Semi-blind channel estimation schemes based on a cooperative form of the cross relation criterion

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***Abstract*—In this paper, two channel estimation schemes are derived for specific cooperative scenarios. Both schemes are based on the cross-relation criterion that has been extensively studied in the (semi-) blind literature. As shown in the paper, in a cooperative system, the channel estimator can be constructed in a natural way either by fractionally-spaced or symbol-spaced samples. We investigate the performance of these two schemes using semi-analytic arguments accompanied by corresponding experimental results.**

I. INTRODUCTION

In the last few years, there have been growing research efforts in the area of cooperative communications [1]. The main idea is that terminals which, due to, mainly, cost and size considerations, can afford only one transmit-receive antenna, cooperate with each other in order to send and/or receive information. The motivation behind their cooperation comes from the gains that are achieved in multi-antenna systems regarding the bandwidth, the consumed power and the error rates because of the provided spatial diversity.

In most of the recently proposed methods for cooperative systems, the involved channels are assumed to be known. Thus, in order the methods to be of practical use, a channel estimation method is needed to provide the desired information. In a wireless environment, as in cooperative communications, semi-blind techniques which combine a blind and a training-based part would be particularly attractive. This combination enjoys the advantages of the purely blind and training methods without suffering from their drawbacks. Thus, among others, on the one hand, the required training sequence is significantly reduced as compared to a purely training-based method. On the other hand, a semi-blind channel estimation method is more robust compared to purely blind methods with respect to frequently encountered problems, such as order overestimation and existence of common roots [2].

Here, we will describe a simple semi-blind least-squares channel estimator that is based on the cross-relation criterion [3]. More specifically, we will see that in a cooperative communications system, such a channel estimator can be constructed naturally either by using fractionally-spaced samples or symbol-spaced samples. We shall study the performance in either case and draw useful conclusions about these two different schemes.

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In the following section, a cooperative communications system is described along with the transmission protocols that are used. In Section III, the CR criterion is presented and, then, a simple least-squares semi-blind channel estimator is derived. In Section IV, two schemes for the construction of the estimator are presented using fractionally-spaced and symbol-spaced samples, respectively. In Section V, the two schemes are compared through semi-analytic arguments that are accompanied by experimental results. Finally, Section VI concludes the paper.

II. COOPERATIVE SYSTEM AND TRANSMISSION PROTOCOLS

We are interested in the communications system that is depicted by Fig. 1. In this system, the source node *S* transmits a signal $s(n)$ to the destination node *D* either directly or through the relay node *R* which operates in a half duplex mode. Node *R* detects and forwards the signal $s(n)$ to node *D*. We assume that node *R* reconstructs perfectly the signal $s(n)$, since, in the scenario under consideration, the source node cooperates with a relay node only when a high SNR channel exists between them and, thus, the detection in the relay node could be considered error-free. However, in Section V, we provide some experimental results in order to demonstrate the impact of errors on the performance of the proposed schemes.

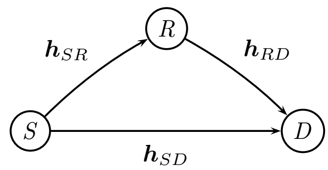

Fig. 1. The cooperative system

We consider the transmission protocols [4] which are presented in Table II. Both protocols need two phases in order to pass the information from *S* to *D*. It is assumed that there is no interference between the two phases, meaning that the signals transmitted are well-separated by some means. In the table, the notation *X,Y* $\rightarrow$ *Z,W* means that nodes *X,Y* transmit information to nodes *Z,W*.

