EFFICIENT DECISION FEEDBACK EQUALIZER FOR SPARSE MULTIPATH CHANNELS

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ABSTRACT

In this paper a computationally efficient Decision Feedback Equalizer (DFE) is proposed. The new equalizer is appropriate for channels with long and sparse impulse response (IR) as those encountered in many wireless communications applications. The main feature of the algorithm is that the actual size of the computationally demanding feedback filter is significantly reduced. This is achieved by exploiting the particular form of the multipath channel to derive a tractable expression for the causal part of the overall discrete channel IR (including the feedforward filter). Based on the above expression the feedback filter can be built so as to act only to a properly selected set of tap positions. The new DFE exhibits considerable computational savings and faster convergence as compared to the conventional DFE, offering the same steady-state performance.

1. INTRODUCTION

In many wireless communication systems, due to the multipath propagation phenomenon, the involved channels exhibit a long time dispersion and delay spreads of up to $40\mu s$ are often encountered. If a wideband signal is transmitted through such a highly dispersive channel then the introduced Intersymbol Interference (ISI) has a span of several tens up to hundreds of symbols. This in turn implies that quite long adaptive equalizers are required at the receiver's end in order to reduce effectively the ISI component of the received signal. Wideband mobile communication systems and digital video terrestrial transmission are typical applications of the kind. In the latter case the involved channel IR may last up to several hundreds of baud intervals. Note that the situation is even more demanding whenever the channel frequency response exhibits deep nulls.

The adaptive DFE has been widely accepted as an effective technique for reducing ISI [1]. Moreover, it has been shown that the DFE structure is particularly suitable for multipath channels, since most part of ISI is due to the long postcursor portion of the IR (see for instance [6]). Recall that an important feature of the DFE is that the postcursor ISI is almost perfectly cancelled by the Feedback (FB) filter, provided of course that the previous decisions are correct. Moreover since noise is involved only in the output of the Feedforward (FF) filter, the DFE exhibits less noise enhancement effects as compared to linear equalizers. In applications of the type described above the DFE has a large number of taps (mainly due to the long FB filter). Thus, in high speed wireless applications, not only the implementation of a real time equalizer becomes a difficult task (due to the very small symbol period) but also the equalizer itself has an increased complexity.

During the last decade there have been many efforts in different directions towards developing efficient implementations of the DFE. As typical examples of such efforts we mention the works in [3], [4], and [5] which are mostly based on a proper exploitation of the discrete sparse form of the channel profile. Another direction has been suggested in [6], [7], and [8], where block implementations of the DFE (in both time and frequency domains) have been derived.

In this paper, an efficient DFE is proposed which is based on a novel time delay estimation technique. Specifically, taking advantage of the particular form of the IR in the frequency domain, the time delays of the undesired echoes are estimated. The resulting information is used as input to a scheme, which yields the estimated time delays of the dominant terms of the inverse channel impulse response up to any desired degree of approximation. Based on the above, a small number of tap positions of the FB filter is selected, which however corresponds to almost the overall amount of the postcursor ISI.

The paper is outlined as follows. In Section 2 the multipath channel is described and a useful formula for the inverse channel is derived. In Section 3 the new DFE is developed and several issues regarding its performance are discussed. Finally, in Section 4 some indicative experimental results are provided.

2. THE MULTIPATH CHANNEL

The impulse response of a multipath channel of the type under consideration is described by

$$h(t) = \delta(t) + \sum_{l=1}^{L} \alpha_l \delta(t - \tau_l) \qquad (1)$$

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where α_l and τ_l are the fading coefficient (complex in general) and the propagation delay, respectively, of the $l-th$ path, L is the number of multipaths and $\delta(\cdot)$ is the Dirac delta function. The dirac function at time $t = 0$ corresponds to the main signal, while the remaining L terms correspond to the undesired echoes. Note that τ_l may be either positive or negative corresponding to a postcursor or a precursor echo, respectively. As mentioned previously, the precursor part of the channel's IR is in general much shorter as compared to the postcursor part. Now, taking the Fourier transform of eqn. (1) we get

$$H(f) = 1 + \sum_{i=1}^{L} \alpha_i e^{-j2\pi f \tau_i} \tag{2}$$

Let us denote as $G(f)$ the frequency response of the inverse channel, and as $S(f)$ the sum of the undesired echoes, i.e. $S(f) = \sum_{i=1}^{L} \alpha_i e^{-j2\pi f \tau_i}$. Then by making the reasonable assumption that $|S(f)| \leq 1$, and after some manipulations, we can derive the following expression for the inverse channel frequency response

$$G(f) = 1 + \sum_{n=1}^{\infty} (-1)^n S^n(f) \tag{3}$$

with

$$S^n(f) = \sum_{n_i} \frac{n!}{n_1! \dots n_L!} \alpha_1^{n_1} \dots \alpha_L^{n_L} e^{-j2\pi f(n_1\tau_1 + \dots + n_L \tau_L)} \tag{4}$$

