Performance Analysis of Mobile Communication Networks in the Presence of Composite Fading, Noise and Interference

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***Abstract*—In this paper, the effects of interference on composite fading environments, where multipath fading coexists with shadowing, are investigated. More specifically, important statistical metrics of a single receiver output signal to interference and noise ratio (SINR) are studied, including the probability density function and the cumulative distribution function. Our analysis is also extended to multi-channel receivers and in particular to selection diversity (SD) receivers. For all scenarios, simplified expressions are also provided for the interference limited cases, where the influence of thermal noise is ignored. The derived expressions are used to analyze the performance, in terms of the average bit error probability and the outage probability. The analysis is accompanied by numerical evaluated results, clearly demonstrating the usefulness of the proposed theoretical framework.**

***Index Terms*—Composite fading channels, bit error probability, minimum number of antennas, outage probability, selection diversity, signal-to-interference and noise ratio.**

I. INTRODUCTION

In many recent wireless communication systems, both licensed or unlicensed, a user quite often shares the same channels with other users and thus in the reception side the signals need to be intelligently separated. This is imperative, since the aggressive frequency reuse that is frequently employed for increasing spectrum efficiency, causes co-channel as well as adjacent channel interference. Both types of interference depend on various physical factors, including interferers' spatial distribution, interfering channels fading, the power of the interferers and the wireless communication system considered. Depending upon the fading characteristics as well as the existence or not of multi-channel transmitters/receivers and/or relays, numerous contributions analyzing the effects of *fading* in conjunction with *interference* have been made, e.g., [1]–[4] and the references therein.

In terrestrial (indoor or outdoor) and satellite land-mobile systems, the link quality is also affected by slow variations of the mean signal level due to the *shadowing* from terrain, buildings and trees [5]. Under these circumstances, where multipath fading coexists with shadowing, the so-called *composite fading/shadowing environment* originates. In the past, this environment has been statistically described by using lognormal-based distributions such as Rayleigh-, Nakagami- and Rice-lognormal [5]–[7]. As a result, rather mathematically cumbersome expressions have been derived for the performance analysis on such communication scenarios. In order to facilitate the communication systems performance evaluation in these environments, new families of distributions that accurately model these composite fading conditions have been proposed, most notably as the $\mathcal{K}$ and the generalized-$\mathcal{K}$ ($\mathcal{K}_G$) distributions, e.g., [8], [9]. Based on the mathematical tractability of these new composite fading models, many research efforts for investigating the influence that interference has to the system performance have been made recently, e.g., [10]–[12]. A common practice in all these works is that the research was restricted to interference-limited wireless communication systems and thus only the statistics of the signal-to-interference-ratio (SIR) was studied.

In this paper extending this approach, the effect of thermal noise is also taken into consideration. This is important since thermal noise may be the main source of system performance degradation especially in cases of weak interfering signals. Therefore, assuming such a complete model, our contribution in this paper can be summarized as follows:

- the signal to interference and noise ratio (SINR) statistics are studied for a single-channel receiver
- the analysis is extended to multi-channel receivers and in particular to selection diversity (SD) receivers that operate in such environments
- the derived expressions are used to investigate the system performance in terms of the average bit error probability (ABEP) and the outage probability (OP).

For all scenarios studied, simplified expressions are also provided for the interference limited cases, where the influence of thermal noise is ignored.

The remainder of this paper is organized as follows. The system model is described in Section II. The statistics of the SINR for single-channel and multi-channel receivers are derived in Sections III and IV, respectively. In Section V the performance analysis of such systems is presented, while numerical evaluated performance results are provided in Section VI. Finally, concluding remarks are given in Section VII.

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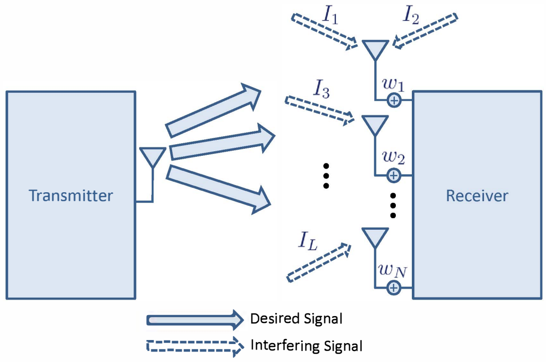

Fig. 1. System model.

