Preamble Design for Channel Estimation in OFDM/OQAM Cooperative Systems

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***Abstract*—Preamble design for LS channel estimation in OFDM/OQAM cooperative systems is investigated in this paper. A simple but important setup is considered, consisting of a pair of single-antenna terminals (source and destination) assisted in their communication by an AF relay and following a well-established two-phase transmission protocol. The so-called sparse preamble case (i.e., pilot tones surrounded by nulls) was recently addressed and the optimal design – in the sense of minimum MSE subject to transmit energy constraints – was shown to coincide with the one for CP-OFDM, thus resulting in no performance gains from the adoption of OFDM/OQAM. In order to complete this study and exhibit the possibilities of OFDM/OQAM to outperform CP-OFDM, the so-called full preamble design (i.e., with all tones carrying pilot symbols) is addressed in this paper. The results are in line with the corresponding design for single-link systems, where the interference among pilots is positively exploited to provide significant performance gains over CP-OFDM. As a byproduct, the solution for cooperative CP-OFDM is also given, through its connection to OFDM/OQAM. The presented simulation results corroborate the analysis, demonstrating superior performance over CP-OFDM for both mildly and highly frequency selective channels and at practical SNR values.**

Keywords*—Channel estimation, filter bank-based multicarrier (FBMC), least squares (LS), offset quadrature amplitude modulation (OQAM), orthogonal frequency division multiplexing (OFDM), relaying networks.

I. Introduction

Cooperative communication systems are able to offer capacity and spatial diversity gains with simple single-antenna terminals [1], [2]. To combat the frequency selectivity of the channels, the cyclic prefix (CP) orthogonal frequency division multiplexing (OFDM) system is commonly adopted which transforms the channels into a set of independent flat ones. This simplifies the receiver's tasks such as channel estimation and equalization [3]. However, the use of CP leads in a power and spectral efficiency loss (as high as 25%). Moreover, the high sidelobes of the OFDM filters are responsible for increased sensitivity to Doppler effects, spectral leakage and difficulties in user synchronization. Notably, the latter are of great importance in cooperative systems, where the synchronization requirements are desired to be minimal.

Multicarrier schemes based on filter banks (FBMC) have recently shown the potential of overcoming such drawbacks, thus providing an alternative to OFDM, at the cost of some additional complexity and delay [4]. When combined with offset quadrature amplitude modulation (OQAM), filters with good localization in both time and frequency are possible, resulting in the so-called OFDM/OQAM modulation scheme [5]. The latter avoids the use of CP and has the potential of a maximum spectral efficiency. Recently, impressive improvements in the throughput of cognitive radio relaying networks employing OFDM/OQAM were demonstrated over their CP-OFDM counterparts [6].

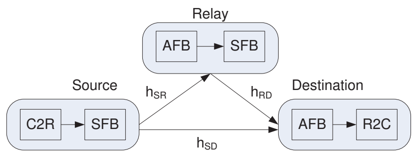

Fig. 1. The cooperative system under consideration.

OFDM/OQAM suffers, however, from an imaginary intercarrier/intersymbol interference, that complicates signal processing tasks at the receiver, including channel estimation. A multitude of training designs and associated channel estimation methods have been proposed for such systems [7]. The design of optimal OFDM/OQAM preambles for the purpose of estimating the channel in single-antenna single-link channels was investigated in [9] (see also [7], [10]). Both full (i.e., with pilots at all the subcarriers) and sparse (i.e., with isolated pilot subcarriers surrounded by nulls) preambles[1] were considered and their performances were analyzed.

Channel estimation for a cooperative OFDM/OQAM system was first studied in [11], for the setup of Fig. 1. In this system, single-antenna terminals are assumed with a single amplify-and-forward (AF) relay assisting a two-phase transmission protocol. As in a CP-OFDM-based system, filter banks are employed at the relay terminal to help amplify the received signal *per subcarrier*. [11] addressed and solved the problem of optimal sparse preamble design for a least squares (LS) channel estimator with optimality being in the sense of minimizing the mean squared error (MSE) subject

[1]Sometimes referred to in the OFDM literature as *block-type* and *comb-type*, respectively.

