SEMI-BLIND ESTIMATION OF MULTIPATH CHANNEL PARAMETERS VIA A SEPARABLE LEAST SQUARES APPROACH

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Abstract: In this paper a novel semi-blind technique is proposed for estimating the time delays and attenuation factors of a multipath channel. The technique is based on a suitable application of the Subchannel Response Matching criterion. The resulting cost function is separable with respect to the two sets of unknown parameters, i.e. time delays and attenuation factors. Thus an efficient two step optimization procedure can be applied. The new method offers computational savings and a lower mean square estimation error as compared to existing semi-blind channel estimation methods. The performance of the new method has been justified theoretically and tested through extensive simulations.

1. INTRODUCTION

The problem of identifying the parameters of a multipath channel is of particular interest in many applications, such as, radar, sonar, underwater acoustics, and wireless communications. In this paper we focus mainly on wireless communications applications in which the multipath phenomenon may cause a severe probability of error due to intersymbol interference (ISI). In such a case, an accurate estimate of the channel impulse response (IR) may be exploited to reduce significantly the introduced ISI.

In high-speed wireless applications, the multipath channel tends to be of a discrete form and consists of a relatively small number of dominant components [1]. Then, provided that the transmitter and receiver filter IRs are known, the channel estimation task is reduced to that of estimating the time delays and attenuation factors of the channel components. There is a number of advantages in using this alternative parameterization. First a significant saving in complexity is achieved and the number of required data is correspondingly reduced. Second, for the same number of data, a lower estimation error is achieved due to the reduced number of involved parameters.

In typical wireless systems, some known symbols are periodically transmitted for synchronization and other control purposes. The length of this known data sequence is often too short to be used as training sequence for effective channel estimation. On the other hand, blind methods which exploit the input signal statistics, ignore the presence of training data and as a result they have also limited performance. For these reasons, semi-blind methods, which combine the advantages of blind and training-based techniques have been recently proposed [2], [3], [4].

A novel semi-blind technique, which exploits the specular channel structure, is proposed in this paper. The technique is based on the well-known Subchannel Response Matching criterion (SRM) [5]. Applying this criterion to the problem at hand we end up with a least squares (LS) problem, which is separable with respect to the unknown parameters, i.e. the time delays and the attenuation factors. The Golub-Pereyra method [6] is then applied in order to separate the optimization problem to two different sub-problems. A sub-problem which is non-linear with respect to the time delays and a sub-problem which is linear with respect to the attenuation parameters. The minimization cost function of the nonlinear problem turns out to possess a special structure, which is exploited for deriving a computationally efficient linear search method for the estimation of the unknown time delays. Subsequently, the Gauss-Newton algorithm may be applied in order to further improve the accuracy of the estimated values. Finally, the attenuation parameters are estimated by solving a linear LS problem. The new method is very simple to implement and for the same degree of estimation accuracy has a computational complexity which is much lower as compared to the existing semi-blind channel parameter estimation methods. Moreover it offers the possibility of trading off performance to complexity in a straightforward manner. The performance of the new method has been justified theoretically and tested through extensive simulations.

The paper is organized as follows. In Section 2 the multipath channel model is defined and the problem is formulated. In Section 3 the new method is derived and a theoretical justification is provided in the appendix. Finally, in Section 4 simulation results verifying the performance of the new method are provided.

2. PROBLEM FORMULATION

In general the channel IR encountered in wireless communication systems has a form which varies significantly depending on several factors. However a common trait in most cases, particularly in high speed applications, is that the multipath channel tends to be of a discrete form (i.e. it consists of a number of dominant multipath components).

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More specifically, if the channel IR is assumed to be time invariant within a small-scale time interval then it may be written as

$$h_c(t) = a_0\delta(t) + \sum_{i=1}^{p-1} a_i\delta(t-\tau_i) \tag{1}$$

where a_i and τ_i are the complex attenuation factor and the delay, respectively, of the $i-th$ multipath component. Without loss of generality, it is assumed that $\tau_0 < \tau_1 < \ldots < \tau_{p-1}$. Thus the problem of multipath channel IR estimation is reduced to the smaller problem of delay and complex attenuation parameters estimation.