II. System and Channel Model

We consider the downlink of a wireless-mobile communication system, with a single-antenna transmitter and (in general) a multiple antennas receiver, with diversity branches experiencing interference coming from L sources, as shown in Fig. 1. We assume that the level of interference at the receiver is such that the effect of thermal noise on system performance cannot be ignored [2]. The complex baseband signal y_n received at the nth antenna, can be expressed as

$$y_n = h_{D_n} s_D + \sum_{i=1}^{L} h_{I_{n,i}} s_{I_i} + w_n \tag{1}$$

where h_{D_n} represents the complex channel gain between the transmitter and the nth receiver antenna and s_D is the desired transmitted complex symbol with energy $E_{s_D} = \mathbb{E}\left\langle |s_D|^2 \right\rangle$, and $\mathbb{E}\left\langle \cdot \right\rangle$ denoting statistical averaging. Furthermore, in (1), $h_{I_{n,i}}$ represents the complex channel gain of the interfering signal s_{I_i} and w_n is the complex additive white Gaussian noise (AWGN) with zero mean and variance N_0. Similar to other studies in the past, e.g., [13], [14], we consider the case where the desired received signal gain is subject only to multipath fading, while interfering signals are subject to multipath fading and shadowing. This represents a quite reasonable assumption, since in general the desired signal is unlikely to experience significant shadowing. On the other hand the interfering signals, which propagate over obstructed paths, due to the larger propagation distances, are experiencing severe shadowing conditions. To proceed, we denote the instantaneous signal-to-noise-ratio (SNR) of the desired signal as $\gamma_{D_n} = |h_{D_n}|^2 E_{s_D}/N_0$, the corresponding average SNR as $\overline{\gamma}_{D_n} = \mathbb{E}\left\langle |h_{D_n}|^2 \right\rangle E_{s_D}/N_0$, whilst the instantaneous interference-to-noise-ratio (INR) of the ith interfering signal is defined as $\gamma_{I_i} = |h_{I_{n,i}}|^2 E_{s_{I_i}}/N_0$, with corresponding average INR equal to $\overline{\gamma}_{I_i} = \mathbb{E}\left\langle |h_{I_{n,i}}|^2 \right\rangle E_{s_{I_i}}/N_0$ and $E_{s_{I_i}} = \mathbb{E}\left\langle |s_{I_i}|^2 \right\rangle$. Since the desired signal is subject only to multipath fading, its instantaneous SNR γ_{D_n} at the nth antenna can be assumed to be exponential distributed with probability density function (PDF) given by

$$f_{\gamma_{D_n}}(x) = \frac{1}{\overline{\gamma}_{D_n}} \exp\left(-\frac{x}{\overline{\gamma}_{D_n}}\right). \tag{2}$$

In this work, the instantaneous INR, γ_{I_i}, of the interfering signals, which are subject to fading/shadowing effects, is assumed to follow a $\mathcal{K}$ distribution with PDF given by [8]

$$f_{\gamma_{I_i}}(x) = 2\frac{\left(k_i/\overline{\gamma}_{I_i}\right)^{\frac{k_i+1}{2}} x^{\frac{k_i-1}{2}}}{\Gamma\left(k_i\right)} K_{k_i-1}\left(2\sqrt{\frac{k_i}{\overline{\gamma}_{I_i}}} x^{1/2}\right). \tag{3}$$

In (3), k_i denotes the shaping parameter of the distribution, which is related to the severity of the shadowing, $\Gamma(\cdot)$ is the Gamma function [15, eq. (8.310/1)] and $K_v(\cdot)$ is the second kind modified Bessel function of vth order [15, eq. (8.407/1)]. The corresponding output SINR, at the nth receiver antenna, can be expressed as [2]

$$\gamma_{\mathrm{out}_n} = \frac{\gamma_{D_n}}{1+\gamma_I} \tag{4}$$

where γ_I denotes the total INR, i.e., $\gamma_I = \sum_{i=1}^{L} \gamma_{I_i}$, and the PDF of γ_{I_i} is given by (3). The PDF of γ_{out_n} can be evaluated as follows

$$f_{\gamma_{\mathrm{out}_n}}(\gamma) = \int_0^\infty (1+x)\, f_{\gamma_{D_n}}((1+x)\gamma) f_{\gamma_I}(x) dx. \tag{5}$$

A. Fully Correlated Interference Shadowing

As far as the interference shadowing is concerned, the case where the L different interfering paths exhibit identical shadowing effects is going to be investigated. In such a scenario the mean values of the corresponding $\mathcal{K}$-distributed random variables are fully correlated. This is the so-called *fully (or totally) correlated shadowing* communication scenario, which has gained considerable interest in the past, e.g., [16]–[18]. This type of shadowing arises in situations where the interferers have approximately the same distance from the receiver, and thus the same obstacles shadow in a quite similar way the various interfering signals. As a result, the local mean powers of the interfering signals become correlated [11], a situation that quite often arises in indoor communication scenarios [19]. Therefore, following such an approach, where the multipath Rayleigh components of the interferers are independent but all of them experience a common local average power, $\overline{\gamma}_I$, it has been shown that the PDF of γ_I can be expressed as [11]

$$f_{\gamma_I}(\gamma) = 2\frac{(k/\overline{\gamma}_I)^{\frac{L+k}{2}} \gamma^{\frac{L+k}{2}-1}}{\Gamma(L)\Gamma(k)} K_{L-k}\left(2\sqrt{\frac{k}{\overline{\gamma}_I}\gamma}\right). \tag{6}$$