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to *sum* transmit energy constraints. In sparse preambles, the pilot symbols are guarded by the surrounding nulls and do not interfere with each other. As a result, no pilot symbol energy increase is present at the received signals (as observed in the case of full-preambles) and the system is similar to CP-OFDM in terms of design conditions and estimation performance. In this paper, the same problem is considered for the full preamble case. Here, neighboring pilot symbols interfere with each other resulting, effectively, to an energy increase of each pilot symbol at the receiver [8]. It has been been shown in the single-link case [10] that equal symbols (optimally) maximize this energy increase and that OFDM/OQAM has a superior estimation performance over CP-OFDM. Here, the conditions that are derived are analogous to the ones observed in [10]. Finally, as a byproduct, the optimal full preamble design for the CP-OFDM-based cooperative system will be derived, through its connection to the OFDM/OQAM system. Simulation results are presented for both mildly and highly frequency selective channels, which corroborate the analysis and demonstrate significant performance gains of OFDM/OQAM over its CP-OFDM counterpart, particularly at practical signal-to-noise ratio (SNR) values.

II. System Description

A. The OFDM/OQAM System

The OFDM/OQAM synthesis filter bank (SFB) output signal is given by [5] $s(l) = \sum_{m=0}^{M-1}\sum_n a(m,n)g_{m,n}(l)$, where $a(m,n)$ are real OQAM symbols, produced by the complex to real OQAM modulator (C2R block in Fig. 1) and

$$g_{m,n}(l) = g\left(l - n\frac{M}{2}\right)e^{j\frac{2\pi}{M}m\left(l-\frac{L_g-1}{2}\right)}e^{j\phi(m,n)}, \quad (1)$$

with g being a real symmetric prototype filter impulse response of length $L_g = MK$ and unit energy. M is the (even) number of subcarriers, K is the overlapping factor and $\phi(m,n) = \phi_0 + \frac{\pi}{2}(m+n)$. ϕ_0 can be arbitrarily chosen, e.g. $\phi(m,n) = (m+n)(\pi/2) - mn\pi$ [5]. Finally, the pair (m,n) is the frequency-time (FT) point with subcarrier index m and time index n.

The main disadvantage of such a system is that $g_{m,n}(\cdot)$ are orthogonal only in the real field. This means that even under perfect transmission conditions, there is an inherent intercarrier/intersymbol interference at the output of receiver's analysis filter bank (AFB) that is purely imaginary, namely $\sum_l g_{m,n}(l)g^*_{p,q}(l) = ju^{p,q}_{m,n}$, where $u^{p,q}_{m,n}$ is real. This is known as *intrinsic* interference [14].

We will make the common assumption that the interference is limited to the first-order neighborhood $\Omega_{p,q}$ around (p,q) and that the channel is (approximately) frequency flat over each subband [7]. These are true for sufficiently well (with respect to the channel coherence bandwidth) time-frequency localized pulses g. If the additional common assumption of a channel whose frequency responce (CFR) is almost constant over $\Omega_{p,q}$ is made – valid for slowly time varying or time invariant channels that are quite short relatively to the filter bank size –, the AFB output at (p,q) can be written as [7], [8]

$$y(p,q) = H(p,q)b(p,q) + \eta(p,q), \quad (2)$$

where $H(p,q)$ is the (M-point) CFR and

$$b(p,q) = a(p,q) + j\sum_{(m,n)\in\Omega_{p,q}} a(m,n)u^{p,q}_{m,n} \quad (3)$$

is the *virtual* transmitted symbol at (p,q). The noise component $\eta(p,q)$ is $\mathcal{CN}(0,\sigma^2)$ if the channel noise is so distributed and is moreover correlated among adjacent subcarriers (see, e.g., [10], [13]).

B. The Cooperative System

The cooperative system under consideration is schematically shown in Fig. 1. A two-phase transmission protocol (first proposed in [15]) to be shortly described is adopted. As shown in [16], this protocol offers the optimal diversity/multiplexing trade-off among all the AF half-duplex protocols. The source S, the destination D, and the relay R are single-antenna terminals. S and R are assumed to be synchronized.

The channel impulse responses $\boldsymbol{h}_i$ are modeled as $L_i \times 1$ complex Gaussian random vectors with independent elements, i.e. $\boldsymbol{h}_i \sim \boldsymbol{\mathcal{CN}}(\boldsymbol{0}, \boldsymbol{C}_i)$, where $\boldsymbol{C}_i$ is diagonal and $i \in \{\mathrm{SD}, \mathrm{SR}, \mathrm{RD}\}$. For the sake of the analysis, these channels are assumed (almost) time invariant for the duration of the two phases. Hence, the time index in the CFRs will be henceforth omitted. Moreover, they are assumed to be short enough to satisfy (2) above.

