overline{\gamma}_{i,j}}+\sum_{p=3}^{k'}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)\gamma_{\text{th}}\right]}{\overline{\gamma}_{i,j}+\sum_{t=1}^{b}\frac{1}{\overline{\gamma}_{n_t}}+\sum_{p=3}^{k'}\frac{1}{\overline{\gamma}_{\lambda_p}}}\right]\right]\right\}.\tag{17}$$

TABLE I
t-SD i.i.d. STATISTICS

CDF	$F_{\gamma_{\text{out}}}(\gamma)=\begin{cases}\exp\left(-\frac{\gamma_{\text{th}}}{\overline{\gamma}_i}\right)-\exp\left(-\frac{\gamma}{\overline{\gamma}_i}\right)+\left[1-\exp\left(-\frac{\gamma_{\text{th}}}{\overline{\gamma}_i}\right)\right]\left[1-\exp\left(-\frac{\gamma}{\overline{\gamma}_i}\right)\right]^{L-1}, & \gamma\geq\gamma_{\text{th}}\\ \left[1-\exp\left(-\frac{\gamma}{\overline{\gamma}_i}\right)\right]^{L}, & \gamma<\gamma_{\text{th}}\end{cases}$
PDF	$f_{\gamma_{\text{out}}}(\gamma)=\begin{cases}\frac{1}{\overline{\gamma}_i}\exp\left(-\frac{\gamma}{\overline{\gamma}_i}\right)+\left[1-\exp\left(-\frac{\gamma_{\text{th}}}{\overline{\gamma}_i}\right)\right]\frac{(L-1)}{\overline{\gamma}_i}\exp\left(-\frac{\gamma}{\overline{\gamma}_i}\right)\left[1-\exp\left(-\frac{\gamma}{\overline{\gamma}_i}\right)\right]^{L-2}, & \gamma\geq\gamma_{\text{th}}\\ \frac{L}{\overline{\gamma}_i}\exp\left(-\frac{\gamma}{\overline{\gamma}_i}\right)\left[1-\exp\left(-\frac{\gamma}{\overline{\gamma}_i}\right)\right]^{L-1}, & \gamma<\gamma_{\text{th}}\end{cases}$
MGF	$M_{\gamma_{\text{out}}}(s)=\sum_{j=0}^{L-1}\binom{L-1}{j}(-1)^j L\frac{1-\exp\left[-\frac{\gamma_{\text{th}}(s\overline{\gamma}_i)_{1+j}}{\overline{\gamma}_i}\right]}{(s\overline{\gamma}_i)_{1+j}}+\frac{\exp\left[-\gamma_{\text{th}}\left(\frac{s\overline{\gamma}_i+1}{\overline{\gamma}_i}\right)\right]}{1+s\overline{\gamma}_i}+F_{\gamma_i}(\gamma_{\text{th}})(L-1)$ $\times\sum_{j=0}^{L-2}\binom{L-2}{j}(-1)^j\frac{\exp\left[-\frac{\gamma_{\text{th}}(s\overline{\gamma}_i)_{1+j}}{\overline{\gamma}_i}\right]}{(s\overline{\gamma}_i)_{1+j}}$
Moments	$\mu_{\gamma_{\text{out}}}(m)=\sum_{j=0}^{L-1}\binom{L-1}{j}(-1)^j\frac{L}{\overline{\gamma}_i}\frac{\gamma\left(m+1,\frac{j+1}{\overline{\gamma}_i}\gamma_{\text{th}}\right)}{[(j+1)/\overline{\gamma}_i]^{m+1}}+\frac{\Gamma\left(m+1,\frac{\gamma_{\text{th}}}{\overline{\gamma}_i}\right)}{(1/\overline{\gamma}_i)^m}+\frac{L-1}{\overline{\gamma}_i}\sum_{i=0}^{L-2}\binom{L-2}{j}$ $\times F_{\gamma_i}(\gamma_{\text{th}})(-1)^j\frac{\Gamma\left(m+1,\frac{j+1}{\overline{\gamma}_i}\gamma_{\text{th}}\right)}{[(j+1)/\overline{\gamma}_i]^{m+1}}$
Capacity	$C_{\gamma_{\text{out}}}=\text{BW}\sum_{j=0}^{L-1}\binom{L-1}{j}(-1)^j\frac{L}{\overline{\gamma}_i}\mathcal{S}_4\left(\frac{j+1}{\overline{\gamma}_i}\right)+\frac{1}{\overline{\gamma}_i}\mathcal{S}_3\left(\frac{1}{\overline{\gamma}_i}\right)+F_{\gamma}(\gamma_{\text{th}})\frac{L-1}{\overline{\gamma}_i}\sum_{j=0}^{L-2}\binom{L-2}{j}(-1)^j\mathcal{S}_3\left(\frac{j+1}{\overline{\gamma}_i}\right)$

$$+\sum_{j,-}^{L}\left(-\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)\frac{\gamma\left(m+1,\sum_{p=1}^{j}\gamma_{\text{th}}/\overline{\gamma}_{\lambda_p}\right)}{\left(\sum_{p=1}^{j}1/\overline{\gamma}_{\lambda_p}\right)^{m+1}} \tag{19}$$

where $\gamma(\cdot,\cdot)$ and $\Gamma(\cdot,\cdot)$ denote the lower and upper incomplete gamma functions [23, eqs. (8.350/1 and 8.350/2)], respectively.

B. Performance Analysis

In this section, using the previously derived expressions for the PDF, CDF, MGF and moments of the output SNR, various performance evaluation criteria will be presented. More specifically, the performance of the proposed diversity scheme is studied using the OP, the ABER, the ASNR and the average channel capacity, while complexity and asymptotic analyses are also included.

1) Outage Probability (OP): OP is defined as the probability that the output SNR falls below a predetermined threshold γ_{T} and is given by $P_{\text{out}}=F_{\gamma_{\text{out}}}(\gamma_{\text{T}})$.

2) Average Bit Error Rate (ABER): Using the previously derived MGF expression in (18) and following the MGF-based approach, the ABER can be readily evaluated for a variety of modulation schemes [2]. In particular, the ABER can be directly calculated for non-coherent differential binary phase shift keying (DBPSK), that is $P_{\text{be}}^{DBPSK}=0.5M_{\gamma_{\text{out}}}(1)$. In addition, for BPSK (and square M-quadrature amplitude modulation (QAM) at high SNRs) the ABER can be evaluated by averaging the conditional symbol error probability, $P_e(\gamma)=A\text{erfc}(\sqrt{B\gamma})$, where erfc$(\cdot)$ is the complementary error function [23, eq. (8.250/4)] and A, B are constants depending on the specific modulation scheme, over the PDF of γ_{out}, i.e., $P_{\text{be}}^{BPSK}=\int_0^\infty P_e(\gamma)f_{\gamma_{\text{out}}}(\gamma)d\gamma$. In this context, substituting (15), in the previous integral, making integration by parts, using [23, eqs. (3.321/1 and 3.321/3)] and after some mathematical manipulations, yields

$$P_{\text{be}}^{BPSK}=A\sum_{i=1}^{L}\pi_i\left\{\frac{1}{\overline{\gamma}_i}\mathcal{S}_1\left(\frac{1}{\overline{\gamma}_i},B\right)+F_{\gamma_i}(\gamma_{\text{th}})\right.$$
$$\left.\times\left[\sum_{j,i}^{L}\left(-\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)\mathcal{S}_1\left(\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}},B\right)\right]\right\}$$
$$+A\sum_{j,-}^{L}\left(-\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)\mathcal{S}_2\left(\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}},B\right) \tag{20}$$

where

$$\mathcal{S}_1(x,y)=\frac{1}{x}\left[\exp(-x\gamma_{\text{th}})-1\right]\text{erfc}\left(\sqrt{y\gamma_{\text{th}}}\right)+\frac{\sqrt{y}}{x}\left[\frac{\text{erfc}\left(\sqrt{y\gamma_{\text{th}}}\right)}{\sqrt{y}}-\frac{\text{erfc}\left(\sqrt{(x+y)\gamma_{\text{th}}}\right)}{\sqrt{x+y}}\right]$$
$$\mathcal{S}_2(x,y)=\frac{1}{x}\left[1-\exp(-x\gamma_{\text{th}})\right]\text{erfc}\left(\sqrt{y\gamma_{\text{th}}}\right)+\frac{\sqrt{y}}{x}\left[\frac{\text{erf}\left(\sqrt{y\gamma_{\text{th}}}\right)}{\sqrt{y}}-\frac{\text{erf}\left(\sqrt{(x+y)\gamma_{\text{th}}}\right)}{\sqrt{x+y}}\right]$$

and erf$(\cdot)$ is the error function [23, eq. (8.250/1)].

3) Average Output SNR: The ASNR is an important performance indicator that is tightly related to the performance metrics of a system, such as the ABER and the asymptotic spectral efficiency and can be directly evaluated by setting $m=1$ in (19).

4) Capacity: Substituting (15) in the definition of the capacity, i.e., $C_{\gamma_{\text{out}}}=\text{BW}\mathbb{E}\langle\log_2(1+\gamma_{\text{out}})\rangle$, where BW is the signal's transmission bandwidth, making an integration by parts and using [23, eqs. (3.352/1 and 4.337/2)], the following closed-form expression can be obtained

$$C_{\gamma_{\text{out}}}=\text{BW}\left\{\sum_{i=1}^{L}\pi_i\left\{\frac{1}{\overline{\gamma}_i}\mathcal{S}_3\left(\frac{1}{\overline{\gamma}_i}\right)+F_{\gamma_i}(\gamma_{\text{th}})\right.\right.$$
$$\left.\times\left[\sum_{j,i}^{L}\left(-\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)\mathcal{S}_3\left(\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)\right]\right\}$$

$$+\sum_{j,-}^{L}\left(-\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)\mathcal{S}_4\left(\sum_{p=1}^{j}\frac{1}{\overline{\gamma}_{\lambda_p}}\right)\Bigg\} \tag{21}$$

where

$$\mathcal{S}_3(x)=\frac{\exp(-x\gamma_{\text{th}})\log_2(1+\gamma_{\text{th}})}{x}-\frac{\exp(x)}{x\ln(2)}\text{Ei}(-x\gamma_{\text{th}}-x)$$

$$\mathcal{S}_4(x)=\frac{\exp(x)}{x\ln(2)}\left[\text{Ei}(-x\gamma_{\text{th}}-x)-\text{Ei}(-x)\right]-\frac{\exp(-x\gamma_{\text{th}})\log_2(1+\gamma_{\text{th}})}{x}$$

with $\text{Ei}(\cdot)$ denoting the exponential integral function [23, eq. (8.211/1)].