Substituting (6) in the definition of the CDF [20, eq. (4.17)], using the Meijer-G function representation for the $K_{L-k}(\cdot)$, i.e., [21, eq. (03.04.26.0006.01)], and then [22, eq. (26)], the CDF of γ_I can be expressed in a simple closed form as

$$F_{\gamma_I}(\gamma) = \frac{(k/\overline{\gamma}_I)^{\frac{L+k}{2}}}{\Gamma(L)\Gamma(k)} \gamma^{\frac{L+k}{2}} \mathcal{G}_{1,3}^{2,1}\left(\frac{k}{\overline{\gamma}_I}\gamma \middle| \begin{matrix} 1-\frac{L+k}{2} \\ \frac{L-k}{2}, -\frac{L-k}{2}, -\frac{L+k}{2} \end{matrix}\right) \tag{7}$$

where $\mathcal{G}_{p,q}^{m,n}[\cdot|\cdot]$ is the Meijer's G-function [15, eq. (9.301)], which is available as built-in function in many commercial mathematical software packages.

III. SINR Statistics for Single-Channel Reception

In this Section, since single-channel reception is considered, the antenna index n used in Section II will be omitted here. Substituting (2) and (6) in (5) yields the following expression for the SINR of $f_{\gamma_{\text{out}}}(\gamma)$

$$f_{\gamma_{\text{out}}}(\gamma) = \frac{2}{\overline{\gamma}_D} \frac{(k/\overline{\gamma}_I)^{\frac{L+k}{2}}}{\Gamma(L)\Gamma(k)} \underbrace{\int_0^\infty x^{\frac{L+k}{2}-1}(1+x)}_{\mathcal{I}_1} \underbrace{\times \exp\left[-\frac{(1+x)\gamma}{\overline{\gamma}_D}\right] K_{L-k}\left(2\sqrt{\frac{k}{\overline{\gamma}_I}}x^{1/2}\right) dx}_{\mathcal{I}_1}. \tag{8}$$

After performing some straightforward mathematical manipulations and using [15, eq. (6.643/3)] a closed-form expression for the PDF of γ_{out} can be derived as

$$\begin{aligned} f_{\gamma_{\text{out}}}(\gamma) = {} & \frac{1}{\overline{\gamma}_D}\left(\frac{k\overline{\gamma}_D}{\gamma\overline{\gamma}_I}\right)^{\frac{L+k-1}{2}} \exp\left(-\frac{\gamma}{\overline{\gamma}_D}\right) \exp\left(\frac{k\overline{\gamma}_D}{2\gamma\overline{\gamma}_I}\right) \\ & \times \left[W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\overline{\gamma}_D}{\gamma\overline{\gamma}_I}\right)\right. \\ & \left. + Lk\left(\frac{\gamma}{\overline{\gamma}_D}\right)^{-1} W_{-\frac{L+k+1}{2},\frac{L-k}{2}}\left(\frac{k\overline{\gamma}_D}{\gamma\overline{\gamma}_I}\right)\right] \end{aligned} \tag{9}$$

where $W_{\lambda,\mu}(\cdot)$ is the Whittaker function [15, eq. (9.220/4)], which is available as built-in function in many commercial mathematical software packages. By definition, the CDF of the instantaneous output SINR is expressed as

$$F_{\gamma_{\text{out}}}(\gamma) = \int_0^\infty F_{\gamma_D}\left[(1+x)\gamma\right] f_{\gamma_I}(x) dx. \tag{10}$$

Substituting the CDF of Rayleigh, i.e.,

$$F_{\gamma_D}(x) = 1 - \exp\left(-\frac{x}{\overline{\gamma}_D}\right) \tag{11}$$

and (6) in (10), following a similar procedure as the one used for deriving (9) and using [15, eqs. (6.561/16 and 6.631/3)] yields the following closed-form expression for $F_{\gamma_{\text{out}}}(\gamma)$

$$\begin{aligned} F_{\gamma_{\text{out}}}(\gamma) = {} & 1 - \exp\left(-\frac{\gamma}{\overline{\gamma}_D}\right)\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I\gamma}\right)^{\frac{L+k-1}{2}} \\ & \times \exp\left(\frac{k\overline{\gamma}_D}{2\overline{\gamma}_I\gamma}\right) W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I\gamma}\right). \end{aligned} \tag{12}$$