TABLE I
TRANSMISSION PROTOCOLS

	Phase A	Phase B
Protocol I	$S \to R, D$	$S, R \to D$
Protocol II	$S \to R, D$	$R \to D$

Here, in each protocol, we focus on the channels whose ending point is node *D*. Hence, in *Protocol I*, the discrete-time signals that are received by node *D* in *Phases A* and *B*, respectively, are

$$y_A(n) = \boldsymbol{h}_{SD}^H \boldsymbol{s}(n) + w(n), \tag{1}$$

$$y_B(n) = (\boldsymbol{h}_{SD}^H + \boldsymbol{h}_{RD}^H)\boldsymbol{s}(n) + w(n), \tag{2}$$

where $\boldsymbol{h}_i^H = [h_i^*(0), h_i^*(1) \dots h_i^*(L)]$, $\boldsymbol{s}(n) = [s(n), s(n-1) \dots s(n-L)]^T$, $i \in \{SD, RD\}$ and $n = L, \dots, N+L-1$. The $(.)^T$, $(.)^H$, $(.)^*$, L, N and $w(n)$ denote the transposition, complex conjugate transposition and complex conjugate operations, the channel order, the number of samples and the noise samples, respectively. The latter are assumed independent and identically distributed zero mean complex Gaussian random variables. In (2), it is assumed that either nodes *S* and *R* transmit in a synchronous manner or any asynchronicity between them is incorporated into the channels. Also, channels $\boldsymbol{h}_{SD}$ and $\boldsymbol{h}_{RD}$ are assumed constant over each phase and, additionally, $\boldsymbol{h}_{SD}$ remains the same for both phases.

The corresponding signals for the case of *Protocol II* are

$$y_A(n) = \boldsymbol{h}_{SD}^H \boldsymbol{s}(n) + w(n), \tag{3}$$

$$y_B(n) = \boldsymbol{h}_{RD}^H \boldsymbol{s}(n) + w(n). \tag{4}$$

The definitions and the assumptions of *Protocol I* are valid here, too. However, in this case, nodes *S* and *R* are not required to transmit synchronously.

III. SEMI-BLIND CHANNEL ESTIMATION BASED ON THE CROSS-RELATION CRITERION

The cross-relation criterion (CR), [3], is used when the communications system can be modeled as a multichannel one. Such a model is obtained either by oversampling the output of a single-input single-output (SISO) system or because the system follows this model by design [2]. Here, we focus on systems with two outputs.

More specifically, let us assume that the two noise-free outputs are given by $\boldsymbol{y}_1 = \boldsymbol{h}_1 * \boldsymbol{s}$ and $\boldsymbol{y}_2 = \boldsymbol{h}_2 * \boldsymbol{s}$, respectively, where $*$ denotes the convolution operation. Also, let us assume that $\boldsymbol{y}_1$, $\boldsymbol{y}_2$ are filtered by the filters $\boldsymbol{w}_1$, $\boldsymbol{w}_2$, respectively. The CR criterion states that if $\boldsymbol{y}_1 * \boldsymbol{w}_1 = \boldsymbol{y}_2 * \boldsymbol{w}_2$, then $\boldsymbol{w}_1 = \alpha \boldsymbol{h}_2$ and $\boldsymbol{w}_2 = \alpha \boldsymbol{h}_1$ and, therefore, the channels can be estimated blindly up to a constant factor.

Using the CR criterion $\boldsymbol{y}_1 * \boldsymbol{h}_2 = \boldsymbol{y}_2 * \boldsymbol{h}_1$, we obtain the following linear system of equations with respect to the unknown channel vectors

$$\begin{bmatrix} \boldsymbol{Y}_2 & -\boldsymbol{Y}_1 \end{bmatrix} \begin{bmatrix} \boldsymbol{h}_1 \\ \boldsymbol{h}_2 \end{bmatrix} = \boldsymbol{0}. \tag{5}$$

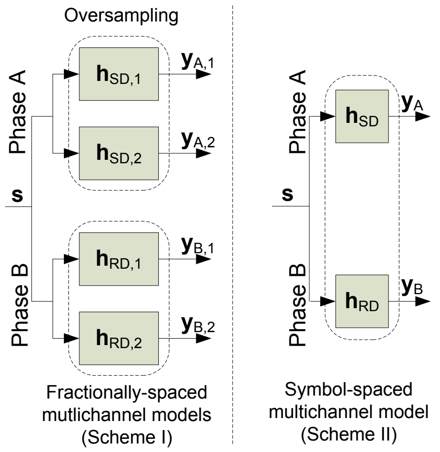

Fig. 2. The two schemes for *Protocol II*

In (5), the $\boldsymbol{Y}_k$'s and $\boldsymbol{h}_k$'s, $k = 1, 2$, are given by