During the first phase, S transmits the symbols $a_1(p,q)$ to R and D. These are received as

$$y_{\mathrm{R}}(p,q) = H_{\mathrm{SR}}(p)b_1(p,q) + \eta_{\mathrm{R}}(p,q), \quad (4)$$

$$y_{\mathrm{D}_1}(p,q) = H_{\mathrm{SD}}(p)b_1(p,q) + \eta_{\mathrm{D}_1}(p,q), \quad (5)$$

respectively. The noise terms are as previously, i.e. correlated complex Gaussian with zero mean and variances σ^2_{R}, σ^2_{D}, respectively. The signal $y_{\mathrm{R}}(p,q)$ is amplified by the relay as

$$x_{\mathrm{R}}(p,q) = \lambda(p,q)y_{\mathrm{R}}(p,q), \quad (6)$$

where the amplification factor $\lambda(p,q)$ is used to "regulate" the transmitted energy per FT point (p,q). In the second phase, S and R send the symbols $a_2(p,q)$ and $x_{\mathrm{R}}(p,q)$ to D, respectively. These are received as

$$y_{\mathrm{D}_2}(p,q) = H_{\mathrm{SD}}(p)b_2(p,q) + H_{\mathrm{RD}}(p)b_{\mathrm{R}}(p,q) + \eta_{\mathrm{D}_2}(p,q), \quad (7)$$

where $\eta_{\mathrm{D}_2}(p,q)$ is statistically described similarly with $\eta_{\mathrm{D}_1}(p,q)$, and $b_2(p,q)$, $b_{\mathrm{R}}(p,q)$ are as in (3) but with $a_2(p,q)$ and (complex) $x_{\mathrm{R}}(p,q)$ at the place of transmitted symbols.

III. Full Preamble-based Channel Estimation

For channel estimation, the source transmits a full preamble at the beginning of each phase, to assist the estimator at D. The preambles will be assumed to consist of two OFDM/OQAM symbols. The first one is a vector of training symbols $a(p,0)$ (which is full) while the second one is a zero vector, i.e. $a(p,1) = 0$, for all p, which serves as a guard against interference from the data. For the sake of the analysis, and without loss of generality, the all zeros FBMC symbol that is also commonly sent before the pilots will be omitted here (as in [10]).[2] In view of the above, the pilot symbol at

[2]Its absence, in practice, can be justified, for example in wireless transmissions that involve inter-frame gaps.

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position p is interfered only by those at $p-1$ and $p+1$. Moreover, with a slight abuse of the OQAM definition, we incorporate, as in [10], the phase factors $e^{j\phi(p,0)}$ in the training symbols, getting $x(p,0) = a(p,0)e^{j\phi(p,0)}$, with the associated minor modification to the $g_{m,0}(\cdot)$ definition. This results in $\sum_l g_{m+1,0}(l)g^*_{m,0}(l) = \sum_l g_{m-1,0}(l)g^*_{m,0}(l) = \beta$, i.e., the interference corresponding to $ju^{p,0}_{m,0}$ in (3) for $m = p \pm 1$ is then purely real with $\beta > 0$ defined in [7]. Hence, the corresponding virtual pilot symbol $b(p,0)$ is given by

$$b(p,0) = x(p,0) + x(p-1,0)\beta + x(p+1,0)\beta. \quad (8)$$

It is the presence of these interfering terms in a full preamble that, with an appropriate choice of the $x(p,0)$'s, can increase (preferably maximize) the energy of the $b(p,0)$'s and permit significant gains in estimation performance over both the OFDM/OQAM sparse preamble and the CP-OFDM full preamble (where $\beta = 0$). Choosing the $x(p,0)$'s to be all equal was shown in [10] to be energy maximizing in single-link OFDM/OQAM systems. Analogous results will be derived here for the cooperative system of interest.

In the following, the time index 0 will be dropped for convenience. The received signals in the first phase are

$$y_{\rm R}(p) = H_{\rm SR}(p)b_1(p) + \eta_{\rm R}(p), \quad (9)$$

$$y_{\rm D_1}(p) = H_{\rm SD}(p)b_1(p) + \eta_{\rm D_1}(p), \quad (10)$$

for R and D, where $b_1(p)$'s are defined according to (8). During the second phase, the source sends a new 2-symbol full preamble of the above structure, using this time the symbols $x_2(p)$. The relay recovers the full preamble structure as follows. It forwards the $x_{\rm R}(p)$'s for the first OFDM/OQAM symbol and the all-zeros one for the second one. For $x_{\rm R}(p)$'s, the following λ factors are used

$$\lambda(p) = \sqrt{\frac{e(p)}{\mathcal{E}(|y_{\rm R}(p)|^2)}} = \sqrt{\frac{e(p)}{\theta^2_{\rm SR}|b_1(p)|^2 + \sigma^2_{\rm R}}}, \quad (11)$$

which set the mean energy per subcarrier p at the input of the relay SFB to $e(p)$. The mean energy $\theta^2_{\rm SR}$ of $H_{\rm SR}(p)$ is given by $\theta^2_{\rm SR} = \mathcal{E}(|H_{\rm SR}(p)|^2) = {\rm Tr}(\boldsymbol{C}_{\rm SR})$, which is independent of p (for channels with uncorrelated scattering, as the ones that are considered here).