Simplified Expressions for the SIR: Considering an interference limited environment, i.e., ignoring the AWGN at the user terminal, the received SIR is given by

$$\gamma_{\text{out}} = \frac{\gamma_D}{\gamma_I} \tag{13}$$

whilst its PDF is

$$f_{\gamma_{\text{out}}}(\gamma) = \int_0^\infty x f_{\gamma_D}(x\gamma) f_{\gamma_I}(x) dx. \tag{14}$$

Substituting (2) and (6) in (14) similar integrals of the form $\mathcal{I}_1$ given in (8) appear. After performing some mathematics and using [15, eq. (6.643/3)] a closed-form expression for the PDF of γ_{out} can be derived as

$$\begin{aligned} f_{\gamma_{\text{out}}}(\gamma) = {} & Lk\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I\gamma}\right)^{\frac{L+k-1}{2}} \exp\left(\frac{k\overline{\gamma}_D}{2\overline{\gamma}_I\gamma}\right) \\ & \times \gamma^{-1} W_{-\frac{L+k+1}{2},\frac{L-k}{2}}\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I\gamma}\right). \end{aligned} \tag{15}$$

Starting from the definition of the instantaneous output SIR $F_{\gamma_{\text{out}}}(\gamma) = \int_0^\infty F_{\gamma_D}(x\gamma) f_{\gamma_I}(x) dx$ and following a similar procedure as that for deriving (12), yields the following simplified expression for the CDF of γ_{out}

$$\begin{aligned} F_{\gamma_{\text{out}}}(\gamma) = {} & 1 - \left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I\gamma}\right)^{\frac{L+k-1}{2}} \exp\left(\frac{k\overline{\gamma}_D}{2\overline{\gamma}_I\gamma}\right) \\ & \times W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I n\gamma}\right). \end{aligned} \tag{16}$$

IV. SINR Statistics for Multi-Channel Reception

We consider SNR-based SD reception in the presence of AWGN and multiple interfering signals. In this case the diversity receiver monitors the available branches continuously and selects the one with the largest instantaneous SNR for data detection. This SD technique requires the separation of the desired signal from the interfering signals, which can be practically achieved by using different pilot signals for each of them [1]. The instantaneous system output SINR of this multi-channel system can be expressed as

$$\gamma_{\text{SD}_{\text{out}}} = \frac{\gamma_{\text{SD}}}{1+\gamma_I} \tag{17}$$

where γ_{SD} represents the instantaneous output SNR of the SD receiver. As mentioned previously, the receiver chooses the branch with the highest SNR, among N available, and thus $\gamma_{SD} = \max\{\gamma_{D_1}, \gamma_{D_2}, \ldots, \gamma_{D_N}\}$, with γ_{D_n} denoting the instantaneous SNR of the nth branch, with PDF given by (2). Assuming independent but non-identical distributed (i.n.d.) fading conditions, the CDF of γ_{SD} is given by

$$F_{\gamma_{\text{SD}}}(\gamma) = \prod_{n=1}^{N} F_{\gamma_n}(\gamma). \tag{18}$$

For independent and identical (i.i.d.) fading conditions, where $F_{\gamma_{D_n}}(\gamma) = F_{\gamma_D}(\gamma), \forall n$, (18) simplifies to $F_{\gamma_{\text{SD}}}(\gamma) = \left[F_{\gamma_D}(\gamma)\right]^N$, with $F_{\gamma_D}(\gamma)$ given in (11). Based on

$$\prod_{n=1}^{n}(1-t_{h,n}) = 1 + \sum_{\substack{n,n \\ \lambda_1,\ldots,\lambda_n}}^{N} \prod_{m=1}^{n} t_{h,\lambda_m} \tag{19}$$

where

$$\sum_{\substack{x,y \\ \lambda_1,\ldots,\lambda_x}}^{z} = \sum_{x=1}^{z}(-1)^y \sum_{\lambda_1=1}^{z-x+1} \sum_{\lambda_2=\lambda_1+1}^{z-x+2} \cdots \sum_{\lambda_x=\lambda_{x-1}+1}^{z}$$

(18) can be re-expressed as

$$F_{\gamma_{\text{SD}}}(\gamma) = 1 + \sum_{\substack{n,n \\ \lambda_1,\ldots,\lambda_n}}^{N} \exp\left(-\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma\right) \tag{20}$$

with $\mathcal{S}^{z}_{x_\bullet,y_\bullet} = \sum_{q=1}^{z} \frac{x_\bullet}{y_\bullet}$. For the case of i.i.d. fading conditions (20) simplifies to

$$F_{\gamma_{\mathrm{SD}}}(\gamma) = \sum_{n=0}^{N} \binom{N}{n} (-1)^n \exp\left(-\frac{n}{\overline{\gamma}_D}\gamma\right). \tag{21}$$