$$\boldsymbol{Y}_k = \begin{bmatrix} y_k(2L) & \dots & y_k(L) \\ y_k(2L+1) & \dots & y_k(L+1) \\ \dots & \dots & \dots \\ y_k(N+L-1) & \dots & y_k(N-1) \end{bmatrix}, \tag{6}$$

$$\boldsymbol{h}_k = [h_k(0), h_k(1) \dots h_k(L)]^T, \tag{7}$$

$$y_k(n) = \boldsymbol{h}_k^H \boldsymbol{s}(n), \tag{8}$$

and

$$\boldsymbol{s}(n) = [s(n), \dots, s(n-L)]^T. \tag{9}$$

A semi-blind estimator can be constructed if (5) is properly extended by the training-based channel outputs $\boldsymbol{y}_k = \boldsymbol{h}_k * \boldsymbol{s}_t$, where $\boldsymbol{s}_t$ is a sequence of M known symbols and $k = 1, 2$. Hence, the extended version of (5) becomes

$$\overbrace{\begin{bmatrix} \boldsymbol{Y}_2 & -\boldsymbol{Y}_1 \\ \boldsymbol{S}_t & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_t \end{bmatrix}}^{\boldsymbol{Y}} \overbrace{\begin{bmatrix} \boldsymbol{h}_1 \\ \boldsymbol{h}_2 \end{bmatrix}}^{\boldsymbol{h}} = \overbrace{\begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \end{bmatrix}}^{\boldsymbol{z}}, \tag{10}$$

where $\boldsymbol{S}_t$ is defined as

$$\boldsymbol{S}_t = \begin{bmatrix} s(L) & \dots & s(0) \\ s(L+1) & \dots & s(1) \\ \dots & \dots & \dots \\ s(M-1) & \dots & s(M-L-1) \end{bmatrix}. \tag{11}$$

Additionally, $\boldsymbol{z}_k = [y_k(L), \dots, y_k(M-1)]^T$, $k = 1, 2$. Finally, it is assumed that $M \ll N$.

In a real scenario, noise is added at the channels' outputs and, in this case, the semi-blind estimator is obtained, using the least-squares criterion, by (12).

$$\hat{\boldsymbol{h}} = (\boldsymbol{Y}^H \boldsymbol{Y})^{-1} \boldsymbol{Y}^H \boldsymbol{z}. \tag{12}$$

TABLE II
PAIRS OF CHANNELS THAT CORRESPOND TO $\boldsymbol{h}_1$, $\boldsymbol{h}_2$ OF EQ. 10

	Scheme I	Scheme II
Protocol I	$\{\boldsymbol{h}_{SD,1}, \boldsymbol{h}_{SD,2}\}$, $\{\boldsymbol{h}_{SD,1}+\boldsymbol{h}_{RD,1}, \boldsymbol{h}_{SD,2}+\boldsymbol{h}_{RD,2}\}$	$\{\boldsymbol{h}_{SD,1}, \boldsymbol{h}_{SD,1}+\boldsymbol{h}_{RD,1}\}$, $\{\boldsymbol{h}_{SD,2}, \boldsymbol{h}_{SD,2}+\boldsymbol{h}_{RD,2}\}$
Protocol II	$\{\boldsymbol{h}_{SD,1}, \boldsymbol{h}_{SD,2}\}$, $\{\boldsymbol{h}_{RD,1}, \boldsymbol{h}_{RD,2}\}$	$\{\boldsymbol{h}_{SD,1}, \boldsymbol{h}_{RD,1}\}$, $\{\boldsymbol{h}_{SD,2}, \boldsymbol{h}_{RD,2}\}$

IV. PROPOSED SCHEMES

The two alternative estimation schemes that are proposed in this section dictate the way that the linear system of (10) is constructed. As we will see later on, this has an impact on the performance of the semi-blind estimator. Let us assume that *Protocol II* is employed. The following description applies also to *Protocol I* in a straightforward way.