5) Complexity: The complexity of the proposed scheme will be evaluated by employing the average number of path estimations (ANPE) and the switching probability (SP) in a guard period as a quantification of the power consumption [5], [24].

a) Average Number of Path Estimations (ANPE): The system complexity increases as the ANPE increases, due to the important amount of information that must be exchanged for performing various tasks, e.g., channel estimation [25]. Moreover, ANPE is directly related with the type and number of comparisons that are required [26]. From the mode of operation of t-SD, the ANPE, N_{out}, is given by

$$N_{\text{out}}=\delta_1+L\delta_L \tag{22}$$

where $\delta_j,\ j=1,L$, denotes the probability that exactly j diversity paths are estimated. It is obvious that $\delta_L=\sum_{i=1}^{L}\pi_i F_{\gamma_i}(\gamma_{\text{th}})$ and $\delta_1=\sum_{i=1}^{L}\pi_i\left[1-F_{\gamma_i}(\gamma_{\text{th}})\right]$, which after substituted in (22) yields

$$N_{\text{out}}=\sum_{i=1}^{L}\pi_i\left[1+(L-1)F_{\gamma_i}(\gamma_{\text{th}})\right]. \tag{23}$$

b) Switching Probability (SP): SP is an important performance measure that is very useful in practical scenarios. In particular, switching among branches not only consumes power, but also reduces the data throughput in a transmit-switched diversity configuration and leads to inaccurate phase estimates [26]. Assuming that at time instance $n-1$ the ith branch was selected, the non-switch probability at time n will be equal to $P_{i,i}$, given in (6). Therefore, it is very easy to show that SP can be expressed as

$$P_{\text{out}}^{s}=\sum_{i=1}^{L}\pi_i(1-P_{i,i}) \tag{24}$$

where $P_{i,i}$ is given in (16) for exponential fading assumption.

6) Asymptotic Analysis: In order to clearly understand important system-design parameters, we focus here on the high SNR regime. This approach help us to quantify the amount of performance variations, which are due to the fading effects, as well as to the scheme's architecture [27]. At the high SNR regime, the exponential PDF and CDF expressions can be approximated by $f_{\gamma_X}(\gamma)\approx\frac{1}{\overline{\gamma}_X}$ and $F_{\gamma_X}(\gamma)\approx\frac{\gamma}{\overline{\gamma}_X}$, respectively. Based on these approximate expressions and by employing the approach presented in Subsection III-A1, the ABER of the DBPSK modulation scheme can be expressed in the high SNR regime as follows

$$P_{\text{be}}^{DBPSK}=\sum_{i=1}^{L}\frac{\pi_i}{2}\left[\frac{\exp(-\gamma_{\text{th}})}{\overline{\gamma}_i}+F_{\gamma_i}(\gamma_{\text{th}})\times\sum_{k=1\neq i}^{L}\prod_{j=1\neq i}^{L}\frac{\Gamma(L,\gamma_{\text{th}})}{\overline{\gamma}_i}\right]+\frac{L}{2}\prod_{i=1}^{L}\frac{\gamma(L,\gamma_{\text{th}})}{\overline{\gamma}_i}. \tag{25}$$

Assuming $\overline{\gamma}_i >> \gamma_{\text{th}}$, the second and the third terms in the right hand side of (25) have negligible effect on the performance. Therefore, $\overline{P}_{\text{be}}^{DBPSK}$ can be expressed as

$$\overline{P}_{\text{be}}^{DBPSK}=\sum_{i=1}^{L}\frac{\pi_i}{2}\frac{\exp(-\gamma_{\text{th}})}{\overline{\gamma}_i}. \tag{26}$$

It is obvious that (26) is of the form $(G_c\text{SNR})^{-G_d}$, where G_d is the diversity gain and G_c is the coding gain. Thus, the coding gain is affected by γ_{th}, while the diversity gain is equal to that of the SEC and SECps schemes [5]. This is a reasonable observation, since when γ_{th} is small as compared with $\overline{\gamma}_i$, then very few switches are expected and thus the diversity gain is lost. On the other hand, for $\overline{\gamma}_i\leq\gamma_{\text{th}}$, $\overline{P}_{\text{be}}^{DBPSK}$ can be expressed as

$$\overline{P}_{\text{be}}^{DBPSK}=\frac{L}{2}\left(\prod_{i=1}^{L}\frac{1}{\overline{\gamma}_i}\right)\gamma(L,\gamma_{\text{th}}). \tag{27}$$

Here, it is noted that G_d is equal to that of SD. This is also a reasonable outcome, since when γ_{th} is large as compared to $\overline{\gamma}_i$ then the receiver will switch to the best channel with increased probability. This means that, as it was also mentioned at the beginning of this section, depending upon the switching threshold, the performance of t-SD ranges between that of a pure SD scheme and that of a single branch scheme.

C. Numerical Results

Based on the previous analysis, comparative numerical results will be presented, in terms of the OP (using (14)), ABER (using (18)), capacity (using (21)), ANPE (using (23)) and SP (using (24)). It is noted that for all multipath channels involved, the well accepted exponentially decaying power delay profile (PDP) has been adopted [2], i.e., $\overline{\gamma}_i=\overline{\gamma}_1\exp[-\delta(i-1)]$, where δ is the power decaying factor. In Fig. 2, assuming $L=3$, $\gamma_T=4$ dB and $\overline{\gamma}_1=20$ dB, the OPs of t-SD and SD[2] are plotted as a function of the switching threshold, γ_{th}, for different values of the power decaying factor δ. In the same figure, the ANPEs of both schemes have been also depicted. We observe that for high enough values of γ_{th}, the OP of t-SD becomes almost equal to that of SD, while for small values of γ_{th}, SD performs better than t-SD. However, for all values of γ_{th}, the ANPEs of t-SD are considerably lower as compared

[2]It should be noted that comparative results with SECps have not been included, since, to the best of the authors' knowledge, an i.n.d. fading analysis does not exist for SECps.

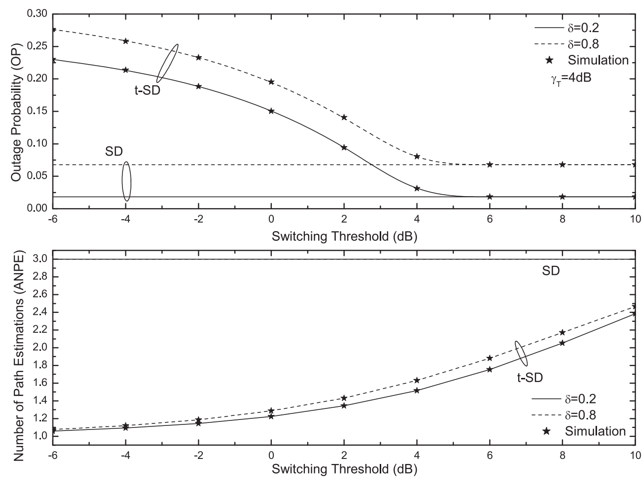

Fig. 2. Outage probability and average number of path estimations of SD and t-SD as a function of γ_{th}.

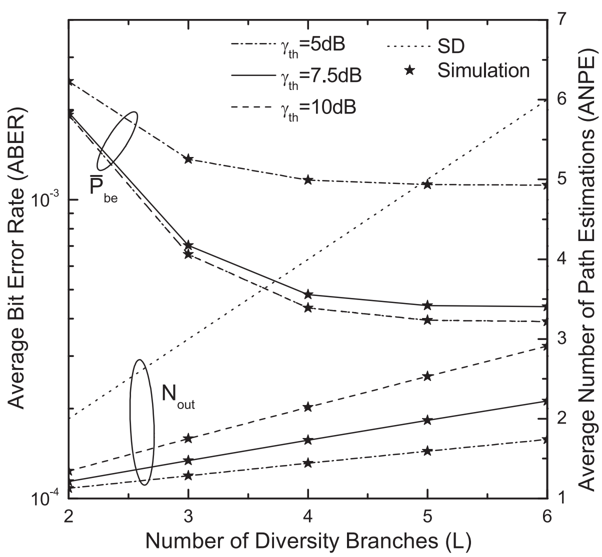

Fig. 3. ABER and average number of path estimations of SD and t-SD as a function of L.

with the corresponding ones of SD, for both values of δ. Thus, important energy savings can be anticipated, due to the need for less channel estimations, without any important loss in performance.

In Fig. 3, assuming $\delta = 0.8$, $\overline{\gamma}_1 = 15$ dB and DBPSK, the ABERs of t-SD and SD are plotted as a function of the number of diversity branches L, for various values of γ_{th}. In the same figure, the ANPEs of both schemes are also included. It is depicted that as γ_{th} increases, the performance of t-SD approaches that of SD. It is very interesting to note that for $\gamma_{\text{th}} = 10$ dB, the ABER of t-SD coincides with that of SD, irrespectively of the number of branches. However, for the same BER, the ANPE of t-SD is almost 50% less than that of SD, a fact that clearly shows the energy savings that can be achieved by employing the proposed scheme. Finally, in Fig. 4, assuming $L = 4$ and $\gamma_{\text{th}} = 10$ dB, the average channel capacities of t-SD and SD have been plotted as a function of the average input SNR of the first branch, $\overline{\gamma}_1$, for different values of δ. In the same figure, the SPs of both schemes have been also included. It is

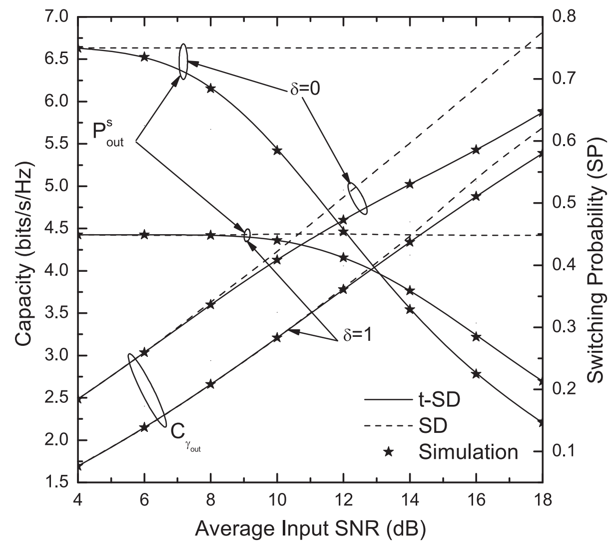

Fig. 4. Average channel capacity and switching probability of SD and t-SD as a function of $\overline{\gamma}_1$.

shown that for lower values of $\overline{\gamma}_1$, the capacity performances are similar, while a performance gap appears when $\overline{\gamma}_1 > \gamma_{\text{th}}$. Thus, for $\delta = 0$, with a relatively low cost on capacity, e.g., 15% for $\overline{\gamma}_1 = 18$ dB, the SP of t-SD is almost 80% less as compared to that of SD. The same trend also holds for the case of $\delta = 1$. A general observation in these figures is that for γ_{th} approaching $\overline{\gamma}_i$, the proposed scheme offers almost similar performance with SD and, at the same time, a significant reduction in ANPE and/or SP. For comparison purposes, computer simulations performance results are also included in Figs. 2, 3 and 4, verifying in all cases the validity of the proposed theoretical approach.