Starting from (17) and substituting (20) and (6) in (10), integrals of the form $\mathcal{I}_1$ appearing in (8), need to be solved. Therefore following the procedure proposed in III, the CDF of $\gamma_{\mathrm{SD}_{\bullet\mathrm{ut}}}$ can be obtained in closed form as

$$\begin{aligned}F_{\gamma_{\mathrm{SD}_{\bullet\mathrm{ut}}}}(\gamma) = 1 + &\sum_{\substack{n,n \\ \lambda_1,\ldots,\lambda_n}}^{N} \left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right)^{\frac{L+k-1}{2}} \exp\left(-\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma\right) \\ \times \exp&\left(\frac{k/\overline{\gamma}_I}{2\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right) W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right).\end{aligned} \tag{22}$$

Its corresponding PDF is given by

$$\begin{aligned}f_{\gamma_{\mathrm{SD}_{\mathrm{out}}}}(\gamma) = &\sum_{\substack{n,n \\ \lambda_1,\ldots,\lambda_n}}^{N} \left(-\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\right)\left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right)^{\frac{L+k-1}{2}} \\ &\times \exp\left(-\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma\right)\exp\left(\frac{k/\overline{\gamma}_I}{2\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right) \\ &\times \left[W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right)\right. \\ &\left.+\frac{Lk}{\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma} W_{-\frac{L+k+1}{2},\frac{L-k}{2}}\left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right)\right].\end{aligned} \tag{23}$$

Considering i.i.d. fading conditions, (22) simplifies to

$$\begin{aligned}F_{\gamma_{\mathrm{SD}_{\mathrm{out}}}}(\gamma) = 1 - &\sum_{n=1}^{N}\binom{N}{n}(-1)^{n+1}\exp\left(-\frac{n}{\overline{\gamma}_D}\gamma\right) \\ \times &\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I n\gamma}\right)^{\frac{L+k-1}{2}} \exp\left(\frac{k\overline{\gamma}_D}{2\overline{\gamma}_I n\gamma}\right) W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I n\gamma}\right)\end{aligned} \tag{24}$$

whilst (23) simplifies to

$$\begin{aligned}f_{\gamma_{\mathrm{SD}_{\mathrm{out}}}}(\gamma) = &\sum_{n=1}^{N}\binom{N}{n}(-1)^n\left(-\frac{n}{\overline{\gamma}_D}\right)\exp\left(-\frac{n}{\overline{\gamma}_D}\gamma\right) \\ \times &\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I n\gamma}\right)^{\frac{L+k-1}{2}} \exp\left(\frac{k\overline{\gamma}_D}{2\overline{\gamma}_I n\gamma}\right)\left\{W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I n\gamma}\right)\right. \\ +Lk&\left.\left(\frac{n}{\overline{\gamma}_D}\gamma\right)^{-1} W_{\frac{-1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I n\gamma}\right)\right\}.\end{aligned} \tag{25}$$

<u>*Simplified Expressions for the SIR:*</u> For the SIR case, the instantaneous system output SIR can be expressed as

$$\gamma_{\mathrm{SD}_{\bullet\mathrm{ut}}} = \frac{\gamma_{\mathrm{SD}}}{\gamma_I}. \tag{26}$$

Starting with (26) and substituting (20) and (6) in (14), again integrals of the form $\mathcal{I}_1$ appearing in (8), need to be solved. Following the procedure proposed in III, (22) simplifies to

$$\begin{aligned}F_{\gamma_{\mathrm{SD}_{\mathrm{out}}}}(\gamma) = 1 + &\sum_{\substack{n,n \\ \lambda_1,\ldots,\lambda_n}}^{N}\left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right)^{\frac{L+k-1}{2}} \\ \times \exp&\left(\frac{k/\overline{\gamma}_I}{2\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right) W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right)\end{aligned} \tag{27}$$

and the corresponding expression for the PDF is given by

$$\begin{aligned}f_{\gamma_{\mathrm{SD}_{\mathrm{out}}}}(\gamma) = &\sum_{\substack{n,n \\ \lambda_1,\ldots,\lambda_n}}^{N} Lk\left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right)^{\frac{L+k-1}{2}}\gamma^{-1} \\ \times \exp&\left(\frac{k/\overline{\gamma}_I}{2\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right) W_{-\frac{L+k+1}{2},\frac{L-k}{2}}\left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^{n}_{1,\overline{\gamma}_{D_{\lambda_m}}}\gamma}\right).\end{aligned} \tag{28}$$