In *Scheme I* (Fig. 2, left part), node *D* treats each phase of the transmission separately and, hence, the corresponding channels are viewed as SISO systems. This means that in order to use the estimator of (12), node *D* has to oversample, at each phase, the corresponding signals.

More specifically, in *Phase A*, the signal of (3) is oversampled and the signals $y_{A,k}(n) = \boldsymbol{h}_{SD,k}^H \boldsymbol{s}(n) + w_k(n)$ for $k = 1, 2$ are produced. These two signals follow a multichannel model, as the one described in the previous section, and are used for the construction of (10) and the subsequent estimation of the subchannels $\boldsymbol{h}_{SD,k}$. So, in this case the following holds.

$$\begin{bmatrix} \boldsymbol{Y}_{A,2} & -\boldsymbol{Y}_{A,1} \\ \boldsymbol{S}_t & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_t \end{bmatrix} \begin{bmatrix} \boldsymbol{h}_{SD,1} \\ \boldsymbol{h}_{SD,2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{z}_{A,1} \\ \boldsymbol{z}_{A,2} \end{bmatrix} \tag{13}$$

Similarly, in *Phase B*, node *D* estimates the subchannels $\boldsymbol{h}_{RD,k}$ and the following holds.

$$\begin{bmatrix} \boldsymbol{Y}_{B,2} & -\boldsymbol{Y}_{B,1} \\ \boldsymbol{S}_t & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_t \end{bmatrix} \begin{bmatrix} \boldsymbol{h}_{RD,1} \\ \boldsymbol{h}_{RD,2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{z}_{B,1} \\ \boldsymbol{z}_{B,2} \end{bmatrix} \tag{14}$$

In *Scheme II* (Fig. 2, right part), node *D* waits until both phases are completed and only then attempts the estimation of the channels. In this case, no oversampling is required because the two signals (i.e. (3), (4)) that are used for the estimation of the channels $\boldsymbol{h}_{SD}$, $\boldsymbol{h}_{RD}$, already follow the multichannel model that is required for the implementation of the estimator of Eq. (12). The linear system of equations in this case is

$$\begin{bmatrix} \boldsymbol{Y}_{B} & -\boldsymbol{Y}_{A} \\ \boldsymbol{S}_t & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_t \end{bmatrix} \begin{bmatrix} \boldsymbol{h}_{SD} \\ \boldsymbol{h}_{RD} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{z}_{A} \\ \boldsymbol{z}_{B} \end{bmatrix}. \tag{15}$$

In the case of *Protocol I*, the two schemes are applied in a similar way. The only difference is in the second phase where the channel to be estimated is the $\boldsymbol{h}_{SD} + \boldsymbol{h}_{RD}$.

V. EXPERIMENTAL STUDY

In this section, the experimental evaluation of the two schemes is presented.

A. Preliminaries

The binary phase shift keying modulation is employed. Table II shows the pairs of channels that are estimated for each protocol and scheme. For example, the channel pairs in (13), (14) appear in the (2,1) cell of the table, i.e. for *Scheme I* and *Protocol II*.

In *Scheme I*, we have to estimate two subchannels for each of the channels of the two phases. In order to be fair in the comparison of the two schemes, we, also, estimate, in the case of *Scheme II*, two subchannels for each of the channels of the two phases. This is done by employing oversampling, although this is not actually required. In this case, the second linear system is constructed from the corresponding channel pairs in a straight-forward way.

Hence, for each scheme, the estimator of (12) is used twice and four subchannel estimates are produced. For our convenience, the two estimators are combined into one producing an augmented version. In what follows, the augmented matrix $\boldsymbol{Y}$ is denoted as $\boldsymbol{Y}^{\mathrm{a}}$. For example, the augmented matrix for (13), (14) is

$$\boldsymbol{Y}^{\mathrm{a}} = \begin{bmatrix} \boldsymbol{Y}_{A,2} & -\boldsymbol{Y}_{A,1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Y}_{B,2} & -\boldsymbol{Y}_{B,1} \\ \boldsymbol{S}_t & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_t & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{S}_t & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{S}_t \end{bmatrix}. \tag{16}$$

The other terms of the linear system and the combined semi-blind estimator are derived in a straight-forward way.