IV. Transmit Antenna Selection with Threshold Based Selection Diversity

In this section, another application example for the proposed selection strategy will be investigated, namely TAS with threshold-based SD (TAS/t-SD). We consider a TAS/t-SD scheme, with N_t transmit and N_r receive antennas, supporting the new selection policy at the reception side and operating over slow faded Rayleigh channels. More specifically, the mode of operation of the proposed scheme is schematically shown in Fig. 5, and can be described as follows. Considering (the signal transmitted by) the first transmit antenna ($k = 1$), the scheme estimates the instantaneous received SNR $\gamma_{i,k}$ of the previously selected branch for this transmit antenna. If its SNR is above γ_{th}, the receiver stays at that branch, otherwise the branch that provides the highest SNR, for the signal transmitted by the first transmit antenna, is selected. The same receiver operation is then repeated for the remaining transmit antennas sequentially. Finally, the transmit antenna that results in the highest SNR at the receiver is selected for transmission. In the TAS scheme, a feedback link is required to inform the transmitter which antenna has been selected for transmission in each frame, [28]. Similar to [9], in the following analysis, it is also assumed that there is no feedback error or delay in the transmission, while the receiver has perfect channel state information.

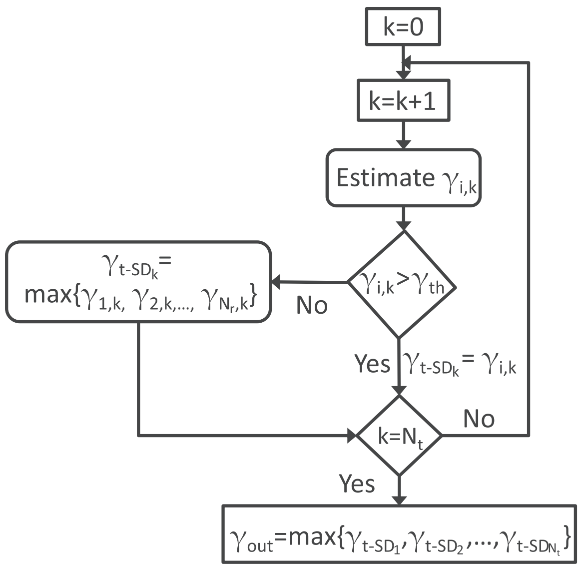

Fig. 5. The mode of operation of the proposed TAS/t-SD scheme.

A. Statistical Characteristics

Based on the previous description, the instantaneous output SNR of the TAS/t-SD scheme can be expressed as

$$\gamma_{\text{out}} = \max_k \left(\gamma_{\text{t}-\text{SD}_k}\right) \tag{28}$$

where $\gamma_{\text{t}-\text{SD}_\text{k}}$ is the instantaneous output SNR of the proposed reception scheme from the kth transmit antenna, with $k = 1, 2, \ldots, N_t$. For i.n.d. fading channels, the CDF of the output SNR can be expressed as

$$F_{\gamma_{\text{out}}}(\gamma) = \prod_{k=1}^{N_t} \left[F_{\gamma_{\text{t}-\text{SD}_k}}(\gamma)\right] \tag{29}$$

where $F_{\gamma_{\text{t}-\text{SD}_k}}(\gamma)$ is the CDF of t-SD for transmit antenna k given in (9). By substituting, this CDF expression in (29), yields

$$F_{\gamma_{\text{out}}}(\gamma) = \begin{cases} \prod_{k=1}^{N_t} \left[F_{\gamma_k}(\gamma) - F_{\gamma_k}(\gamma_{\text{th}}) \right. \\ \left. +F_{\gamma_k}(\gamma_{\text{th}}) F_{\gamma_k}(\gamma)^{N_r-1}\right], \gamma \geq \gamma_{\text{th}} \\ \prod_{k=1}^{N_t} F_{\gamma_k}(\gamma)^{N_r}, \gamma < \gamma_{\text{th}}. \end{cases} \tag{30}$$

For exponentially distributed fading channels and based on the analysis presented in Appendix B, the CDF expression of the TAS/t-SD system output SNR is given by

$$F_{\gamma_{\text{out}}}(\gamma) = \begin{cases} \sum_{\ell=0}^{N_t} \sum_{\omega_1(\ell,N_t)} \sum_{\nu=0}^{N_t} \sum_{\omega_3(\nu,N_t)} P_1 \sum_{\mu=0}^{(N_r-1)\sum_{k=1}^{N_t}\ell_k} \\ \sum_{\omega_4(\mu,N_t)} P_2 \exp\left(-\sum_{k=1}^{N_t} \frac{\mu_k+\nu_k}{\overline{\gamma}_k}\gamma\right), \gamma \geq \gamma_{\text{th}} \\ \sum_{\ell=0}^{N_t N_r} \sum_{\omega_2(\ell,N_t)} P_3 \exp\left(-\sum_{k=1}^{N_t} \frac{\gamma \ell_k}{\overline{\gamma}_k}\right), \gamma < \gamma_{\text{th}} \end{cases} \tag{31}$$

where $\omega_1(\ell, N_t), \omega_2(\ell, N_t), \omega_3(\nu, N_t)$ are defined in Appendix B and $\omega_4(\mu, N_t) = \left\{\left(\mu_1, \ldots, \mu_{N_t}\right) : \mu_k \in \{0, 1, \ldots, \ell_k(N_r-1)\}, \sum_{k=1}^{N_t} \mu_k = \mu\right\}$. Moreover, $P_1 = \prod_{k=1}^{N_t} F_{\gamma_k}(\gamma_{\text{th}})^{\ell_k} \exp(-\frac{\gamma_{\text{th}}(1-\ell_k-\nu_k)}{\overline{\gamma}_k})$, $P_2 = (-1)^{\mu+\nu} \prod_{k=1}^{N_t} \binom{\ell_k(N_r-1)}{\mu_k}$ and $P_3 = (-1)^{\ell} \prod_{k=1}^{N_t} \binom{N_r}{\ell_k}$. The corresponding PDF expression can be directly evaluated by differentiating (31), resulting to

$$f_{\gamma_{\text{out}}}(\gamma) = \begin{cases} \sum_{\ell=0}^{N_t} \sum_{\omega_1(\ell,N_t)} \sum_{\nu=0}^{N_t} \sum_{\omega_3(\nu,N_t)} P_1 \sum_{\mu=0}^{(N_r-1)\sum_{k=1}^{N_t}\ell_k} \\ \times \sum_{\omega_4(\mu,N_t)} P_2 \left(-\sum_{k=1}^{N_t} \frac{\mu_k+\nu_k}{\overline{\gamma}_k}\right) \\ \times \exp\left(-\sum_{k=1}^{N_t} \frac{\mu_k+\nu_k}{\overline{\gamma}_k}\gamma\right), \gamma \geq \gamma_{\text{th}} \\ \sum_{\ell=0}^{N_t N_r} \sum_{\omega_2(\ell,N_t)} P_3 \left(-\sum_{k=1}^{N_t} \frac{\ell_k}{\overline{\gamma}_k}\right) \\ \times \exp\left(-\sum_{k=1}^{N_t} \frac{\gamma \ell_k}{\overline{\gamma}_k}\right), \gamma < \gamma_{\text{th}}. \end{cases} \tag{32}$$

1) Moments Generating Function (MGF): Substituting (32) in the definition of the MGF, using [23, eq. (3.310)], and after some mathematics the following closed-form expression for the MGF of γ_{out} is derived

$$\begin{aligned} M_{\gamma_{\text{out}}}(s) &= \sum_{\ell=0}^{N_t} \sum_{\omega_1(\ell,N_t)} \sum_{\nu=0}^{N_t} \sum_{\omega_3(\nu,N_t)} P_1 \sum_{\mu=0}^{(N_r-1)\sum_{k=1}^{N_t}\ell_k} \\ &\times \sum_{\omega_4(\mu,N_t)} P_2 \left(-\sum_{k=1}^{N_t} \frac{\mu_k+\nu_k}{\overline{\gamma}_k}\right) \frac{\exp\left[-\gamma_{\text{th}}\left(\sum_{k=1}^{N_t} \frac{\mu_k+\nu_k}{\overline{\gamma}_k}\right)_s\right]}{\left(\sum_{k=1}^{N_t} \frac{\mu_k+\nu_k}{\overline{\gamma}_k}\right)_s} \\ &+ \sum_{\ell=0}^{N_t N_r} \sum_{\omega_2(\ell,N_t)} P_3 \left(-\sum_{k=1}^{N_t} \frac{\ell_k}{\overline{\gamma}_k}\right) \frac{1-\exp\left[-\gamma_{\text{th}}\left(\sum_{k=1}^{N_t} \frac{\ell_k}{\overline{\gamma}_k}\right)_s\right]}{\left(\sum_{k=1}^{N_t} \frac{\ell_k}{\overline{\gamma}_k}\right)_s}. \end{aligned} \tag{33}$$

2) Moments: Substituting (32) in the definition of the moments, using [23, eqs. (8.350/1 and 8.350/2)], and after some mathematics the following closed-form expression for the moments of γ_{out} is obtained

$$\begin{aligned} \mu_{\gamma_{\text{out}}}(m) &= \sum_{\ell=0}^{N_t} \sum_{\omega_1(\ell,N_t)} \sum_{\nu=0}^{N_t} \sum_{\omega_3(\nu,N_t)} P_1 \sum_{\mu=0}^{(N_r-1)\sum_{k=1}^{N_t}\ell_k} \\ &\times \sum_{\omega_4(\mu,N_t)} P_2 \left(-\sum_{k=1}^{N_t} \frac{\mu_k+\nu_k}{\overline{\gamma}_k}\right) \frac{\Gamma\left(1+m, \sum_{k=1}^{N_t} \frac{\mu_k+\nu_k}{\overline{\gamma}_k}\gamma_{\text{th}}\right)}{\left(\sum_{k=1}^{N_t} \frac{\mu_k+\nu_k}{\overline{\gamma}_k}\right)^{m+1}} \\ &+ \sum_{\ell=0}^{N_t N_r} \sum_{\omega_2(\ell,N_t)} P_3 \left(-\sum_{k=1}^{N_t} \frac{\ell_k}{\overline{\gamma}_k}\right) \frac{\gamma\left(m+1, \sum_{k=1}^{N_t} \frac{\ell_k}{\overline{\gamma}_k}\gamma_{\text{th}}\right)}{\left(\sum_{k=1}^{N_t} \frac{\ell_k}{\overline{\gamma}_k}\right)^{m+1}}. \end{aligned} \tag{34}$$