The received signal at D, during the second phase, can be written as

$$y_{\rm D_2}(p) = H_{\rm SD}(p)b_2(p) + H_{\rm R}(p)b_3(p) + w_2(p), \quad (12)$$

where $H_{\rm R}(p) = H_{\rm SR}(p)H_{\rm RD}(p)$ is the CFR of the S-R-D channel.[3] The $b_2(p)$'s are defined according to (8) and $b_3(p) = \lambda(p)b_1(p) + \lambda(p-1)b_1(p-1)\beta + \lambda(p+1)b_1(p+1)\beta$, for all p. Finally, $w_2(p) = H_{\rm RD}(p)\lambda(p)\eta_{\rm R}(p) + H_{\rm RD}(p)\lambda(p-1)\eta_{\rm R}(p-1)\beta + H_{\rm RD}(p)\lambda(p+1)\eta_{\rm R}(p+1)\beta + \eta_{\rm D_2}(p)$ is a zero mean random variable with variance equal to

$$\sigma^2_{w_2}(p) = \theta^2_{\rm RD}\sigma^2_{\rm R}\big[\lambda^2(p) + \lambda^2(p-1)\beta^2 + \lambda^2(p+1)\beta^2 + 2\lambda(p)\lambda(p-1)\beta^2 + 2\lambda(p)\lambda(p+1)\beta^2\big] + \sigma^2_{\rm D}, \quad (13)$$

where $\theta^2_{\rm RD} = \mathcal{E}(|H_{RD}(p)|^2) = {\rm Tr}(\boldsymbol{C}_{\rm RD})$ and our knowledge of the correlation of adjacent noise components (equal to $\sigma^2_{\rm R}\beta$ [10]) has been used.

[3]It is tacitly assumed here that the composite channel is short enough to meet the assumptions stated in Sec. II-A that validate (2).

Putting eqs. (10) and (12) together results in

$$\begin{bmatrix} \boldsymbol{y}_{\rm D_1} \\ \boldsymbol{y}_{\rm D_2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{0} \\ \boldsymbol{B}_2 & \boldsymbol{B}_3 \end{bmatrix} \begin{bmatrix} \boldsymbol{H}_{\rm SD} \\ \boldsymbol{H}_{\rm R} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\eta}_{\rm D_1} \\ \boldsymbol{w}_2 \end{bmatrix}, \quad (14)$$

where $\boldsymbol{y}_{{\rm D}_k} = [\, y_{{\rm D}_k}(0) \;\; y_{{\rm D}_k}(1) \;\; \cdots \;\; y_{{\rm D}_k}(M-1)\,]^T$, $k = 1,2$, and similarly for $\boldsymbol{H}_i, \boldsymbol{\eta}_{\rm D_1}, \boldsymbol{w}_2$, and $\boldsymbol{B}_l = {\rm diag}(b_l(0), b_l(1), \ldots, b_l(M-1))$, for $l = 1,2,3$. Equivalently, with straightforward matching of terms,

$$\boldsymbol{y} = \boldsymbol{B}\boldsymbol{H} + \boldsymbol{w}, \quad (15)$$

where it is natural to assume the matrix $\boldsymbol{B}$ to be non-singular. The noise term $\boldsymbol{w}$ is zero mean with covariance $\boldsymbol{C}_{\boldsymbol{w}} = {\rm diag}\left(\boldsymbol{C}_{\boldsymbol{\eta}_{\rm D_1}}, \boldsymbol{C}_{\boldsymbol{w}_2}\right)$. The diagonal blocks of $\boldsymbol{C}_{\boldsymbol{w}}$ are not diagonal matrices. However, as it will be observed later on, we are only interested in their diagonal elements, which are $[\boldsymbol{C}_{\boldsymbol{\eta}_{\rm D_1}}]_{pp} = \sigma^2_{\rm D}$ and $[\boldsymbol{C}_{\boldsymbol{w}_2}]_{pp} = \sigma^2_{w_2}(p)$, respectively. The LS estimate of $\boldsymbol{H}$ and the associated error covariance matrix are obviously given by[4]

$$\hat{\boldsymbol{H}} = \boldsymbol{B}^{-1}\boldsymbol{y}, \quad \boldsymbol{C}_{\hat{\boldsymbol{H}}} = \boldsymbol{B}^{-1}\boldsymbol{C}_{\boldsymbol{w}}\boldsymbol{B}^{-H} \quad (16)$$