Let $g(t)$ be the pulse shaping filter (convolution of transmitter and receiver filters). The overall impulse response of the communication system is then given as

$$h(t) = a_0 g(t) + \sum_{i=1}^{p-1} a_i g(t-\tau_i) \tag{2}$$

In this paper, it is assumed that the pulse shaping filter $g(t)$ is a known raised cosine function and has finite support, i.e. $g(t) = 0$ for $t \notin [0, L_g T]$, where T is the symbol period. Also, we consider the multichannel model, in which the channel output is oversampled by a factor of N_s samples per symbol period. For simplicity in our derivations we take $N_s = 2$. Thus the sampled channel IR is expressed by two vectors, one for each subchannel, i.e. for $i = 1, 2$

$$\boldsymbol{h}_i^T = \left[\, h(\tfrac{(i-1)}{2}T)\ \ h(T + \tfrac{(i-1)}{2}T)\ \ldots\ h(LT + \tfrac{(i-1)}{2}T)\ \right]$$

where LT is the span of the overall channel IR. It can be easily shown that the subchannels' IRs can be written in the form

$$\boldsymbol{h}_i = \boldsymbol{G}_i(\boldsymbol{\tau})\mathbf{a}, \quad i = 1, 2 \tag{3}$$

where $\boldsymbol{\tau} = [\tau_0\ \tau_1\ \ldots\ \tau_{p-1}]^T$ and $\mathbf{a} = [a_0\ a_1\ \ldots\ a_{p-1}]^T$. Finally $\boldsymbol{G}_i(\boldsymbol{\tau})$ is an $(L+1) \times p$ matrix whose columns are delayed versions of $g(t)$ depending on the unknown parameters τ_i. Specifically this matrix has the form

$$\boldsymbol{G}_i(\boldsymbol{\tau}) = \begin{bmatrix} g(\frac{i-1}{2}T - \tau_0) & \ldots & g(\frac{i-1}{2}T - \tau_{p-1}) \\ g(T + \frac{i-1}{2}T - \tau_0) & \ldots & g(T + \frac{i-1}{2}T - \tau_{p-1}) \\ \vdots & \ddots & \vdots \\ g(LT + \frac{i-1}{2}T - \tau_0) & \ldots & g(LT + \frac{i-1}{2}T - \tau_{p-1}) \end{bmatrix}$$

The subchannels' output samples can be written as follows

$$y_i(n) = \mathbf{h}_i^T \mathbf{s}_L(n) + w_i(n), \quad i = 1, 2 \tag{4}$$

where $\mathbf{s}_L^T(n) = [s(n), s(n-1), \ldots, s(n-L)]$ is the common input data vector and $w_i(n)$ stands for the output noise of subchannel i at time n. The input sequence is assumed to be i.i.d. and independent of the noise sequences. In the next section the problem of estimating the unknown parameters' vectors $\boldsymbol{\tau}$ and $\mathbf{a}$ is treated, using a small number of information symbols which are assumed to be known at the receiver (semi-blind identification).

3. THE PROPOSED ESTIMATION METHOD

3.1. Derivation of the Method

The starting point for the derivation of the new parametric algorithm is the subchannel response matching concept, which wisely exploits the multichannel structure [5]. More specifically, in the two channels, noise-free case, the following relation holds

$$\{y_1\} \star \{h_2\} = \{y_2\} \star \{h_1\} \tag{5}$$

where $\star$ stands for convolution and for $i = 1, 2$, $\{y_i\}$ denotes the output sequence of the subchannel i. If we assume that output samples $y_i(k), y_i(k+1), \ldots, y_i(k+N-1)$, $i = 1, 2$, $N > L$, are available, Eq. (5) leads to the following identity

$$\begin{bmatrix} Y_2 & -Y_1 \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \end{bmatrix} = \mathbf{0} \tag{6}$$

where

$$Y_i = \begin{bmatrix} y_i(L+k) & \cdots & y_i(k) \\ y_i(L+k+1) & \cdots & y_i(k+1) \\ \vdots & \vdots & \vdots \\ y_i(k+N-1) & \cdots & y_i(k+N-L-1) \end{bmatrix} \tag{7}$$

with $i = 1, 2$. Let us further assume that the information symbols $s(k-L), s(k-L+1), \ldots, s(k+M)$ are known at the receiver, where M is of the order of L and $M \ll N$. Then, if we ignore noise, the system of equations in (6) can be augmented as follows

$$\begin{bmatrix} Y_2 & -Y_1 \\ S_{ML} & \mathbf{0} \\ \mathbf{0} & S_{ML} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{y}_{1M} \\ \mathbf{y}_{2M} \end{bmatrix} \tag{8}$$

where $\mathbf{y}_{iM}^T = [y_i(k), y_i(k+1), \ldots, y_i(k+M)]$ for $i = 1, 2$ and

$$S_{ML} = \begin{bmatrix} s(k) & \cdots & s(k-L) \\ s(k+1) & \cdots & s(k+1-L) \\ \vdots & \vdots & \vdots \\ s(k+M) & \cdots & s(k+M-L) \end{bmatrix} \tag{9}$$