TABLE II
TAS/t-SD i.i.d. STATISTICS

CDF	$F_{\gamma_{\text{out}}}(\gamma)=\begin{cases}\sum_{\ell=0}^{N_t}\binom{N_t}{\ell}F_{\gamma_k}(\gamma_{\text{th}})^{\ell}\sum_{\mu=0}^{N_t-\ell}\sum_{\nu=0}^{\ell(N_r-1)}\binom{N_t-\ell}{\mu}\binom{\ell(N_r-1)}{\nu}(-1)^{\nu+\mu}\\ \quad\times\exp\left(-\frac{\gamma_{\text{th}}(N_t-\ell-\mu)}{\overline{\gamma}}\right)\exp\left(-\frac{\mu+\nu}{\overline{\gamma}}\gamma\right), \gamma\geq\gamma_{\text{th}}\\ \sum_{\ell=0}^{N_rN_t}\binom{N_rN_t}{\ell}(-1)^{\ell}\exp\left(-\frac{\ell\gamma}{\overline{\gamma}}\right), \ \gamma<\gamma_{\text{th}}\end{cases}$
PDF	$f_{\gamma_{\text{out}}}(\gamma)=\begin{cases}\sum_{\ell=0}^{N_t}\binom{N_t}{\ell}F_{\gamma_k}(\gamma_{\text{th}})^{\ell}\sum_{\mu=0}^{N_t-\ell}\sum_{\nu=0}^{\ell(N_r-1)}\binom{N_t-\ell}{\mu}\binom{\ell(N_r-1)}{\nu}(-1)^{\nu+\mu}\\ \quad\times\exp\left(-\frac{\gamma_{\text{th}}(N_t-\ell-\mu)}{\overline{\gamma}}\right)\left(-\frac{\mu+\nu}{\overline{\gamma}}\right)\exp\left(-\frac{\mu+\nu}{\overline{\gamma}}\gamma\right), \gamma\geq\gamma_{\text{th}}\\ \sum_{\ell=0}^{N_rN_t}\binom{N_rN_t}{\ell}(-1)^{\ell}\left(-\frac{\ell}{\overline{\gamma}}\right)\exp\left(-\frac{\ell\gamma}{\overline{\gamma}}\right), \quad \gamma<\gamma_{\text{th}}\end{cases}$
MGF	$M_{\gamma_{\text{out}}}(s)=\sum_{\ell=0}^{N_t}\binom{N_t}{\ell}F_{\gamma_k}(\gamma_{\text{th}})^{\ell}\sum_{\mu=0}^{N_t-\ell}\sum_{\nu=0}^{\ell(N_r-1)}\binom{N_t-\ell}{\mu}\binom{\ell(N_r-1)}{\nu}(-1)^{\nu+\mu}\exp\left(-\frac{\gamma_{\text{th}}(N_t-\ell-\mu)}{\overline{\gamma}}\right)$ $\times\left(-\frac{\mu+\nu}{\overline{\gamma}}\right)\frac{\exp\left(-\gamma_{\text{th}}\left(s+\frac{\mu+\nu}{\overline{\gamma}}\right)\right)}{s+\frac{\mu+\nu}{\overline{\gamma}}}+\sum_{\ell=0}^{N_rN_t}\binom{N_rN_t}{\ell}(-1)^{\ell}\left(-\frac{\ell}{\overline{\gamma}}\right)\frac{1-\exp\left(-\gamma_{\text{th}}\left(s+\frac{\ell}{\overline{\gamma}}\right)\right)}{s+\frac{\ell}{\overline{\gamma}}}$
Moments	$\mu_{\gamma_{\text{out}}}(m)=\sum_{\ell=0}^{N_t}\binom{N_t}{\ell}F_{\gamma_k}(\gamma_{\text{th}})^{\ell}\sum_{\mu=0}^{N_t-\ell}\sum_{\nu=0}^{\ell(N_r-1)}\binom{N_t-\ell}{\mu}\binom{\ell(N_r-1)}{\nu}(-1)^{\nu+\mu}$ $\times\exp\left(-\frac{\gamma_{\text{th}}(N_t-\ell-\mu)}{\overline{\gamma}}\right)\left(-\frac{\mu+\nu}{\overline{\gamma}}\right)\frac{\Gamma\left(m+1,\frac{\mu+\nu}{\overline{\gamma}}\gamma_{\text{th}}\right)}{\left(\frac{\mu+\nu}{\overline{\gamma}}\right)^{m+1}}+\sum_{\ell=0}^{N_rN_t}\binom{N_rN_t}{\ell}(-1)^{\ell}\left(-\frac{\ell}{\overline{\gamma}}\right)\frac{\gamma\left(m+1,\frac{\ell}{\overline{\gamma}}\gamma_{\text{th}}\right)}{\left(\frac{\ell}{\overline{\gamma}}\right)^{m+1}}$
Capacity	$C_{\gamma_{\text{out}}}=\text{BW}\left\{\sum_{\ell=0}^{N_t}\binom{N_t}{\ell}F_{\gamma_k}(\gamma_{\text{th}})^{\ell}\sum_{\mu=0}^{N_t-\ell}\sum_{\nu=0}^{\ell(N_r-1)}\binom{N_t-\ell}{\mu}\binom{\ell(N_r-1)}{\nu}(-1)^{\nu+\mu}\right.$ $\left.\times\exp\left(-\frac{\gamma_{\text{th}}(N_t-\ell-\mu)}{\overline{\gamma}}\right)\left(-\frac{\mu+\nu}{\overline{\gamma}}\right)\mathcal{S}_3\left(\frac{\mu+\nu}{\overline{\gamma}}\right)+\sum_{\ell=0}^{N_rN_t}\binom{N_rN_t}{\ell}(-1)^{\ell}\left(-\frac{\ell}{\overline{\gamma}}\right)\mathcal{S}_4\left(\frac{\ell}{\overline{\gamma}}\right)\right\}$

B. Performance Analysis

Following the performance analysis framework presented in the previous section, the OP, ABER and ASNR can be directly evaluated by using the CDF, MGF and the moments expressions given in (31), (33) and (34), respectively.

1) Capacity: Substituting (35) in the definition of the capacity, making an integration by parts, using [23, eqs. (3.352/1 and 4.337/2)], and after some mathematics, the following closed-form expression for the capacity of γ_{out} is derived

$$C_{\gamma_{\text{out}}}=\text{BW}\left\{\sum_{\ell=0}^{N_t}\sum_{\omega_1(\ell,N_t)}\sum_{\nu=0}^{N_t}\sum_{\omega_3(\nu,N_t)}P_1\sum_{\mu=0}^{(N_r-1)\sum_{k=1}^{N_t}\ell_k}\right.$$
$$\times\sum_{\omega_4(\mu,N_t)}P_2\left(-\sum_{k=1}^{N_t}\frac{\mu_k+\nu_k}{\overline{\gamma}_k}\right)\mathcal{S}_3\left(\sum_{k=1}^{N_t}\frac{\mu_k+\nu_k}{\overline{\gamma}_k}\right)$$
$$\left.+\sum_{\ell=0}^{N_tN_r}\sum_{\omega_2(\ell,N_t)}P_3\left(-\sum_{k=1}^{N_t}\frac{\ell_k}{\overline{\gamma}_k}\right)\mathcal{S}_4\left(\sum_{k=1}^{N_t}\frac{\ell_k}{\overline{\gamma}_k}\right)\right\}. \tag{35}$$

For i.i.d. fading conditions, simplified expressions for the CDF, PDF, MGF, the moments and the capacity of γ_{out} are given in Table II.

2) Complexity: Based on (23), the complexity of the proposed scheme evaluated in terms of the ANPE can be expressed as

$$N_{\text{out}}=\left[\sum_{k=1}^{N_t}1+(L-1)F_{\gamma_k}(\gamma_{\text{th}})\right]. \tag{36}$$

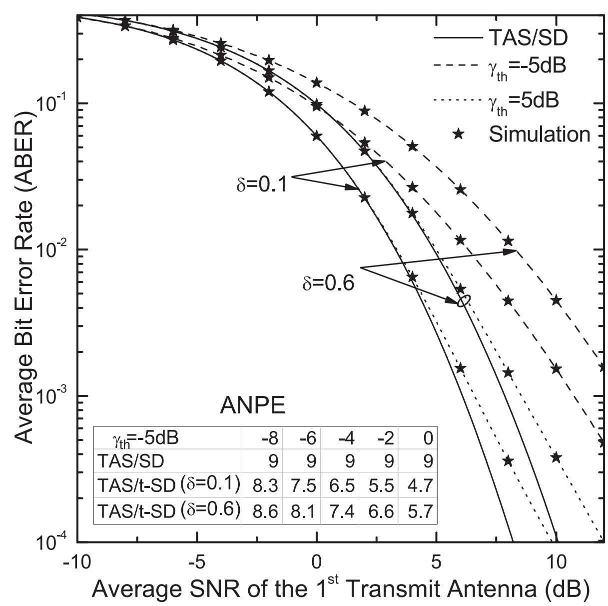

Fig. 6. ABER and ANPE of TAS/SD and TAS/t-SD as a function of $\overline{\gamma}_1$.