where for each n the sum in (4) is over all possible combinations n_i of nonnegative integers $n_1, n_2, \dots n_L$ for which $n_1 + n_2 + \dots + n_L = n$. Obviously the larger the n the more the terms we take into account in constructing the frequency response of the inverse channel. Taking inverse Fourier transform of eqn. (3) and keeping only up to second order terms (higher order terms are negligible) we get the following approximation formula for the inverse channel IR

$$\begin{aligned} g(t) \approx 1 &- \sum_{i=1}^{L} \alpha_i \delta(t-\tau_i) + \sum_{i=1}^{L} \alpha_i^2 \delta(t - 2\tau_i) \\ &+ \sum_{i} \sum_{j \neq i} \alpha_i \alpha_j \delta(t - \tau_i - \tau_j) \end{aligned} \tag{5}$$

The above formula leads to several useful observations regarding the inverse channel IR. Some of its properties will be pointed out and exploited later on in the paper.

3. DERIVATION OF THE FAST DFE

Taking into account (1), the sampled output of the multipath channel can be written as (assuming without loss of generality that $\tau_l = n_l T$, T being the symbol period)

$$x(n) = u(n) + \sum_{l=1}^{L} \alpha_l u(n - n_l) + w(n) \tag{6}$$

where $\{u(n)\}$ is the i.i.d. symbol sequence with variance σ_u^2 and $\{w(n)\}$ is zero-mean complex white Gaussian noise independent of the input sequence. The channel span is considered to be N. As mentioned previously, in a typical multipath channel, the postcursor part is much longer that the precursor part of the channel. The latter, in practice, consists of a small number of strong echoes (usually 1 or 2), located very close to the main signal (see for instance [3]).

To start our derivation, let us formulate the conventional LMS-based DFE algorithm in the decision-directed mode as follows

$$z(n) = \mathbf{a}_M^T(n) \mathbf{x}_M(n + M - 1) \tag{7}$$

$$v(n) = \mathbf{b}_N^T(n) \mathbf{d}_N(n-1) \tag{8}$$

$$y(n) = z(n) + v(n) \tag{9}$$

$$d(n) = f\{y(n)\} \tag{10}$$

$$e(n) = d(n) - y(n) \tag{11}$$

$$\mathbf{a}_M(n+1) = \mathbf{a}_M(n) + \mu^a \mathbf{x}_M^*(n + M - 1) e(n) \tag{12}$$

$$\mathbf{b}_N(n+1) = \mathbf{b}_N(n) + \mu^b \mathbf{d}_N^*(n-1) e(n) \tag{13}$$

where $\mathbf{x}_M(n + M - 1) = [x(n + M - 1) \cdots x(n)]^T$ and $\mathbf{d}_N(n-1) = [d(n-1) \cdots d(n-N)]^T$ are the input vector to the equalizer and the decision vector respectively. Vectors $\mathbf{a}_M(n)$ and $\mathbf{b}_N(n)$ denote the M-th order FF and the N-th order FB filter respectively. $f\{\cdot\}$ stands for the decision device function. To facilitate comparison of complexities, the FB filter order is taken equal to the channel length (this order is commonly used in practice [3]). In order to remove the introduced ISI, a DFE applied to the multipath channel output, should comprise a long FB filter and a relatively short FF filter (typical figures in HDTV are 100-300 and 30-60 coefficients respectively). However, these figures make the implementation of the DFE a difficult task, because the required computational burden would be prohibitive.

The main idea behind the technique described in this paper is to take advantage of the special form of the channel and reduce the computational complexity by considering only the effect of the non-zero coefficients of the channel. Such an approach, however, presumes a technique for determining the echo positions. In the following, we present a method for obtaining the echo locations n_l, based on a frequency domain expression for the cross-correlation between the channel input and output.

3.1. Estimation of the time delays

The $2N$-DFT's of the input and output signals can be expressed as

$$U(k) = \sum_{m=0}^{2N-1} u(n+m) e^{-j\frac{2\pi}{2N}mk} \tag{14}$$

$$X(k) = \sum_{i=0}^{2N-1} x(n+i) e^{-j\frac{2\pi}{2N}ik} \tag{15}$$

If we consider the expected value of the product of the above sequences we get

$$\mathcal{E}\{X(k)U^*(k)\} =$$

$$\sum_{i=0}^{2N-1} \sum_{m=0}^{2N-1} \mathcal{E}\{x(n+i)u^*(n+m)\} e^{-j\frac{\pi}{N}(i-m)k} \tag{16}$$