By imposing the channel parametric structure, Eq. (8) is written as

$$Y_S G(\boldsymbol{\tau})\mathbf{a} = \mathbf{z} \tag{10}$$

where

$$Y_S = \begin{bmatrix} Y_2 & -Y_1 \\ S_{ML} & \mathbf{0} \\ \mathbf{0} & S_{ML} \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} \mathbf{0} \\ \mathbf{y}_{1M} \\ \mathbf{y}_{2M} \end{bmatrix}, \quad G(\boldsymbol{\tau}) = \begin{bmatrix} G_1(\boldsymbol{\tau}) \\ G_2(\boldsymbol{\tau}) \end{bmatrix}$$

When the channel is corrupted by noise, we can estimate the channel parameters $\mathbf{a}, \boldsymbol{\tau}$ by solving the following least squares (LS) problem

$$\min_{\mathbf{a}, \boldsymbol{\tau}} ||\mathbf{z} - \Phi(\boldsymbol{\tau})\mathbf{a}||^2, \quad \Phi(\boldsymbol{\tau}) = Y_S G(\boldsymbol{\tau}) \tag{11}$$

The non-linear LS problem in (11) is separable with respect to the unknown parameters $\boldsymbol{\tau}$ and $\mathbf{a}$. In particular, the problem is nonlinear with respect to $\boldsymbol{\tau}$ and linear with respect to $\mathbf{a}$. As a result, the optimization process can be conducted separately with respect to the distinct parameter sets $\boldsymbol{\tau}$ and $\mathbf{a}$ respectively, [6]. More specifically

- The delay parameters $\boldsymbol{\tau}$ are obtained from the solution of the following non-linear optimization problem

  $$\boldsymbol{\tau}_{opt} = arg \min_{\boldsymbol{\tau}}\{f(\boldsymbol{\tau})\} \tag{12}$$

  where $f(\boldsymbol{\tau}) = ||(I - \Phi(\boldsymbol{\tau})\Phi^{\dagger}(\boldsymbol{\tau}))\mathbf{z}||^2$ and $\dagger$ denotes the pseudoinverse of a matrix

- The attenuation parameters $\mathbf{a}$ are determined by the linear LS method as

$$\mathbf{a}_{opt} = \Phi^{\dagger}(\boldsymbol{\tau}_{opt})\mathbf{z} \qquad (13)$$

The non-linear problem in Eq. (12) can be treated either by performing a multidimensional search in the space of parameter set $\hat{\boldsymbol{\tau}}$ or by applying a non-linear optimization search method, e.g. a Newton type method. In the former case the computational burden may be prohibitive, in the latter the procedure may be trapped in a local minimum, away from the global solution. An alternative approach is to investigate the form of the function to be minimized, in order to restrict the search for the global minimum in a subspace of the whole parameter space. Indeed, it is shown in the Appendix that the function under consideration has a specific form which allows for a linear search to be performed for the estimation of the global minimum. More specifically, it is proven that this function is decoupled with respect to the delay parameters $\tau_i, i = 0, 1, \ldots, p-1$, i.e., the optimization search can be performed separately for each τ_i and independently of the other delay parameters. The special form of the cost function is illustrated in Fig. 1 for a two ray multipath channel. In this figure, function $f(\boldsymbol{\tau})$ is plotted versus τ_0, τ_1. It is clear from the figure that instead of a 2-dimensional search, two 1-dimensional searches are adequate for the location of the global minimum. The basic steps of the new technique are summarized below.

1. Set values $\hat{L}, \hat{p}$ for unknown L, p, respectively.
2. Initialize τ_i, $i = 0, 1, \ldots, p-1$ with distinct random values in the interval $[0, LT]$.
3. Choose a linear search step size, δ and set $i = 1$.
4. Minimize $f(\boldsymbol{\tau})$ with respect to τ_i. Find $\tau_{i,opt}$ by evaluating the function at $\tau_i = j\delta$, $j = 0, 1, \ldots, \frac{\hat{L}T}{\delta}$
5. Set $\tau_i = \tau_{i,opt}$, $i = i + 1$ and repeat from step 3 until $i = \hat{p}$.
6. Run a Gauss-Newton search in the neighborhood of $\boldsymbol{\tau}_{opt}$ to improve the estimation accuracy.
7. Obtain the attenuation parameters from (13).