C. Numerical Results

Here, using the previously presented analysis, comparative numerical evaluated results will be provided, in terms of the ABER (using (33)), the ASNR (using (34)) and the ANPE (using (36)). In Fig. 6, assuming $N_t = N_r = 3$, the ABERs of TAS/SD and TAS/t-SD are plotted with respect to $\overline{\gamma}_1$, for different values of δ. It is shown in this figure that the two ABER performances are almost similar for $\overline{\gamma}_1 \leq \gamma_{\text{th}}$, while the performance gap slightly reduces with the decrease of δ. In the same

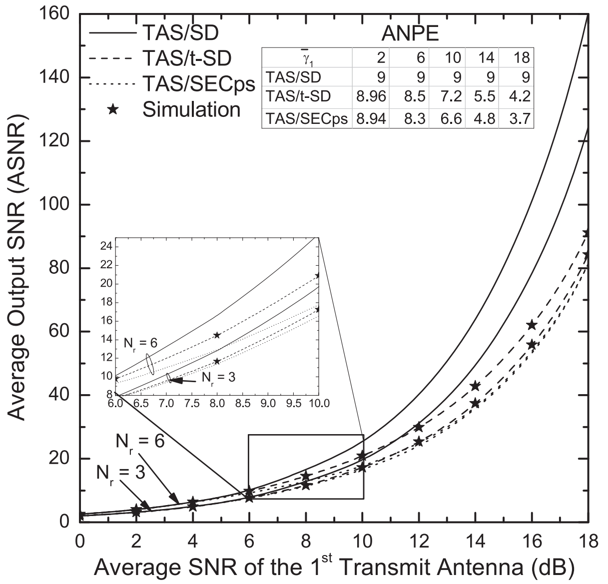

Fig. 7. ASNR and ANPE of TAS/SD, TAS/t-SD and TAS/SECps as a function of $\overline{\gamma}_1$.

figure, assuming $\gamma_{\text{th}} = -5$ dB, a table containing ANPEs, for various values of $\overline{\gamma}_1$, of the previously tested antenna selection schemes, is also included. By comparing the figure and table results, it is obvious that for the same parameters, the ANPE of t-SD is much lower than that of SD, especially for higher values of $\overline{\gamma}_1$. In Fig. 7, assuming $N_t = 3$, $\delta = 0.7$ and $\gamma_{\text{th}} = 8$ dB, the ASNR is plotted as a function of $\overline{\gamma}_1$ for different values of N_r. It is shown in this figure that TAS/t-SD has always better performance as compared to TAS/SECps, especially for higher values of N_r and/or $\overline{\gamma}_1$. In the same figure, a table containing the ANPE of the three schemes under consideration is also included, for various values of $\overline{\gamma}_1$. Comparing TAS/SD with TAS/t-SD, it is interesting to note that with a relatively small cost on the ASNR, e.g., 12% for $\overline{\gamma}_1 = 10$dB, the ANPE of t- SD is 20% less as compared to that of SD. Therefore, in this figure it is also verified that as far as the performance and complexity trade-off is concerned, TAS/t-SD represents an excellent compromise, as compared to TAS/SD and TAS/SECps. For comparison purposes, computer simulations performance results are also included in Figs. 6 and 7, verifying in all cases the validity of the proposed theoretical approach.

V. Cooperative Communications Framework

We consider a cooperative communication scenario, where a source node (S) communicates with the destination node (D), directly and/or with the aid of L relay nodes R_ℓ, with $\ell = 1, 2, \ldots, L$, using the amplify-and-forward protocol, as shown in Fig. 8. Each node is equipped with a single antenna and the transmission is realized in two phases under the half-duplex constraint. We assume slow fading and that channel reciprocity can be exploited. At the guard interval of each transmission period, the source and the destination send ready-to-send (RTS) and clear to send (CTS) packets. Based on these packets, the previously selected relay estimates the instantaneous SNRs of the source-relay and relay-destination links. The resulting SNR estimate is compared with a

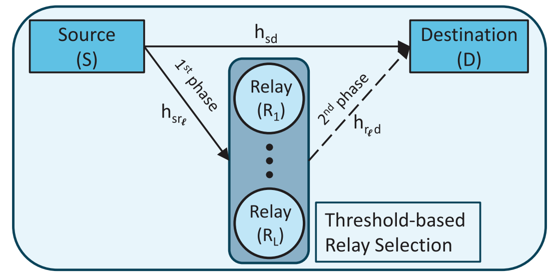

Fig. 8. The threshold-based relay selection.

predefined threshold γ_{th} and if it exceeds it, then this relay transmits a short duration flag packet, signaling its presence. In that case, as soon as the remaining relays "hear" this relay to flag its presence or forward information, they backoff. Otherwise, if the predefined threshold is not exceeded, the previously selected relay sends a flag packet to the other relays and the destination asking for the initiation of a best relay procedure similar to the one proposed in [29].

Let us denote the instantaneous SNR of the received signal as $\gamma_X = h_X^2 E_s/N_0$, where $X \in \{\text{sd}, \text{sr}_\ell, \text{r}_\ell\text{d}\}$ when referring to the instantaneous SNR of the S-D, S-R_ℓ and R_ℓ-D received signals, respectively. Moreover, h_X represents the Rayleigh faded channel coefficients, E_s the average signal energy, N_0 the variance of the additive white Gaussian noise (AWGN). The corresponding average SNRs are $\overline{\gamma}_X = \mathbb{E}\langle h_X^2\rangle E_s/N_0$, with $\mathbb{E}\langle\cdot\rangle$ being the statistical averaging. In the following analysis, we have assumed exponentially distributed instantaneous SNRs, with PDF and CDF given in (11) and (12), respectively.

In the proposed selection strategy, in each transmission period, the previously selected relay, R_n, estimates its current instantaneous channels, $h_{\text{sr}_n}, h_{\text{r}_n\text{d}}$, in both links. Based on them, the end-to-end SNR of the overall $\text{S} \to \text{R}_n \to \text{D}$ channel can be evaluated as [30]

$$\gamma_{\text{sr}_n\text{d}} = \frac{\gamma_{\text{sr}_n}\gamma_{\text{r}_n\text{d}}}{\gamma_{\text{sr}_n} + \gamma_{\text{r}_n\text{d}} + 1}. \tag{37}$$

Subsequently, $\gamma_{\text{sr}_n\text{d}}$ is compared with a predefined threshold γ_{th}, and if it exceeds it, then R_n remains as the selected relay. Otherwise, the best relay, in terms of the provided end-to-end SNR, is selected. Let the total output SNR at the proposed relay selection scheme be denoted as $\gamma_{t-\text{rs}}$. Let also assume that maximal ratio combining (MRC) of the two received signals takes place at D. Then, the total output SNR at D is given as

$$\gamma_{\text{out}} = \gamma_{t-\text{rs}} + \gamma_{\text{sd}}. \tag{38}$$

The use of the exact expression of $\gamma_{\text{sr}_n\text{d}}$, provided in (37), to compute $\gamma_{t-\text{rs}}$ in (38) will result to a cumbersome mathematical analysis. A mathematically more convenient approach is to employ instead a tight upper bound of $\gamma_{\text{sr}_n\text{d}}$, as in [31], i.e.,

$$\gamma_{\text{sr}_n\text{d}} \le \gamma_n = \min\{\gamma_{\text{sr}_n}, \gamma_{\text{r}_n\text{d}}\} \tag{39}$$

where $f_{\gamma_n}(\gamma) = \frac{1}{\overline{\gamma}_n}\exp(-\frac{\gamma}{\overline{\gamma}_n})$, with $\overline{\gamma}_n = \frac{\overline{\gamma}_{\text{sr}_n}\overline{\gamma}_{\text{r}_n\text{d}}}{\phi(\overline{\gamma}_{\text{sr}_n}+\overline{\gamma}_{\text{r}_n\text{d}})}$ and $\phi = 1$. From (38) and (39), the upper bound of the total SNR is given by

$$\gamma_{\text{out}} \le \gamma_{\text{out}_\text{b}} = \gamma_\text{b} + \gamma_{\text{sd}} \tag{40}$$

where $\gamma_{\rm b}$ represents a bound on the output SNR of the relay selection scheme, $\gamma_{\rm t-rs}$, which is based on (39). Moreover, a lower bound of $\gamma_{{\rm sr}_n{\rm d}}$ can be formulated as in [32] $\gamma_{{\rm sr}_n{\rm d}} \geq \gamma_n = 0.5\min\{\gamma_{{\rm sr}_n}, \gamma_{{\rm r}_n{\rm d}}\}$, which results to $\phi = 2$ at the definition of $\overline{\gamma}_n$.

A. Statistical Framework

In the following analysis, we assume i.i.d. conditions, i.e., $\overline{\gamma}_{{\rm sr}_\ell} = \overline{\gamma}_{\rm sr}$ and $\overline{\gamma}_{{\rm r}_\ell{\rm d}} = \overline{\gamma}_{\rm rd}, \forall \ell$.

Lemma 1: Based on the selection scheme proposed in Section II, the MGF of $\gamma_{{\rm out}_{\rm b}}$ is given by

$$M_{\gamma_{{\rm out}_{\rm b}}}(s) = \frac{\phi}{\overline{\gamma}_n\overline{\gamma}_{\rm sd}}\left\{\frac{\exp\left[-\left(\overline{\gamma}_n^{-1}\right)_s\gamma_{{\rm th}_\phi}\right]}{\left(\overline{\gamma}_n^{-1}\right)_s\left(\overline{\gamma}_{\rm sd}^{-1}\right)_s} - F_{\gamma_n}\left(\gamma_{{\rm th}_\phi}\right)\right.$$
$$\times\sum_{\ell=1}^{L-1}\binom{L-1}{\ell}\frac{\exp\left[-\left(\ell/\overline{\gamma}_n\right)_s\gamma_{{\rm th}_\phi}\right]}{\left(\ell/\overline{\gamma}_n\right)_s\left(\overline{\gamma}_{\rm sd}^{-1}\right)_s}(-1)^\ell\ell + \sum_{\ell=1}^{L}\binom{L}{\ell}\ell$$
$$\left.\times(-1)^{\ell-1}\left[\frac{1}{\left(\ell/\overline{\gamma}_n\right)_s\left(\overline{\gamma}_{\rm sd}^{-1}\right)_s} - \frac{\exp\left[-\left(\ell/\overline{\gamma}_n\right)_s\gamma_{{\rm th}_\phi}\right]}{\left(\ell/\overline{\gamma}_n\right)_s\left(\overline{\gamma}_{\rm sd}^{-1}\right)_s}\right]\right\} \quad (41)$$

where $\gamma_{{\rm th}_\phi} = \gamma_{\rm th}/\phi$.

Proof: See Appendix C. ■

Lemma 2: The PDF of $\gamma_{{\rm out}_{\rm b}}$ is given by

$$f_{\gamma_{{\rm out}_{\rm b}}}(\gamma) = \frac{f_{\gamma_n}\left(\gamma_{{\rm th}_\phi}\right)}{\overline{\gamma}_{\rm sd}}\left\{U\left(\gamma-\gamma_{{\rm th}_\phi}\right)\left[\frac{\exp\left(-\frac{\gamma-\gamma_{{\rm th}_\phi}}{\overline{\gamma}_{\rm sd}}\right)}{\overline{\gamma}_{d,1}}\right.\right.$$
$$\left.-\frac{\exp\left(-\frac{\gamma-\gamma_{{\rm th}_\phi}}{\overline{\gamma}_n}\right)}{\overline{\gamma}_{d,1}}\right] - \sum_{\ell=1}^{L-1}\frac{\binom{L-1}{\ell}(-1)^\ell\ell}{\overline{\gamma}_{d,\ell}}U\left(\gamma-\gamma_{{\rm th}_\phi}\right)$$
$$\times F_{\gamma_n}\left(\gamma_{{\rm th}_\phi}\right)\left[\exp\left(-\frac{\gamma-\gamma_{{\rm th}_\phi}}{\overline{\gamma}_{\rm sd}}\right) - \exp\left[-\frac{\left(\gamma-\gamma_{{\rm th}_\phi}\right)\ell}{\overline{\gamma}_n}\right]\right]$$
$$\times\sum_{\ell=1}^{L}\frac{\binom{L}{\ell}(-1)^{\ell-1}\ell}{\overline{\gamma}_n\overline{\gamma}_{d,\ell}}\left[\left[\exp\left(-\frac{\gamma}{\overline{\gamma}_{\rm sd}}\right) - \exp\left(-\frac{\ell\gamma}{\overline{\gamma}_n}\right)\right]\right.$$
$$-\exp\left(-\frac{\ell\gamma_{{\rm th}_\phi}}{\overline{\gamma}_n}\right)U\left(\gamma-\gamma_{{\rm th}_\phi}\right)$$
$$\left.\left.\times\left[\exp\left(-\frac{\gamma-\gamma_{{\rm th}_\phi}}{\overline{\gamma}_{\rm sd}}\right) - \exp\left[-\frac{(\gamma-\gamma_{{\rm th}_\phi})\ell}{\overline{\gamma}_n}\right]\right]\right]\right\} \quad (42)$$

where $\overline{\gamma}_{\rm d,x} = \frac{x}{\overline{\gamma}_n} - \frac{1}{\overline{\gamma}_{\rm sd}}$ and $U(\cdot)$ denotes the unit step function.