For i.i.d. fading conditions the CDF of $\gamma_{\mathrm{SD}_{\bullet\mathrm{ut}}}$ can be expressed as

$$\begin{aligned}F_{\gamma_{\mathrm{SD}_{\mathrm{out}}}}(\gamma) = 1 + &\sum_{n=1}^{N}\binom{N}{n}(-1)^n\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I n\gamma}\right)^{\frac{L+k-1}{2}} \\ &\times \exp\left(\frac{k\overline{\gamma}_D}{2\overline{\gamma}_I n\gamma}\right) W_{\frac{1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I n\gamma}\right)\end{aligned} \tag{29}$$

whilst (28) simplifies to

$$\begin{aligned}f_{\gamma_{\mathrm{SD}_{\bullet\mathrm{ut}}}}(\gamma) = Lk&\sum_{n=1}^{N}\binom{N}{n}(-1)^{n+1}\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I n\gamma}\right)^{\frac{L+k-1}{2}} \\ \times \exp&\left(\frac{k\overline{\gamma}_D}{2\overline{\gamma}_I n\gamma}\right)\gamma^{-1} W_{\frac{-1-L-k}{2},\frac{L-k}{2}}\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I n\gamma}\right).\end{aligned} \tag{30}$$

V. PERFORMANCE ANALYSIS

In this Section, the performance is evaluated using the ABEP and the OP criteria. More specifically, the OP is defined as the probability that the SINR falls below a predetermined threshold γ_{th} and is given by $P_{\mathrm{out}} = F_{\gamma_{\bullet\mathrm{ut}}}(\gamma_{\mathrm{th}})$. Therefore, the OP can be obtained by using (12), (22) (or (24) for i.i.d. fading) for the single channel and the multi-channel technique, respectively. As far as the ABEP is concerned, it will be evaluated by using the MGF based approach.

A. Single-Channel Reception

Substituting (9) in the definition of the MGF [20, eq. (4.17)], using [21, eq. (07.45.26.0005.01)], and employing [15, eq.

(7.813/1)] yields the following closed-form expression for the MGF of the SINR

$$
\begin{aligned}
M_{\gamma_{\text{out}}}(s) = {} & \frac{1}{\overline{\gamma}_D}\frac{\left(k\overline{\gamma}_D/\overline{\gamma}_I\right)^{\frac{L+k}{2}}}{\Gamma(L)\Gamma(k)}\left\{\left(\frac{1}{\overline{\gamma}_D}+s\right)^{\frac{L+k}{2}-1}\right.\\
& \times\mathcal{G}_{3,1}^{1,3}\left(\frac{\overline{\gamma}_I/\left(k\overline{\gamma}_D\right)}{1/\overline{\gamma}_D+s}\middle|\begin{matrix}\frac{L+k}{2},1-\frac{L-k}{2},1+\frac{L-k}{2}\\ \frac{L+k}{2}\end{matrix}\right)+\left(\frac{1}{\overline{\gamma}_D}+s\right)^{\frac{L+k}{2}}\\
& \left.\times\overline{\gamma}_D\mathcal{G}_{3,1}^{1,3}\left(\frac{\overline{\gamma}_I/\left(k\overline{\gamma}_D\right)}{1/\overline{\gamma}_D+s}\middle|\begin{matrix}\frac{L+k}{2}+1,1-\frac{L-k}{2},1+\frac{L-k}{2}\\ \frac{L+k}{2}+1\end{matrix}\right)\right\}.
\end{aligned}
\tag{31}
$$

Simplified Expressions for the SIR: Using again (15) in the definition of the MGF and employing [21, eq. (07.45.26.0005.01)], (31) simplifies to

$$
\begin{aligned}
M_{\gamma_{\text{out}}}(s) = {} & \frac{1}{\Gamma(k)\Gamma(L)}\left(\frac{k\overline{\gamma}_D}{\overline{\gamma}_I}\right)^{\frac{L+k-1}{2}}s^{\frac{L+k-1}{2}}\\
& \times\mathcal{G}_{3,1}^{1,3}\left(\frac{\overline{\gamma}_I}{k\overline{\gamma}_D s}\middle|\begin{matrix}\frac{L+k+1}{2},1-\frac{L-k+1}{2},1+\frac{L-k-1}{2}\\ 1+\frac{L+k-1}{2}\end{matrix}\right).
\end{aligned}
\tag{32}
$$