IV. Preamble Design

The training design consists of a) determining the source training symbols $x_k(p) = a_k(p)e^{j\phi_k(p)}$, $k = 1,2$, b) the relay energies $e(p)$ per subcarrier in the second phase, and c) the source training energy allocation between the two transmission phases. In a manner analogous to [11], the preamble optimization criterion will be to minimize the ${\rm MSE} = \frac{1}{2M}{\rm Tr}(\boldsymbol{C}_{\hat{\boldsymbol{H}}})$ subject to *sum* energy constraints at the source and the relay, namely

$$\min_{\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{e},\mathcal{E}_1,\mathcal{E}_2} \frac{1}{2M}{\rm Tr}(\boldsymbol{C}_{\hat{\boldsymbol{H}}}) \quad (17)$$

subject to (s.t.)

$$\sum_{p=0}^{M-1}\big[|x_k(p)|^2 + \beta x_k(p)x^*_k(p-1) + \beta x_k(p)x^*_k(p+1)\big] \le \mathcal{E}_k,\ k = 1,2 \quad (18)$$

$$\mathcal{E}\Big\{\sum_{p=0}^{M-1}\big[|x_{\rm R}(p)|^2 + \beta x_{\rm R}(p)x^*_{\rm R}(p-1) + \beta x_{\rm R}(p)x^*_{\rm R}(p+1)\big]\Big\} \le \mathcal{E}_{\rm R} \quad (19)$$

$$\mathcal{E}_1 + \mathcal{E}_2 = \mathcal{E}_{\rm S}, \quad (20)$$

where $\boldsymbol{x}_k$ and $\boldsymbol{e}$ are $M \times 1$ vectors containing the $x_k(p)$'s and $e(p)$'s, respectively, $\mathcal{E}_k$ is the source energy allocated to training in phase k and $\mathcal{E}_{\rm R}, \mathcal{E}_{\rm S}$ are given energy budgets referring to the SFB outputs (see also [9], [10]). In (19), the relay energy is constrained in a mean sense.

It will be convenient to re-write the cost function above in an alternative form. Specifically, by applying the matrix inversion lemma to the 2×2 block matrix $\boldsymbol{B}^{-1}$ with diagonal blocks, it can be shown that the trace in (17) is applied on a sum of diagonal matrices. The MSE can then be written as ${\rm MSE} = \frac{1}{2M}\sum_{p=0}^{M-1} v_p$, where

$$v_p = \frac{\sigma^2_{\rm D}}{|b_1(p)|^2} + \frac{\sigma^2_{\rm D}|b_2(p)|^2}{|b_3(p)|^2|b_1(p)|^2} + \frac{\sigma^2_{w_2}(p)}{|b_3(p)|^2} \quad (21)$$

[4]In view of the color of the noise $\boldsymbol{w}$, one could instead consider the Gauss-Markov estimator. It is obvious however that this would lead here to the same error covariance as in (16).

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Some comments concerning the energies are in order. First, the constraints of the minimization problem correspond to the energies at the *output* of the SFBs of the source or the relay. Second, due to the stability of the SFBs, the energies at the inputs of the SFBs can be constrained as $\sum_{p=0}^{M-1}|x_1(p)|^2 \leq E_1$ and $\sum_{p=0}^{M-1}|x_2(p)|^2 \leq E_2$ for the source and $\sum_{p=0}^{M-1} e(p) \leq E_{\rm R}$ for the relay (see also [9], [10]). Furthermore, it can be shown [10] that $\sum_{p=0}^{M-1}|b_k(p)|^2 \leq (1+2\beta)^2 \sum_{p=0}^{M-1}|x_k(p)|^2$, $k=1,2$.

A. Optimal Energy Allocation Between the Phases

First, the optimal allocation of the total energy $\mathcal{E}_{\rm S}$ (equivalently $E_{\rm S}$) at the source between the two phases is investigated. All terms in (21) are functions of the symbols $x_k(p)$. Based on $\sum_{p=0}^{M-1}|x_k(p)|^2 \leq E_k$, the symbol energies $|x_k(p)|^2$ can be written as fractions of E_k, namely as $|x_k(p)|^2 = f_k(p)E_k$, where $\sum_p f_k(p) \leq 1$. Setting also $E_2 = E_1 - E_{\rm S}$, (21) can be written as function of E_1 only and can thus be minimized for $0 \leq E_1 \leq E_{\rm S}$. One can then readily show that the MSE is a decreasing function of E_1 and its minimum value is attained for $E_1 = E_{\rm S}$, implying that $E_2 = 0$ and $\boldsymbol{x}_2 = \mathbf{0}$.