Proof: Based on the Laplace transform identity [33, eq. (1.1.1/25)] and using [23, eqs. (17.13/9a and 17.13/95)], the inverse Laplace transform of (41) yields (42). ■

Lemma 3: The CDF of $\gamma_{{\rm out}_{\rm b}}$ is given by

$$F_{\gamma_{{\rm out}_{\rm b}}}(\gamma) = \frac{1}{\overline{\gamma}_{\rm sd}}\left\{U\left(\gamma-\gamma_{{\rm th}_\phi}\right)\left[\frac{\exp\left(-\frac{\gamma_{{\rm th}_\phi}}{\overline{\gamma}_n}\right)}{\overline{\gamma}_{d,1}}\left[\frac{\overline{\gamma}_{\rm sd}}{\overline{\gamma}_n}\right.\right.\right.$$
$$\left.\left.\times F_{\gamma_{\rm sd}}\left(\gamma-\gamma_{{\rm th}_\phi}\right) - F_{\gamma_n}\left(\gamma-\gamma_{{\rm th}_\phi}\right)\right]\right] - F_{\gamma_n}\left(\gamma_{{\rm th}_\phi}\right)$$
$$\times\sum_{\ell=1}^{L-1}\frac{\binom{L-1}{\ell}(-1)^\ell}{\overline{\gamma}_{d,\ell}}\exp\left(-\frac{\ell\gamma_{{\rm th}_\phi}}{\overline{\gamma}_n}\right)\left[\frac{\ell\overline{\gamma}_{\rm sd}}{\overline{\gamma}_n}F_{\gamma_{\rm sd}}\left(\gamma-\gamma_{{\rm th}_\phi}\right)\right.$$
$$\left.\left.-F_{\gamma_n}\left[\left(\gamma-\gamma_{{\rm th}_\phi}\right)\ell\right]\right]\right] + \sum_{\ell=1}^{L}\binom{L}{\ell}\frac{(-1)^{\ell-1}}{\overline{\gamma}_{d,\ell}}$$
$$\times\left[\left[\frac{\ell\overline{\gamma}_{\rm sd}}{\overline{\gamma}_n}F_{\gamma_{\rm sd}}(\gamma) - F_{\gamma_n}(\gamma\ell)\right] - \exp\left(-\frac{\ell\gamma_{{\rm th}_\phi}}{\overline{\gamma}_n}\right)U\left(\gamma-\gamma_{{\rm th}_\phi}\right)\right.$$
$$\left.\left.\times\left[\frac{\ell\overline{\gamma}_{\rm sd}}{\overline{\gamma}_n}F_{\gamma_{\rm sd}}\left(\gamma-\gamma_{{\rm th}_\phi}\right) - F_{\gamma_n}\left[\left(\gamma-\gamma_{{\rm th}_\phi}\right)\ell\right]\right]\right]\right\}. \quad (43)$$

Proof: The CDF expression can be directly evaluated by integrating (42). ■

Lemma 4: The mth order moment of $\gamma_{{\rm out}_{\rm b}}$ is given by

$$\mu_{\gamma_{{\rm out}_{\rm b}}}(m) = \frac{1}{\overline{\gamma}_{\rm sd}}\left\{\frac{1/\overline{\gamma}_n}{\overline{\gamma}_{\rm d,1}}\left[\exp\left(-\overline{\gamma}_{\rm d,1}\gamma_{{\rm th}_\phi}\right)\overline{\gamma}_{\rm sd}^{m+1}\right.\right.$$
$$\left.\times\Gamma\left(m+1, \frac{\gamma_{{\rm th}_\phi}}{\overline{\gamma}_{\rm sd}}\right) - \overline{\gamma}_n^{m+1}\Gamma\left(m+1, \frac{\gamma_{{\rm th}_\phi}}{\overline{\gamma}_n}\right)\right] - F_{\gamma_n}\left(\gamma_{{\rm th}_\phi}\right)\sum_{\ell=1}^{L-1}$$
$$\times\binom{L-1}{\ell}\frac{(-1)^\ell\ell}{\overline{\gamma}_n\overline{\gamma}_{\rm d,\ell}}\left[\exp\left(-\overline{\gamma}_{\rm d,\ell}\gamma_{{\rm th}_\phi}\right)\overline{\gamma}_{\rm sd}^{m+1}\Gamma\left(m+1, \frac{\gamma_{{\rm th}_\phi}}{\overline{\gamma}_{\rm sd}}\right)\right.$$
$$\left.-\frac{\Gamma\left(m+1, \gamma_{{\rm th}_\phi}\ell/\overline{\gamma}_n\right)}{\left(\ell/\overline{\gamma}_n\right)^{m+1}}\right] + \sum_{\ell=1}^{L}\binom{L}{\ell}(-1)^{\ell-1}\frac{\ell/\overline{\gamma}_n}{\overline{\gamma}_{\rm d,\ell}}\left[\overline{\gamma}_{\rm sd}^{m+1}\right.$$
$$\times\Gamma(m+1) - \frac{\Gamma(m+1)}{\left(\ell/\overline{\gamma}_n\right)^{m+1}} - \exp\left(-\overline{\gamma}_{\rm d,\ell}\gamma_{{\rm th}_\phi}\right)\overline{\gamma}_{\rm sd}^{m+1}$$
$$\left.\left.\times\Gamma\left(m+1, \frac{\gamma_{{\rm th}_\phi}}{\overline{\gamma}_{\rm sd}}\right) + \frac{\Gamma\left(m+1, \gamma_{{\rm th}_\phi}\ell/\overline{\gamma}_n\right)}{\left(\ell/\overline{\gamma}_n\right)^{m+1}}\right]\right\}. \quad (44)$$

Proof: The mth order moment of $\gamma_{{\rm out}_{\rm b}}$ can be evaluated by substituting (42) in the definition of the moments. Then, using [23, eqs. (3.351/2 and 3.351/3)], (44) is obtained. ■

B. Performance Analysis

Using the previously derived expressions for the PDF, CDF, MGF and moments of the output SNR, various performance evaluation criteria are presented next such as the OP, ABER, ASNR and the capacity, while complexity and asymptotic analyses are also provided.

1) Outage Probability (OP): Based on (43), OP can be directly evaluated as

$$P_{{\rm out}_{\rm b}} = F_{\gamma_{{\rm out}_{\rm b}}}(\gamma_{\rm T}). \quad (45)$$

2) Average Bit Error Rate (ABER): Using the previously derived MGF expression in (41), the ABER can be calculated directly for DBPSK as $P_{\text{be}}^{DBPSK} = 0.5M_{\gamma_{\text{out}_\text{b}}}(1)$. In addition, for BPSK (and square M-QAM at high SNRs) and following a similar approach as in Subsection III-B2, the ABER in the proposed scheme is given by

$$P_{\text{be}}^{BPSK} = \frac{A}{\overline{\gamma}_n}\left\{\exp\left(-\frac{\gamma_{\text{th}_\phi}}{\overline{\gamma}_n}\right)\mathcal{S}_5(1) - F_{\gamma_n}\left(\gamma_{\text{th}_\phi}\right)\sum_{\ell=1}^{L-1}\binom{L-1}{\ell}\right.$$
$$\times \ell(-1)^\ell \exp\left(-\frac{\ell\gamma_{\text{th}_\phi}}{\overline{\gamma}_n}\right)\mathcal{S}_5(\ell) + \sum_{\ell=1}^{L}\binom{L}{\ell}(-1)^{\ell-1}\ell\left[\frac{1}{\overline{\gamma}_{\text{d},\ell}}\right.$$
$$\left.\left. - \frac{\sqrt{\frac{B\overline{\gamma}_{\text{sd}}}{1+B\overline{\gamma}_{\text{sd}}}}}{\overline{\gamma}_{\text{d},\ell}} - \frac{\overline{\gamma}_n}{\ell\overline{\gamma}_{\text{sd}}}\frac{1-\sqrt{\frac{B}{\ell/\overline{\gamma}_n + B}}}{\overline{\gamma}_{\text{d},\ell}} - \exp\left(-\frac{\ell\gamma_{\text{th}_\phi}}{\overline{\gamma}_n}\right)\mathcal{S}_5(\ell)\right]\right\} \tag{46}$$

where

$$\mathcal{S}_5(\ell) = \frac{\overline{\gamma}_{\text{sd}}\overline{\gamma}_n}{\overline{\gamma}_{\text{sd}}\ell - \overline{\gamma}_n}\left[\text{erfc}\left(\sqrt{B\gamma_{\text{th}_\phi}}\right) - \frac{\sqrt{B\overline{\gamma}_{\text{sd}}}\exp\left(\gamma_{\text{th}_\phi}/\overline{\gamma}_{\text{sd}}\right)}{\sqrt{1+B\overline{\gamma}_{\text{sd}}}}\right.$$
$$\left.\times\text{erfc}\left(\sqrt{\frac{\gamma_{\text{th}_\phi}}{\overline{\gamma}_{\text{sd}}} + B\gamma_{\text{th}_\phi}}\right)\right] - \frac{\overline{\gamma}_n^2}{\ell\left(\overline{\gamma}_{\text{sd}}\ell - \overline{\gamma}_n\right)}\left[\text{erfc}\left(\sqrt{B\gamma_{\text{th}_\phi}}\right)\right.$$
$$\left. - \frac{\sqrt{B\overline{\gamma}_n}\exp\left(\gamma_{\text{th}_\phi}\ell/\overline{\gamma}_n\right)}{\sqrt{\ell + B\overline{\gamma}_n}}\text{erfc}\left(\sqrt{\frac{\ell\gamma_{\text{th}_\phi}}{\overline{\gamma}_n} + B\gamma_{\text{th}_\phi}}\right)\right].$$

3) Average Output SNR (ASNR): ASNR can be directly evaluated by setting $m = 1$ in (44).