B. Multi-Channel Reception

Considering i.n.d. fading conditions, based on (23) and following a procedure similar to that used in the single-channel reception case, the MGF of $\gamma_{\text{SD}_{\text{out}}}$ can be expressed as

$$
\begin{aligned}
M_{\gamma_{\text{SD}_{\text{out}}}}(s) = {} & \sum_{\substack{n,n\\ \lambda_1,\ldots,\lambda_n}}^{N}\frac{\left(-\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}\right)}{\Gamma(k)\Gamma(L)}\left(\frac{\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}+s}{\overline{\gamma}_I\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}/k}\right)^{\frac{L+k-1}{2}}\\
& \times\left[\frac{\mathcal{G}_{3,1}^{1,3}\left(\frac{\frac{\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}\overline{\gamma}_I}{k}}{\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}+s}\middle|\begin{matrix}\frac{L+k-1}{2},\frac{1-L+k}{2},\frac{1+L-k}{2}\\ \frac{L+k-1}{2}\end{matrix}\right)}{\left(\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}+s\right)}\right.\\
& \left.+\frac{\mathcal{G}_{3,1}^{1,3}\left(\frac{\frac{\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}\overline{\gamma}_I}{k}}{\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}+s}\middle|\begin{matrix}\frac{L+k+1}{2},\frac{1-L+k}{2},\frac{1+L-k}{2}\\ \frac{L+k+1}{2}\end{matrix}\right)}{\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}}\right].
\end{aligned}
\tag{33}
$$

For the i.i.d. fading case, based on (25), (33) simplifies to

$$
\begin{aligned}
M_{\gamma_{\text{SD}_{\text{out}}}}(s) = {} & \sum_{n=1}^{N}\binom{N}{n}(-1)^{n+1}\left(1+\frac{s\overline{\gamma}_D}{n}\right)\\
& \times\left[\frac{\left(n/\overline{\gamma}_D+s\right)k/\overline{\gamma}_I}{n/\overline{\gamma}_D}\right]^{\frac{L+k-1}{2}}\frac{1}{\Gamma(k)\Gamma(L)}\\
& \times\left[\mathcal{G}_{3,1}^{1,3}\left(\frac{n\overline{\gamma}_I/k}{n+s\overline{\gamma}_D}\middle|\begin{matrix}\frac{L+k-1}{2},\frac{k-L+1}{2},\frac{1+L-k}{2}\\ \frac{L+k-1}{2}\end{matrix}\right)\right.\\
& \left.+\left(1+\frac{s\overline{\gamma}_D}{n}\right)\mathcal{G}_{3,1}^{1,3}\left(\frac{n\overline{\gamma}_I/k}{n+s\overline{\gamma}_D}\middle|\begin{matrix}\frac{L+k+1}{2},\frac{k-L+1}{2},\frac{1+L-k}{2}\\ \frac{L+k+1}{2}\end{matrix}\right)\right].
\end{aligned}
\tag{34}
$$

Simplified Expressions for the SIR: For the SIR and i.n.d. fading conditions, based on (28) the MGF of $\gamma_{\text{SD}_{\text{out}}}$ can be

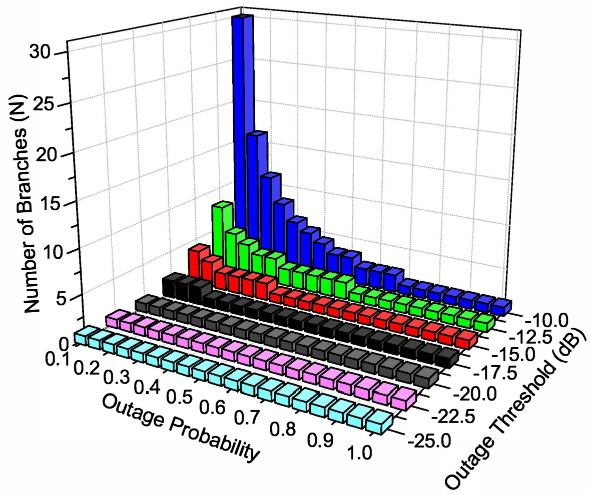

Fig. 2. The number of branches as a function of the outage probability and the normalized outage threshold for SNR-based SD reception.