B. Solution for $x_1(p)$'s and $e(p)$'s

The optimization problem is now written as follows

$$\min_{\boldsymbol{x}_1,\boldsymbol{e}} \frac{1}{2M}\sum_{p=0}^{M-1}\left[\frac{\sigma_{\rm D}^2}{|b_1(p)|^2} + \frac{\sigma_{w_2}^2(p)}{|b_3(p)|^2}\right], \quad (22)$$

$$\text{s.t.} \sum_{p=0}^{M-1}\big[|x_1(p)|^2 + \beta x_1(p)x_1^*(p-1) + \beta x_1(p)x_1^*(p+1)\big] \leq \mathcal{E}_{\rm S} \quad (23)$$

$$\mathcal{E}\{\sum_{p=0}^{M-1}\big[|x_{\rm R}(p)|^2 + \beta x_{\rm R}(p)x_{\rm R}^*(p-1) + \beta x_{\rm R}(p)x_{\rm R}^*(p+1)\big]\} \leq \mathcal{E}_{\rm R} \quad (24)$$

The cost function in (22) has a complicated form with respect to the unknown parameters. This is due to the fact that the amplification factors $\lambda(m)$, for $m = p-1, p, p+1$, which appear at both the numerator and the denominator of the second term in (22), are in turn functions of the b_1's and e's. It thus seems that an analytical, closed-form expression for the optimal parameters is difficult to be found. However, targeting such a solution, we first derive a lower bound, which will suggest a simplification allowing us to come up with an analytical solution.

Indeed, by using the triangle inequality at the denominator of the second term in (22), we can write $|b_3(p)|^2 \leq (|\lambda(p)b_1(p)| + |\lambda(p-1)b_1(p-1)\beta| + |\lambda(p+1)b_1(p+1)\beta|)^2 = L_1(p)$, where the equality holds iff the b_1's have the same phase. Applying the Cauchy-Schwartz inequality in its numerator leads to $\sigma_{w_2}^2(p) \geq \theta_{\rm RD}^2\sigma_{\rm R}^2\{\lambda^2(p) + \frac{\beta^2}{6}[\lambda(p-1) + \lambda(p+1) + 2\sqrt{\lambda(p)\lambda(p-1)} + 2\sqrt{\lambda(p)\lambda(p+1)}]^2\} + \sigma_{\rm D}^2 = L_2(p)$, with equality iff $\lambda(p) = \lambda(p-1) = \lambda(p+1)$. The cost function in (22) can then be lower bounded as $\text{MSE} \geq \frac{1}{2M}\sum_{p=0}^{M-1}\left[\frac{\sigma_{\rm D}^2}{|b_1(p)|^2} + \frac{L_2(p)}{L_1(p)}\right]$, where the equality holds under the aforementioned conditions.

The above suggests that letting the λ's at a subcarrier p and its immediate neighbors be equal is a plausible choice. This is a first approach to be analyzed below. In a second approach to simplifying the problem, the relay is assumed to operate at a high SNR regime, i.e., $\sigma_{\rm R}^2 \approx 0$.

1) Assuming $\lambda(p) = \lambda(p-1) = \lambda(p+1)$*:* We then have $L_2(p) = \theta_{\rm RD}^2\sigma_{\rm R}^2\lambda^2(p)(1+6\beta^2) + \sigma_{\rm D}^2$ and $L_1(p) = \lambda^2(p)|b_3'(p)|^2$, where $b_3'(p) = b_1(p) + b_1(p-1)\beta + b_1(p+1)\beta$. The MSE can then be lower bounded as

$$\text{MSE} \geq \frac{1}{2M}\sum_{p=0}^{M-1}\left[\frac{\sigma_{\rm D}^2}{|b_1(p)|^2} + \frac{\theta_{\rm RD}^2\sigma_{\rm R}^2\left(1+6\beta^2\right)}{|b_3'(p)|^2} + \frac{\sigma_{\rm R}^2\sigma_{\rm D}^2}{e(p)|b_3'(p)|^2} + \frac{\sigma_{\rm D}^2\theta_{\rm SR}^2|b_1(p)|^2}{e(p)|b_3'(p)|^2}\right]. \quad (25)$$