4) Ergodic Capacity: Substituting (42) in the definition of the capacity, making an integration by parts and using [23, eqs. (3.352/1 and 4.337/1)], the following closed-form expression can be derived

$$C_{\gamma_{\text{out}}} = \frac{BW}{2\ln(2)}\left\{\exp\left(-\frac{\gamma_{\text{th}_\phi}}{\overline{\gamma}_n}\right)\frac{\mathcal{S}_6\left(1,\gamma_{\text{th}_\phi}\right)}{\overline{\gamma}_n\overline{\gamma}_{\text{sd}}} - F_{\gamma_n}\left(\gamma_{\text{th}_\phi}\right)\right.$$
$$\times\sum_{\ell=1}^{L-1}\frac{\ell}{\overline{\gamma}_n}\binom{L-1}{\ell}\frac{(-1)^\ell}{\overline{\gamma}_{\text{sd}}}\exp\left(-\frac{\ell\gamma_{\text{th}_\phi}}{\overline{\gamma}_n}\right)\mathcal{S}_6\left(\ell,\gamma_{\text{th}_\phi}\right)$$
$$+\sum_{\ell=1}^{L}\binom{L}{\ell}\frac{(-1)^{\ell-1}\ell}{\overline{\gamma}_n\overline{\gamma}_{\text{sd}}}\left[\mathcal{S}_6\left(\ell,0\right)\right.$$
$$\left.\left. - \exp\left(-\frac{\ell\gamma_{\text{th}_\phi}}{\overline{\gamma}_n}\right)\mathcal{S}_6\left(\ell,\gamma_{\text{th}_\phi}\right)\right]\right\} \tag{47}$$

where

$$\mathcal{S}_6(x,y) = \frac{\overline{\gamma}_{\text{sd}}}{\overline{\gamma}_{\text{d},x}}\left[\ln(1+y) - \exp\left(\frac{1+y}{\overline{\gamma}_{\text{sd}}}\right)\text{Ei}\left(-\frac{1+y}{\overline{\gamma}_{\text{sd}}}\right)\right]$$
$$-\frac{\overline{\gamma}_n}{x\overline{\gamma}_{\text{d},x}}\left[\ln(1+y) - \exp\left(\frac{(1+y)x}{\overline{\gamma}_n}\right)\text{Ei}\left(-\frac{(1+y)x}{\overline{\gamma}_n}\right)\right].$$

5) Complexity Analysis: Complexity will be analyzed using the average number of active relays (ANAR) and the SP.

a) Average Number of Active Relays (ANAR): The system complexity increases as the ANAR increases, due to the important amount of information that must be exchanged for performing various operations, e.g., channel estimation [25]. Following the approach presented in Subsection III-B5a, it can be shown that ANAR is given by

$$N_{\text{out}} = 1 + (L-1)F_{\gamma_n}\left(\gamma_{\text{th}_\phi}\right). \tag{48}$$

b) Switching Probability (SP): Switching among relays consumes power, reduces the data throughput as well as leads to inaccurate phase estimates [16], [26]. For i.i.d. fading, following a similar approach as that of Subsection III-B5b, the relay SP, P_{out}^s, is also given by

$$P_{\text{out}}^s = F_{\gamma_n}\left(\gamma_{\text{th}_\phi}\right)\left[1 - \frac{1}{L}F_{\gamma_n}\left(\gamma_{\text{th}_\phi}\right)^{L-1}\right]. \tag{49}$$

6) Asymptotic Analysis: In the high SNR regime, using the asymptotic analysis provided in Subsection III-B6 to the analytical framework presented in Appendix C, the ABER of the DBPSK modulation scheme can be expressed as follows

$$\overline{P}_{\text{be}} = \frac{\exp\left(-\gamma_{\text{th}_\phi}\right)}{2\overline{\gamma}_{\text{sd}}}\left(\frac{1}{\overline{\gamma}_n} - \frac{1+\gamma_{\text{th}_\phi}}{\overline{\gamma}_{\text{sr}}\overline{\gamma}_{\text{rd}}}\right). \tag{50}$$

Assuming $\overline{\gamma}_{\text{sr}} = \overline{\gamma}_{\text{rd}} = \overline{\gamma}_{\text{sd}} = \overline{\gamma}$ and $\overline{\gamma} > \gamma_{\text{th}}$, the second term in the right hand side of (50) has a negligible effect on the performance. Therefore, $\overline{P}_{\text{be}}$ can be expressed as

$$\overline{P}_{\text{be}} = \exp\left(-\gamma_{\text{th}_\phi}\right)\overline{\gamma}^{-2}. \tag{51}$$

It is obvious that (51) is also of the form $(G_c\text{SNR})^{-G_d}$. Thus, the coding gain is affected by the switching threshold, while the diversity gain is equal to 2 as in the SEC and SECps relay selection schemes [18]. On the other hand, for $\overline{\gamma} \leq \gamma_{\text{th}}$, by following a similar procedure as previously, it can be proved that $\overline{P}_{\text{be}}$ can be expressed as

$$\overline{P}_{\text{be}} = 2^{L-1}L!\overline{\gamma}^{-L-1}. \tag{52}$$

In this case, it is interesting to note that the diversity gain is equal to that of BRS, [31]. This also means that depending upon the switching threshold, t-RS switches operation between a pure BRS and a SECps.

C. Numerical Results and Simulations

Here, numerical performance evaluation results will be presented for the t-RS scheme, in terms of the OP (using (43)), the ABER (using (46)), the ANAR (using (48)) and the SP (using (49)). Additionally, for comparison purposes, the corresponding performances of other relay selection schemes are also included.

In Fig. 9, assuming $L = 4$ and normalized outage threshold, $\gamma_T/\overline{\gamma}_X = -6$dB, the OPs of two cooperative relaying schemes, namely BRS, obtained using [30, eq. (18)], and t-RS, are plotted as a function of the switching threshold, γ_{th}. In the same figure, the relay SP and the average ANAR are also included. It is shown that for higher values of γ_{th}, the OP of t-RS becomes

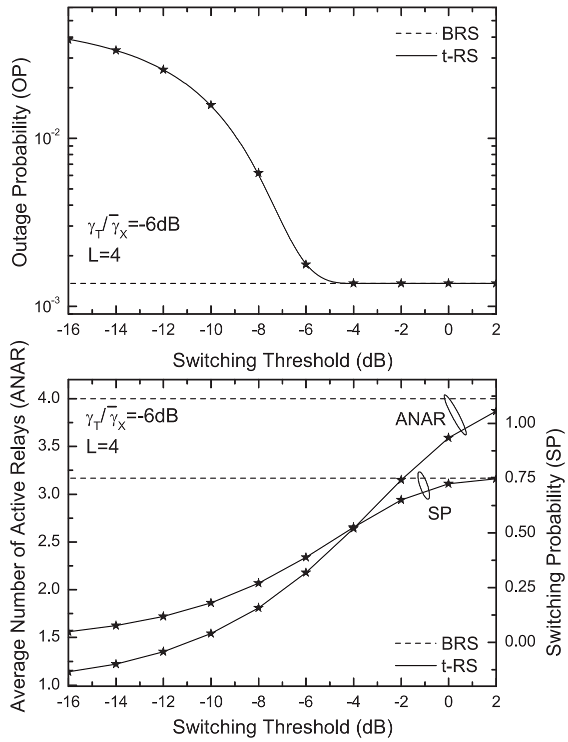

Fig. 9. OP, ANAR and SP of BRS and t-RS as a function of γ_{th}.

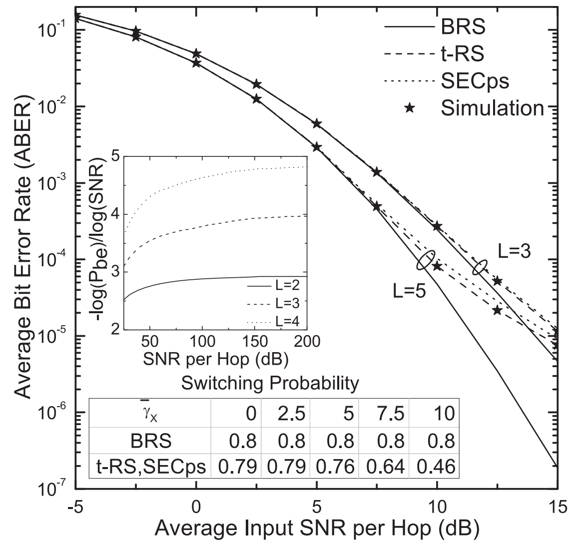

Fig. 10. ABER and SP of BRS, SECps and t-RS as a function of $\overline{\gamma}_X$.

almost equal to that of BRS. However, for these high values of γ_{th}, ANAR as well as SP of t-RS are considerably lower as compared with the corresponding ones of BRS. Therefore, without having any major loss in performance, important energy savings can be achieved. In Fig. 10, considering the BPSK modulation scheme and $\gamma_{\text{th}} = 5$dB, the ABERs of BRS (obtained using [30, eq. (15)]), SECps (obtained using [18, eq. (15)]) and t-RS are plotted as a function of the average input SNR per hop, $\overline{\gamma}_X$, and for various values of L. It is depicted that for lower values of $\overline{\gamma}_X$, the ABER performances are equal. A performance gap appears when $\overline{\gamma}_X > \gamma_{\text{th}}$, with BRS having always the best performance, as expected, and SECps the worst, while the performance gap between t-RS and SECps increases as L increases. In the same figure, assuming also $L = 5$, a table where the SP of the previously examined relay selection schemes is also included. Comparing the figure and table results of t-RS and BRS, it is obvious that for $\overline{\gamma}_X \approx \gamma_{\text{th}}$, t-RS offers a considerable reduction on the SP, without any important loss on the ABER. Finally also in the same figure, assuming $\gamma_{\text{th}} = 20$dB and DBPSK modulation, the diversity order, defined as $d = -\log\left(\overline{P}_{\text{be}}\right)/\log(\text{SNR})$, is plotted as a function of the SNR for different values of the number of relays L. As depicted in this figure, as the SNR tends to infinity, the diversity order approaches $L + 1$, verifying the analysis of Section V.B.6. Computer simulation performance results are also included in Figs. 9 and 10, verifying the validity of the proposed theoretical approach.