3.2. Efficiency issues

The evaluation of the function in (12) at each step of optimization search can be realized in an efficient manner. In the following we deal with the delay parameter τ_0. It can be easily shown that the same analysis is also applied for the remaining parameters $\tau_1, \tau_2, \ldots, \tau_{p-1}$. As shown in the Appendix, the optimization criterion in (12) is equivalent to the following

$$\boldsymbol{\tau}_{opt} = arg \max_{\boldsymbol{\tau}} F(\boldsymbol{\tau}) \qquad (14)$$

where $F(\boldsymbol{\tau})$ is defined in (19). Then starting from the definitions of $\mathbf{q}(\boldsymbol{\tau})$ and $A(\boldsymbol{\tau})$ in (20) we write these quantities as

$$\mathbf{q}^H(\boldsymbol{\tau}) = \begin{bmatrix} q_0^* & \tilde{\mathbf{q}}^H \end{bmatrix} \quad A(\boldsymbol{\tau}) = \begin{bmatrix} b & \mathbf{u}^H \\ \mathbf{u} & R \end{bmatrix} \qquad (15)$$

We can easily see that q_0 is a function of τ_0, and $\tilde{\mathbf{q}}$, R, depend on $\tau_1, \tau_2, \ldots, \tau_{p-1}$, which remain fixed during the optimization process for τ_0. By applying the matrix inversion lemma, it follows that matrix

$$A^{-1}(\boldsymbol{\tau}) = \begin{bmatrix} c & \mathbf{v}^H \\ \mathbf{v} & S \end{bmatrix} \qquad (16)$$

can be expressed in terms of the quantities in (15). From (19), (20), (15), (16) after some algebra we obtain

$$F(\boldsymbol{\tau}) = \frac{1}{c}|cq_0^* + \tilde{\mathbf{q}}^H\mathbf{v}|^2 + \tilde{\mathbf{q}}^H R^{-1}\tilde{\mathbf{q}} \qquad (17)$$

Note that the second term in the right hand side of (17) is computed only once, at the beginning of the optimization search for τ_0. Such an approach leads to a significant reduction of the number of operations required and can be generalized for every τ_i, $i = 1, 2, \ldots, p-1$.

4. SIMULATION RESULTS

The performance of the new method was tested via extensive simulations. In our experiments, the input sequence was taken from a binary alphabet and passed through different multipath channels. At the channel output, AWG noise was added to yield an SNR varying between 16 and 26 dB. The new method was compared with the non-parametric semi-blind SRM-based method presented in [2]. In the latter method the channel IR is estimated using a linear combination of a blind and a non-blind cost function. A parameter, say α, is used in order to control the linear mixing of the two cost functions. The estimation is performed completely blindly if $\alpha = 1$. Note that the channel IR is estimated in a non-parametric manner, i.e. without taking into account its particular structure, as in the proposed method for the case of multipath channels. For each experiment the data sequence consisted of 250 samples.

In Fig. 2 the results for a medium length channel are depicted. Specifically, the number of multipath components is assumed to be $p = 5$ with delays $\boldsymbol{\tau} = [0\ 6.4\ 15.7\ 22.3\ 33.1]$ and amplitudes $\mathbf{a} = [1\ 0.5\ -0.3\ 0.18\ 0.27]$. The overall channel IR was the convolution of the multipath channel IR with a raised cosine filter with a roll-off factor equal to 0.3 and $L_g = 6$, i.e. the total channel span is $L = 40$. It was taken $M = L - 1$, i.e. the training was equal to $2L$. The linear search step size was 0.1. The two methods were compared using the root-mean-square-error between actual and estimated channel IRs, i.e.

$$RMSE = (1/Q)\sqrt{\sum_{k=1}^{Q}\sum_{i=1}^{2L}(h_{act}(i) - h_{est}^k(i))^2}$$

where Q is the number of independent simulations performed, $h_{act}(i)$ is the i-th element of the actual $T/2$-spaced channel IR and $h_{est}^k(i)$ is the i-th element of the estimated channel IR during the k-th simulation. Both IR sequences $\{h_{act}\}$ and $\{h_{est}^k\}$ have been previously normalized so as to have a unity norm. For both methods a number of 100 independent experiments was carried out, i.e. Q=100. In Fig. 2 the RMSE of the new parametric semi-blind channel estimation method is compared to the RMSE of the algorithm proposed in [2] with $\alpha = 0.3$. Two curves for the new parametric algorithm are plotted corresponding to different weightings of the blind and non-blind part in the computation of the attenuation parameters. The new parametric method outperforms significantly the SRM method for all SNRs considered. Moreover the performance of the new algorithm is independent of the weighting factor α, which means that the training symbols are sufficient for the estimation of the attenuation parameters, after the delays are estimated. Note that the performance of the new method could have been improved by applying a few steps of the Gauss-Newton (GN) method to get more accurate estimates of the delays.