In view of the constraints $\sum_{p=0}^{M-1}|b_1(p)|^2 \leq E_{\rm S}(1+2\beta)^2$, $\sum_{p=0}^{M-1}|b_3'(p)|^2 \leq E_{\rm S}(1+2\beta)^4$ and $\sum_{p=0}^{M-1} e(p) \leq E_{\rm R}$, and resorting to the Arithmetic-Harmonic mean (AHM) inequality for the first term and to Lagrange multipliers for the rest, the above lower bound is seen to be minimized for $|b_1(p)|^2 = E_{\rm S}(1+2\beta)^2/M$, $|b_3'(p)|^2 = E_{\rm S}(1+2\beta)^4/M$ and $e(p) = E_{\rm R}/M$. These values can be achieved for $x_1(p) = \sqrt{E_{\rm S}/M}e^{j\phi}$ and $e(p) = E_{\rm R}/M$, for all p. Finally, the constraints in (23), (24) hold with equality if $E_{\rm S}(1+2\beta) = \mathcal{E}_{\rm S}$ and $E_{\rm R}\left[1 + 2\beta + \frac{2\beta(\beta-1)\sigma_{\rm R}^2 M}{\theta_{\rm SR}^2 E_{\rm S}(1+2\beta)^2 + M\sigma_{\rm R}^2}\right] = \mathcal{E}_{\rm R}$, respectively.

2) The relay operates at high SNR: With $\sigma_{\rm R}^2 \approx 0$, we have $L_2(p) \approx \sigma_{\rm D}^2$ and $L_1(p) = \theta_{\rm SR}^{-2}[\sqrt{e(p)} + \beta\sqrt{e(p-1)} + \beta\sqrt{e(p+1)}]^2$. Moreover, the constraint $\sum_{p=0}^{M-1}[\sqrt{e(p)} + \beta\sqrt{e(p-1)} + \beta\sqrt{e(p+1)}]^2 \leq E_{\rm R}(1+2\beta)^2$ can be written, in a similar way as $\sum_{p=0}^{M-1}|b_1(p)|^2 \leq E_{\rm S}(1+2\beta)^2$. The MSE can then be bounded as

$$\begin{aligned}\text{MSE} &\geq \frac{\sigma_{\rm D}^2 M^2}{2M\sum_{p=0}^{M-1}|b_1(p)|^2} + \frac{\sigma_{\rm D}^2 M^2}{2M\sum_{p=0}^{M-1}L_1(p)}\\ &\geq \frac{\sigma_{\rm D}^2 M^2}{2ME_{\rm S}(1+2\beta)^2} + \frac{\sigma_{\rm D}^2\theta_{\rm SR}^2 M^2}{2ME_{\rm R}(1+2\beta)^2},\end{aligned}$$

where the first inequality is due to the AHM inequality and the second to the above constraints. The bound holds with equality for $|b_1(p)|^2 = \frac{E_{\rm S}(1+2\beta)^2}{M}$ and $[\sqrt{e(p)} + \beta\sqrt{e(p-1)} + \beta\sqrt{e(p+1)}]^2 = \frac{E_{\rm R}(1+2\beta)^2}{M}$. These values can be obtained for $x_1(p) = \sqrt{E_{\rm S}/M}e^{j\phi}$ and $e(p) = E_{\rm R}/M$, for all p, a choice that also minimizes (22). This is the desired solution for (22) if the constraints (23), (24) hold with equality, which is true if, additionally, $E_{\rm S}(1+2\beta) = \mathcal{E}_{\rm S}$ and $E_{\rm R}(1+2\beta) = \mathcal{E}_{\rm R}$, respectively.

Remark: In both of the cases analyzed above, the choice of all equal $x_1(p)$ is shown to be a solution. It is interesting to recall that this is in line with the optimal preamble design in single-link OFDM/OQAM systems [7], where it was shown to maximize the virtual pilot energies in (8). Moreover, this choice of the pilot symbols, in conjuction with the uniform energy allocation at the relay, also leads to all equal λ's, something that was only assumed in the first approach. Note also that the matrix $\boldsymbol{B}$ in (15) becomes diagonal (and indeed nonsingular).

C. The CP-OFDM Case

In this subsection, we will describe the preamble design for the cooperative system under study when CP-OFDM is utilized

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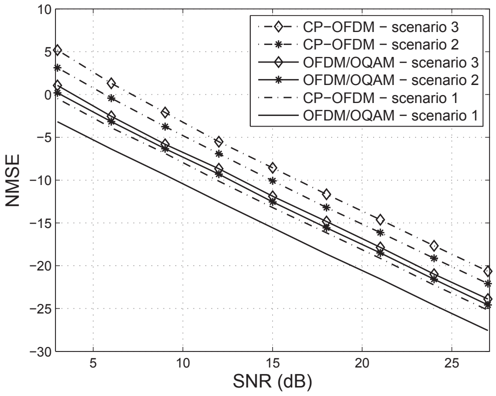

Fig. 2. Estimation performance for channels of low to mild frequency selectivity. $M = 64$, $K = 3$.