VI. Conclusions

In this paper a novel threshold-based channel selection strategy is proposed and analyzed. In particular, assuming not identical channel statistics and based on the Markov chain theory, a generic analytical framework is proposed and then used to derive important statistical metrics such as the PDF and CDF of the output SNR for the scheme under consideration. The proposed strategy can be applied to different communication scenarios. As representative examples, in this paper three application cases have been considered, namely i) threshold-based diversity reception (t-SD), ii) transmit antenna selection with t-SD (TAS/t-SD) and iii) threshold-based relay selection (t-RS). In all cases, closed-form expressions for the OP, ABER, ASNR and the capacity have been obtained and employed to analyze the performance of the systems under consideration. It was shown that the proposed scheme provides excellent compromise between performance and complexity as compared to previously known selection schemes.

$$
\begin{aligned}
P_{i,j} &= \Pr\left[\gamma_j = \max(\mathbb{G}), \gamma_i < \gamma_{\text{th}}\right] = \sum_{b=0}^{L-2} \underbrace{\Pr\left[\gamma_j \geq \gamma_{(1)} \geq \gamma_{(2)} \geq \cdots \geq \gamma_{(b)} \geq \gamma_i\,,\ \gamma_i \geq \gamma_{\text{L}_{\max}}\,,\ \gamma_i < \gamma_{\text{th}}\right]}_{P_{i,j}^{b}} \\
&= \underbrace{\Pr\left[\gamma_j \geq \gamma_i\,,\ \gamma_i \geq \gamma_{\text{L}_{\max}}\,,\ \gamma_i < \gamma_{\text{th}}\right]}_{P_{i,j}^{0}} + \underbrace{\Pr\left[\gamma_j \geq \gamma_{(1)} \geq \gamma_i, \gamma_i \geq \gamma_{\text{L}_{\max}}, \gamma_i < \gamma_{\text{th}}\right]}_{P_{i,j}^{1}} + \cdots \\
&\quad + \underbrace{\Pr\left[\gamma_j \geq \gamma_{(1)} \geq \gamma_{(2)} \geq \cdots \geq \gamma_{(L-2)} \geq \gamma_i\,,\ \gamma_i \geq \gamma_{\text{L}_{\max}}\,,\ \gamma_i < \gamma_{\text{th}}\right]}_{P_{i,j}^{L-2}}. \qquad \text{(A-2)}
\end{aligned}
$$

APPENDIX A
THE TRANSITION PROBABILITIES $P_{i,j}$

Let us define the sets

$$\mathbb{F} = \mathbb{G} - \{\gamma_i, \gamma_j\}, \mathbb{B} \subseteq \mathbb{F}, \mathbb{L} = \mathbb{F} - \mathbb{B}. \quad \text{(A-1)}$$

Note that $\mathbb{B}$ is an arbitrary subset of $\mathbb{F}$ with cardinality $|\mathbb{B}| = b \leq L - 2$. We consider that $\mathbb{B}$ may contain any combination of b elements from $\mathbb{F}$. Next, we re-arrange the elements of $\mathbb{B}$ in descending order, i.e., $\gamma_{(1)} \geq \gamma_{(2)} \geq \ldots \geq \gamma_{(b)}$. Hence, assuming $\gamma_{\mathbb{L}_{\max}} = \max\{\mathbb{L}\}$ and for all $b \in [0, L-2]$, the transition probabilities $P_{i,j}$ can be expressed as in (A-2) (shown at the bottom of the previous page). In (A-2), $P_{i,j}^0$ can be directly evaluated as

$$P_{i,j}^0 = \int_0^{\gamma_{\text{th}}} \left[\int_x^\infty f_{\gamma_j}(y)dy\right] F_{\gamma_{\mathbb{L}_{\max}}}(x) f_{\gamma_i}(x)dx. \quad \text{(A-3)}$$

Moreover, it should be noted that the joint PDF of the ordered statistical set $\{\gamma_{(1)}, \gamma_{(2)}, \cdots, \gamma_{(b)}\}$ can be expressed as [34], [35]

$$f_{\gamma_{(1)},\gamma_{(2)},\ldots,\gamma_{(b)}}(x_1, x_2, \ldots, x_b) = \sum_{n_{1:b}=1}^{L} f_{\gamma_{n_1}}(x_1) f_{\gamma_{n_2}}(x_2) \cdots f_{\gamma_{n_b}}(x_b) \quad \text{(A-4)}$$

where $\sum_{n_{1:b}=1}^{L} \equiv \sum_{\substack{n_1=1 \\ n_1 \neq i \neq j}}^{L} \sum_{\substack{n_2=1 \\ n_2 \neq n_1}}^{L} \cdots \sum_{\substack{n_b=1 \\ n_b \neq n_{b-1}}}^{L}$ and $x_1 \geq x_2 \geq \ldots \geq x_b \geq 0$. Therefore, for $b \neq 0$, it is straightforward to show that the probabilities $P_{i,j}^b$ in (A-2) are given by

$$P_{i,j}^b = \sum_{n_{1:b}=1}^{L} \int_0^{\gamma_{\text{th}}} \left[\int_x^\infty \int_{x_1}^\infty \cdots \int_{x_{b-1}}^\infty \int_{x_b}^\infty f_{\gamma_j}(y) \times f_{\gamma_{n_b}}(x_b) f_{\gamma_{n_{b-1}}}(x_{b-1}) \cdots f_{\gamma_{n_1}}(x_1) dy dx_b dx_{b-1} \ldots dx_1\right] \times \left[\int_0^x f_{\gamma_{\mathbb{L}_{\max}}}(z)dz\right] f_{\gamma_i}(x)dx. \quad \text{(A-5)}$$

Using (A-3) and (A-5) in (A-2) leads to (7).

APPENDIX B
PROOF OF EQ. (31)

Using (12) in (30), employing [36, eq. (45)] and after some mathematical manipulations yields

$$F_{\gamma_{\text{out}}}(\gamma) = \begin{cases} \sum_{\ell=0}^{N_t} \sum_{\omega_1(\ell,N_t)} \left[\prod_{k=1}^{N_t} \left(\exp\left(-\frac{\gamma_{\text{th}}}{\overline{\gamma}_k}\right) - \exp\left(-\frac{\gamma}{\overline{\gamma}_k}\right)\right)^{1-\ell_k}\right] \\ \times \left[\prod_{k=1}^{N_t} F_{\gamma_k}(\gamma_{\text{th}})^{\ell_k}\right] \\ \times \left[\prod_{k=1}^{N_t} \left(1 - \exp\left(-\frac{\gamma}{\overline{\gamma}_k}\right)\right)^{\ell_k(N_r-1)}\right], \gamma \geq \gamma_{\text{th}} \\ \sum_{\ell=0}^{N_t N_r} \sum_{\omega_2(\ell,N_t)} (-1)^\ell P_3 \exp\left(-\sum_{k=1}^{N_t} \frac{\gamma \ell_k}{\overline{\gamma}_k}\right), \gamma < \gamma_{\text{th}}. \end{cases} \quad \text{(B-1)}$$

where $\omega_1(\ell, N_t) = \left\{(\ell_1, \ldots, \ell_{N_t}) : \ell_k \in \{0, 1\}, \sum_{k=1}^{N_t} \ell_k = \ell\right\}$, $\omega_2(\ell, N_t) = \left\{(\ell_1, \ldots, \ell_{N_t}) : \ell_k \in \{0, 1, \ldots, N_r\}, \sum_{k=1}^{N_t} \ell_k = \ell\right\}$.

Using again [36, eq. (45)] in (B-1) and after some mathematical manipulation yields

$$F_{\gamma_{\text{out}}}(\gamma) = \begin{cases} \sum_{\ell=0}^{N_t} \sum_{\omega_1(\ell,N_t)} \sum_{\nu=0}^{N_t} \sum_{\omega_3(\nu,N_t)} (-1)^\nu P_1 \exp\left(-\sum_{k=1}^{N_t} \frac{\gamma \nu_k}{\overline{\gamma}_k}\right) \\ \times \left[\prod_{k=1}^{N_t} \left(1 - \exp\left(-\frac{\gamma}{\overline{\gamma}_k}\right)\right)^{\ell_k(N_r-1)}\right], \gamma \geq \gamma_{\text{th}} \\ \sum_{\ell=0}^{N_t N_r} \sum_{\omega_2(\ell,N_t)} (-1)^\ell P_3 \exp\left(-\sum_{k=1}^{N_t} \frac{\gamma \ell_k}{\overline{\gamma}_k}\right), \gamma < \gamma_{\text{th}}. \end{cases} \quad \text{(B-2)}$$

where $\omega_3(\nu, N_t) = \{(\nu_1, \ldots, \nu_{N_t}) : \nu_k \in \{0, 1 - \ell_k\}, \sum_{k=1}^{N_t} \nu_k = \nu\}$. Finally, employing the binomial identity and using [36, eq. (45)] in (B-2), yields the closed-form expression given in (31).

APPENDIX C
PROOF OF LEMMA 1

Based on the PDF expression shown in Table I, the PDF of γ_b is given by

$$f_{\gamma_b}(\gamma) = \begin{cases} \overline{\gamma}_n \exp(-\overline{\gamma}_n \gamma) - F_{\gamma_n}(\gamma_{\text{th}}) \sum_{\ell=1}^{L-1} \binom{L-1}{\ell} \\ \times (-1)^\ell \overline{\gamma}_n \ell \exp(-\overline{\gamma}_n \ell \gamma), \gamma \geq \gamma_{\text{th}_\phi} \\ \sum_{\ell=1}^{L} \binom{L}{\ell} (-1)^{\ell-1} \overline{\gamma}_n \ell \exp(-\overline{\gamma}_n \ell \gamma), \gamma < \gamma_{\text{th}_\phi}. \end{cases} \quad \text{(C-1)}$$

Substituting (C-1) in the definition of the MGF and using [23, eqs. (2.311 and 3.310)] yields the following expression for the MGF of γ_b

$$M_{\gamma_b}(s) = \frac{\overline{\gamma}_n}{(\overline{\gamma}_n)_s} \exp\left[-(\overline{\gamma}_n)_s \gamma_{\text{th}_\phi}\right] - F_{\gamma_n}(\gamma_{\text{th}_\phi}) \times \sum_{\ell=1}^{L-1} \binom{L-1}{\ell} (-1)^\ell \frac{\overline{\gamma}_n \ell}{(\overline{\gamma}_n \ell)_s} \exp\left[-(\overline{\gamma}_n \ell)_s \gamma_{\text{th}_\phi}\right] + \sum_{\ell=1}^{L} \binom{L}{\ell} (-1)^{\ell-1} \frac{\overline{\gamma}_n \ell}{(\overline{\gamma}_n \ell)_s} \left[1 - \exp\left[-(\overline{\gamma}_n \ell)_s \gamma_{\text{th}_\phi}\right]\right]. \quad \text{(C-2)}$$

Finally, the MGF of γ_{out_b} can be easily evaluated as $M_{\gamma_{\text{out}_b}}(s) = M_{\gamma_b}(s) M_{\gamma_{\text{sd}}}(s)$, where $M_{\gamma_{\text{sd}}}(s) = \frac{1}{1+\overline{\gamma}_{\text{sd}} s}$, resulting to (41), which also completes the proof.

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