5. CONCLUSION

A new semi-blind technique for estimating the time delays and attenuation factors of a multipath channel has been developed. By suitably applying the Subchannel Response Matching criterion, a two step optimization procedure arises. The first one, concerning the time delays, is a complicated nonlinear minimization problem. Due to the rich structure of the underlying cost function the computational procedure can be greatly simplified. The second step is a simple linear LS problem. The new method is very simple to implement and for the same degree of accuracy has a computational complexity which is much lower as compared to the existing semi-blind channel IR estimation methods. Extensive simulation results have confirmed the theoretically expected performance of the method.

Appendix: Structure of the Minimization Cost Function

In this appendix the particular structure of the cost function $f(\boldsymbol{\tau})$, defined in (12), will be investigated, leading to the efficient time delays finding procedure described in Section 3. From the definition of $f(\boldsymbol{\tau})$ in (12) and of $\Phi(\boldsymbol{\tau})$ in (11) we obtain

$$f(\boldsymbol{\tau}) = \mathbf{z}^H\mathbf{z} - F(\boldsymbol{\tau}) \tag{18}$$

where

$$F(\boldsymbol{\tau}) = \mathbf{q}^H(\boldsymbol{\tau})A^{-1}(\boldsymbol{\tau})\mathbf{q}(\boldsymbol{\tau}) \tag{19}$$

with

$$\mathbf{q}(\boldsymbol{\tau}) = G^H(\boldsymbol{\tau})Y_S^H\mathbf{z}, \;\; A(\boldsymbol{\tau}) = G^H(\boldsymbol{\tau})Y_S^HY_SG(\boldsymbol{\tau}) \tag{20}$$

Thus, it is evident from (18) that the problem of minimizing $f(\boldsymbol{\tau})$ with respect to $\boldsymbol{\tau}$ is equivalent to maximizing $F(\boldsymbol{\tau})$ with respect to the same parameters' vector. To proceed further, we focus on the middle part of $F(\boldsymbol{\tau})$, i.e., $G(\boldsymbol{\tau})A^{-1}(\boldsymbol{\tau})G^H(\boldsymbol{\tau})$, which is the one depending on the unknown parameters τ_i, $i = 0, \ldots, p-1$. It can be proven after some lengthy algebra, not shown here due to space limitations, that matrix $A^{-1}(\boldsymbol{\tau})$ is a positive definite matrix with a diagonally dominant form and positive diagonal elements. In practice, its non-diagonal elements have a magnitude much less than the diagonal ones. Taking advantage of this particular form we may approximate this matrix with a positive diagonal one. Thus the above middle part can be written as

$$G(\boldsymbol{\tau})A^{-1}(\boldsymbol{\tau})G^H(\boldsymbol{\tau}) \approx \sum_{i=0}^{p-1} \beta_i \mathbf{g}(\tau_i)\mathbf{g}^H(\tau_i) \tag{21}$$

where $\mathbf{g}(\tau_i)$ is the $i-th$ column of $G(\boldsymbol{\tau})$ and β_i is a nonnegative quantity. Therefore the function to be maximized can be written as

$$F(\boldsymbol{\tau}) = \sum_{i=0}^{p-1} \mathbf{z}^H Y_S \beta_i \mathbf{g}(\tau_i)\mathbf{g}^H(\tau_i)Y_S^H\mathbf{z} \tag{22}$$

i.e., it is decomposed into p terms with each one being depended on a respective unknown parameter τ_i. This fact justifies the use of a separate search for each τ_i, independently of the other delay parameters.

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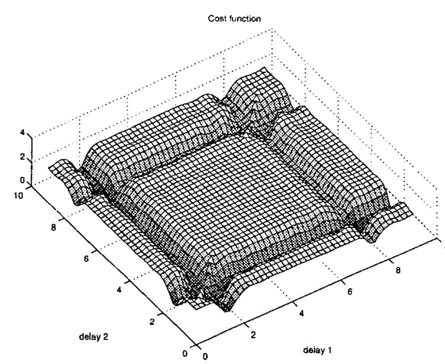

Fig. 1. Cost function, $\boldsymbol{\tau} = [1.2 \;\; 7.4]^T$ and $\boldsymbol{a} = [0.5 \;\; 0.5]^T$

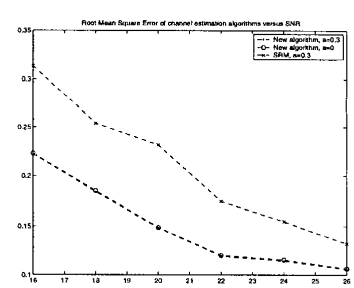

Fig. 2. RMSE curves for different SNRs