where $\mathcal{E}\{\cdot\}$ denotes the expectation operator. If we now substitute eqn. (6) to eqn. (16) and take into consideration that the input sequence is i.i.d. and independent of the noise, then after some manipulations we get

$$\mathcal{E}\{U^*(k)X(k)\} = 2N\sigma_u^2 + \sigma_u^2 \sum_{l=1}^{L} \alpha_l (2N - |n_l|) e^{-j\frac{\pi}{N} n_l k} \tag{17}$$

for $k = 0, 1, \ldots, 2N-1$. That is, we end up with a sum of complex sinusoids at frequencies n_l. Applying the $2N$-IDFT to the resulting sequence, we can easily determine the echo locations n_l at the non-zero points of the IDFT. Note that with an appropriate scaling of the IDFT, we can remove the influence of the factor $(2N - |n_l|)$, which weights less the far echoes.

In a practical situation, time averaging is used instead of $\mathcal{E}\{\cdot\}$ and the above formula is computed as follows

$$C_{UX}^{(R)}(k) = \sum_{r=0}^{R-1} \lambda^{R-1-r} X_r(k) U_r^*(k) \tag{18}$$

where $\lambda \leq 1$ and

$$X_r(k) = \sum_{i=0}^{2N-1} x(n+rN+i) e^{-j\frac{\pi}{N} ik}$$

$$U_r(k) = \sum_{m=0}^{2N-1} u(n+rN+m) e^{-j\frac{\pi}{N} mk}$$

That is, R half-overlapped blocks of $2N$ samples of $\{u\}$ and $\{x\}$ are used to compute $C_{UX}^{(R)}(k)$. The $L+1$ IDFT points of (18) having the highest amplitude are then chosen as the desired locations. The number L of the dominant undesired echoes can be preset by the designer by taking into account a worst case scenario for the specific application. Alternatively, number L can be computed from the data using rank determination techniques. Another strategy would be to set a threshold and select the locations of the IDFT points of (18) having amplitude which exceeds this threshold.

3.2. The algorithm

In the proposed DFE, we focus our attention to the demanding FB part and reduce the computational load by properly selecting $O(L)$ number of taps out of N taps. That is, instead of using an N-length FB filter, an $O(L)$-length filter proves to be sufficient (note that $N >> L$).

In the initial stage of the algorithm, the method described in the previous section is used for an adequate number of blocks $R = R_0$ and the time delays n_l are determined. Such an approach introduces a delay to the algorithm which increases as the number of blocks R_0 increases. However the greater the parameter R_0, the higher is the degree of accuracy in selecting the correct positions. As it will be shown below, the initial delay of the algorithm is fully compensated by the fast convergence achieved by the new DFE. After obtaining the parameters n_l, the FB filter of the equalizer is constructed as explained below.

In [2] it was proved that in the optimum DFE the feedback filter coefficients are in fact the causal samples of the overall impulse response (including the FF filter). Taking this into account we proceed to derive an expression for the FB filter in terms of the FF filter. Assuming that the previous decisions are correct and minimizing the MSE $\mathcal{E}\{|e(n)|^2\}$ with respect to the feedback coefficients we can easily obtain the following expression for the optimum FB filter

$$\mathbf{b}_N = H\mathbf{a}_M \tag{19}$$

where matrix $H = \{\mathbf{d}_N \mathbf{x}_M^T\}$. This $N \times M$ matrix contains the causal coefficients of the channel IR and has a sparse Toeplitz structure. Since the FF filter approximates the anticausal part of the inverse channel IR, it has a special form as dictated by eqn. (5). Keeping only up to first order terms of $\mathbf{a}_M$ and right-multiplying by H we get the coefficients of the FB filter up to second order approximation. It turns out that the positions where the FB filter has nonzero taps depend on the echo locations in the channel IR. Specifically:
a) there are "primary" non-zero taps at the positions where $n_l > 0$ in the channel IR (causal echoes) , and
b) for each "primary" non-zero tap $n_l > 0$, there are "secondary" non-zero taps at the positions $n_l - |n_k|$, where $n_k < 0$ are positions of the anticausal echoes in the channel IR.
Consequently, the FB filter can be restricted to act only to the above positions.

Under time-varying conditions, the changes of the multipath environment can be detected by using a proper forgetting factor λ and updating $C_{UX}^{(R)}(k)$, every N samples, according to the following expression

$$C_{UX}^{(R+1)}(k) = \lambda C_{UX}^{(R)}(k) + X_R(k) U_R^*(k) \tag{20}$$

Note that in the decision-directed mode of the DFE, the DFT values of the decision samples replace $U_R^*(k)$ in (20). A deeper investigation of the new DFE in a time varying multipath environment is the subject of current research. The basic steps of the proposed algorithm are given below:

1. Compute $C_{UX}^{(R)}(k)$ from (18).
2. Compute the IDFT of $C_{UX}^{(R)}(k)$.
3. Estimate the primary echo locations.
4. Compute the secondary echo locations (point b above).
5. For the next N iterations, apply the DFE with the FB filter acting to the above positions.
6. Update $C_{UX}^{(R)}(k)$ and repeat from step 2.