expressed in closed form as follows

$$
\begin{aligned}
M_{\gamma_{\text{SD}_{\text{out}}}}(s) = {} & \sum_{\substack{n,n+1\\ \lambda_1,\ldots,\lambda_n}}^{N}\left(\frac{k/\overline{\gamma}_I}{\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}}\right)^{\frac{L+k-1}{2}}\frac{s^{\frac{L+k-3}{2}}}{\Gamma(k)\Gamma(L)}\\
& \times\mathcal{G}_{3,1}^{1,3}\left(\frac{\overline{\gamma}_I\mathcal{S}^n_{1,\overline{\gamma}_{D_{\lambda_m}}}}{ks}\middle|\begin{matrix}\frac{L+k+1}{2},\frac{1-L+k}{2},\frac{1+L-k}{2}\\ \frac{L+k+1}{2}\end{matrix}\right)
\end{aligned}
\tag{35}
$$

which for i.i.d. fading conditions simplifies to

$$
\begin{aligned}
M_{\gamma_{\text{SD}_{\text{out}}}}(s) = {} & \sum_{n=1}^{N}\binom{N}{n}(-1)^{n+1}\left(\frac{k\overline{\gamma}_D}{n\overline{\gamma}_I}\right)^{\frac{L+k-1}{2}}\\
& \times\frac{s^{\frac{L+k-3}{2}}}{\Gamma(k)\Gamma(L)}\mathcal{G}_{3,1}^{1,3}\left(\frac{n\overline{\gamma}_I}{k\overline{\gamma}_D s}\middle|\begin{matrix}\frac{L+k+1}{2},\frac{k-L+1}{2},\frac{1+L-k}{2}\\ \frac{L+k+1}{2}\end{matrix}\right).
\end{aligned}
\tag{36}
$$

Using the previously derived MGF expressions and following the MGF-based approach, the ABEP can be readily evaluated for a variety of modulation schemes, e.g., phase shift keying (PSK) or quadrature amplitude modulation (QAM) [5].

VI. Numerical Results and Discussion

In this Section, results related to single as well as multi-channel reception and different $\mathcal{K}$-distributed i.i.d. fading and shadowing conditions will be presented and discussed. Considering multi-channel reception (i.e., based on (24)) and assuming $L = 5, k = 1.6, \overline{\gamma}_I = 5$dB, the number of the required branches for achieving a predefined target OP is plotted in Fig. 2, for different values of the normalized outage threshold, $\gamma_{\text{th}}/\overline{\gamma}_D$. As it is shown in this figure the number of branches increases as $\gamma_{\text{th}}/\overline{\gamma}_D$ increases and/or the target OP decreases. Moreover, it is easily verified that for relatively low values of $\gamma_{\text{th}}/\overline{\gamma}_D$ and/or high target OP it is not necessary to employ SD, since even with single-channel reception the target OP is achieved. This means that the overall power consumption of the receiver side can be reduced by avoiding unnecessary circuity and channel estimations.

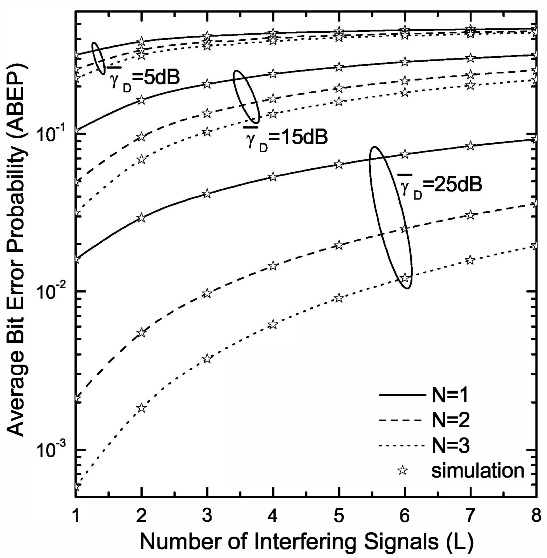

Fig. 3. The ABEP for multi-channel receivers with SNR-based SD reception versus the number of interfering signals.

In Fig. 3, considering SD and $\overline{\gamma}_I = 10$dB, $k = 2$, the ABEP is plotted as a function of the number of interfering signals L, for various values of the number of branches N and different average SNRs $\overline{\gamma}_D$ of the desired signal. We observe that as $\overline{\gamma}_D$ and/or N increase the ABEP decreases, whilst in all cases the performance worsens with the increase of L. It is interesting to note that for higher values of $\overline{\gamma}_D$, the line gap of the performances obtained using different values of N increases. For comparison purposes, computer simulation performance results are also included in Fig. 3, verifying the validity of the proposed theoretical approach.

VII. Conclusions

An analytical framework for evaluating important statistical metrics of the instantaneous output SINR of single as well as multi-channel diversity receivers operating over composite fading channels is presented. Focusing on the fully correlated shadowing case, various statistical characteristics of single-channel and SD receivers are derived in closed form, which are then used to study system performance in terms of ABEP and OP. The obtained results indicate that the combination of fading/shadowing and interference disrupts seriously the system performance. Furthermore, it is shown that a power efficient solution to improve this situation is to employ SD reception with a relatively small number of diversity branches.

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