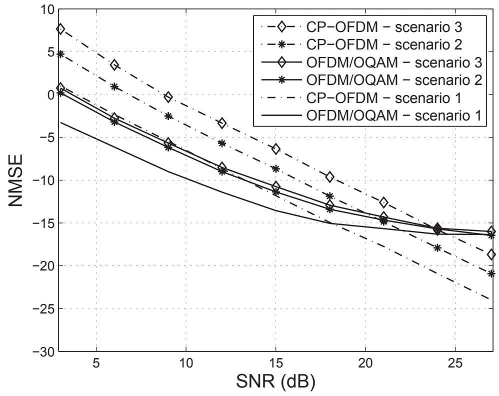

Fig. 3. As in Fig. 2, for highly frequency selective channels.

instead of OFDM/OQAM. It is easy to verify that the system equations (14) are directly applicable to the CP-OFDM based system by setting $\beta = 0$. Here is how the optimization problem also results from (17). Denoting the energies $|x_k(p)|^2$ by $\alpha_k(p)$ for $k = 1, 2$, the associated cost function and constraints can be written as $\mathrm{MSE} = \frac{1}{2M} \sum_{p=0}^{M-1} \Big[\frac{\sigma_{\mathrm{D}}^2 + \sigma_{\mathrm{R}}^2 \theta_{\mathrm{RD}}^2}{\alpha_1(p)} + \frac{\sigma_{\mathrm{D}}^2 \theta_{\mathrm{SR}}^2 \alpha_2(p)}{e(p)\alpha_1(p)} + \frac{\sigma_{\mathrm{D}}^2 \sigma_{\mathrm{R}}^2 \alpha_2(p)}{e(p)\alpha_1^2(p)} + \frac{\sigma_{\mathrm{D}}^2 \theta_{\mathrm{SR}}^2}{e(p)} + \frac{\sigma_{\mathrm{D}}^2 \sigma_{\mathrm{R}}^2}{e(p)\alpha_1(p)} \Big]$ and $\sum_{p=0}^{M-1} \alpha_k(p) \leq E_k$, for $k = 1, 2$, $\sum_{p=0}^{M-1} e(p) \leq E_{\mathrm{R}}$ and $E_1 + E_2 = E_{\mathrm{S}}$, respectively. As observed, in the CP-OFDM case, the preamble design is focused only on the energies and not the values of the pilot symbols. This problem can be optimally solved and the result is that S uniformly allocates all its energy to the first phase and R forwards the pilot signals by assigning uniform energy per subcarrier. A similar problem can be found in [12] although there the relay plays no significant role in the design as its amplification is not performed per subcarrier as it is here.

V. Simulation Results

In this section, simulation results are reported to test the previous analysis. Filter banks designed as in [17] were employed, with $M = 64$ and $K = 3$. Results are shown for two cases, corresponding to low/mild and severe frequency selectivity. In the first case, all channels were generated to undergo Rayleigh block fading with an exponential profile and lengths $L_{\mathrm{SD}} = 4$, $L_{\mathrm{SR}} = 3$, and $L_{\mathrm{RD}} = 2$, that is, quite small compared to M. The ITU Veh-A profile was assumed in the second case for all channels involved, giving rise to channels of lengths $L_{\mathrm{SR}} = L_{\mathrm{RD}} = L_{\mathrm{SD}} = 29 \approx \frac{M}{2}$. (The S-R-D channel has length $57 \approx M$.) The energy budgets were chosen as $\mathcal{E}_{\mathrm{S}} = \mathcal{E}_{\mathrm{R}} = M$, so that the mean energy per pilot symbol be equal to 1. Moreover, as usually assumed, $\sigma_{\mathrm{D}}^2 = \sigma_{\mathrm{R}}^2$. The performance of the corresponding CP-OFDM system was also tested, where a CP of minimum length (equal to the channel order) was assumed.

Three scenarios were examined. In the first one, the derived optimal training conditions were respected. In the second and third scenarios, $E_1 = E_2$. The third scenario also permits the relay to depart from the uniform energy allocation and employ randomly chosen λ's. The results are depicted in Figs. 2 and 3, for the two channel models, respectively. The normalized MSE (NMSE) ($\mathcal{E}(\|\boldsymbol{H} - \hat{\boldsymbol{H}}\|^2 / \|\boldsymbol{H}\|^2)$) performance is plotted versus SNR. As expected, the OFDM/OQAM performance is considerably better at practical SNRs. At weak noise regimes, the inaccuracies of the model (2), which relies on the assumption of relatively low channel frequency selectivity, become more apparent, resulting in the well-known error floors in the OFDM/OQAM performance [7] (see Fig. 3). Finally, one can see that the violation of the training conditions deteriorates the performance for both OFDM systems.

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