In Table 1, the computational complexity (number of complex multiplications per sample) of the new algorithm is compared to that of the conventional DFE, under the assumption that N is a power o 2. In Table 1, L_1, L_2 stand for the number of causal and non-causal echoes respectively ($L_1 + L_2 = L$). Since $L << N$, the proposed DFE achieves a significant reduction of the computational burden. For instance, if $M = 32$, $N = 128$, $L_1 = 4$, $L_2 = 2$, the required multiplications for the new algorithm and the conventional

New DFE	Conventional DFE
$2M + 2L_1(L_2 + 1) + \log_2(N)$	$2M + 2N$

Table 1: Number of complex multiplications

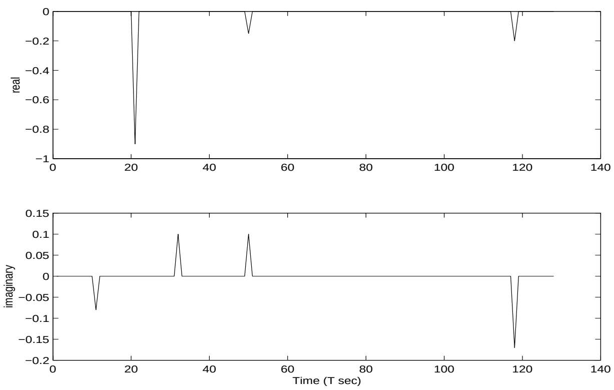

Figure 1: The multipath channel

DFE would be 95 and 320 respectively. As illustrated in the next section this saving is obtained without sacrificing the performance of the DFE.

4. SIMULATIONS AND DISCUSSION

Fig. 1 shows a typical terrestrial HDTV channel IR [3], which is used in our experiments. The input to the channel is a QPSK signal, while white complex Gaussian noise is added to the channel output. The SNR is 30db. Three mean squared error curves are depicted in Fig. 2. Curve a corresponds to the conventional DFE ($M = 15, N = 128$), and curves b, c to the proposed DFE. More specifically, curve c results without considering the effect of the "secondary echoes" - as explained in section 3 - which is the case in curve b. The decrease of the mean squared error in steady state demonstrates the significance of these secondary terms. We also observe from Fig. 2 that, despite the initial delay ($R_0 = 12$ in this case), the overall convergence of the new algorithm is faster, compared to the conventional DFE. Evidently, this is due to the very short FB filter employed in the new scheme.

In Fig. 3, the tracking behavior of the proposed DFE is compared to that of the standard DFE in a time-varying environment. More specifically, after $n = 6000$ iterations the location of the first causal echo suddently changes. This corresponds to an abrupt modification of the multipath channel response. We observe that the new DFE (with $\lambda = 0.88$) reacts immediately and converges faster to the steady state, compared to the conventional DFE.

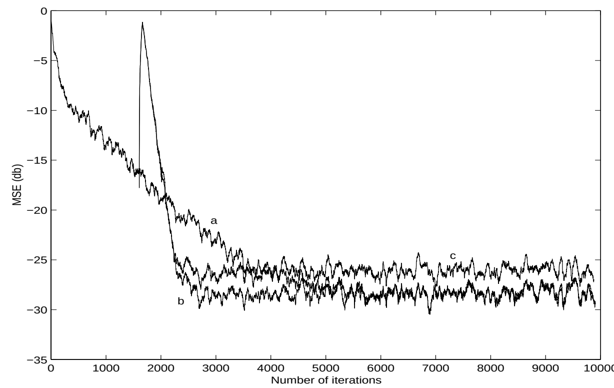

Figure 2: Mean squared error curves

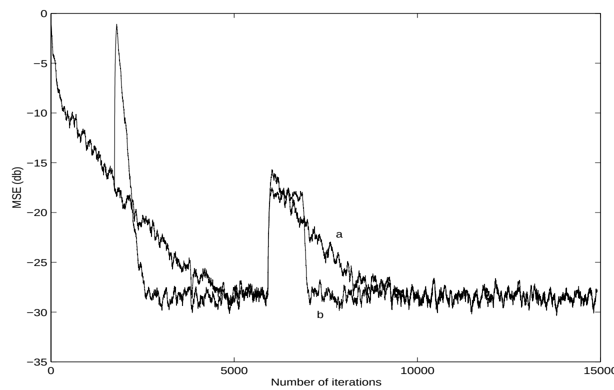

Figure 3: Mean squared error curves under time-varying conditions

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