Three types of channels have been used in our experiments. In the first type, which is called *random*, the elements of the subchannels $\boldsymbol{h}_{SD,k}$, $\boldsymbol{h}_{RD,k}$ are complex Gaussian random variables with zero mean and variance equal to 2. In the experiments, the length of each subchannel is set to 26.

The second type (so-called *random-rc*) is produced by taking the convolution of a raised cosine pulse $g(.)$ with a random vector whose elements are distributed as above (the length of the vector with symbol spaced elements is set to 26). The pulse has roll-of factor 0.3, extends to 6 symbol intervals and is oversampled by 2. In this case, the length of each subchannel is 32.

Finally, the third one is a *multipath* channel of the form $\sum_j a_j g(t-\tau_j)$, where a_j's are complex Gaussian random variables distributed as above and τ_j's are uniformly distributed on prespecified intervals. More specifically, $\boldsymbol{h}_{SD}$ is assumed to have four paths with delays 0, $U(6,9)$, $U(13,16)$, $U(20,22)$ and $\boldsymbol{h}_{RD}$ three paths with delays $U(1,4)$, $U(10,12)$, and $U(17.5,19)$, where $U(a,b)$ denotes that the corresponding time delay lies uniformly in the interval (a,b).

TABLE III
MINIMUM SINGULAR VALUES OF THE AUGMENTED MATRIX OF (16)

Protocol I	Random	Random-RC	Parametric
Scheme I	2.3577	0.0510	0.5039
Scheme II	1.9027	1.2324	1.7068
Protocol II	**Random**	**Random-RC**	**Parametric**
Scheme I	2.3415	0.0519	0.5187
Scheme II	2.3284	1.6164	2.2507

B. Semi-analytic arguments

In Table III, the minimum singular values of the extended noise-free matrix $\boldsymbol{Y}^{\mathrm{a}}$ are shown for different protocol, scheme and channel combinations. It is known that (e.g., [5], page 72) the minimum singular value of a matrix is the 2-norm distance of that matrix from the set of all rank-deficient matrices. Although $\boldsymbol{Y}^{\mathrm{a}}$ is full rank, it may be close to the set of rank-deficient matrices depending on the channel and the scheme.

The above problem can be explained if we consider the columns of $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$ in (5). The elements $y_1(n)$, $y_2(n)$ of these matrices are highly correlated if the channels $\boldsymbol{h}_1$, $\boldsymbol{h}_2$ are highly correlated and, hence, the corresponding columns are near dependent. In *Scheme I*, $\boldsymbol{h}_1$, $\boldsymbol{h}_2$ are derived by oversampling the channel that is estimated at each phase for each protocol (see Table II). When the channel to be estimated incorporates a raised cosine pulse (or any other smooth pulse), the derived subchannels are correlated because of the pulse and, hence, this has an impact on the condition of the matrix $[\boldsymbol{Y}_2 - \boldsymbol{Y}_1]$ (e.g. this is the case for the random-rc and parametric channels). In *Scheme II*, the correlation of the column elements of the two matrices is smaller because the subchannels are derived from the oversampling of different channels. However, in this scheme, when *Protocol I* is used, there is a correlation because the subchannels to be estimated have common parts, e.g. both subchannels $\boldsymbol{h}_{SD,1}$, $\boldsymbol{h}_{SD,1} + \boldsymbol{h}_{SD,2}$ have the term $\boldsymbol{h}_{SD,1}$. The augmented matrix $\boldsymbol{Y}^{\mathrm{a}}$ has two pairs of such matrices and, if we take into account the previous remarks, $\boldsymbol{Y}^{\mathrm{a}}$ can be, as mentioned earlier, closer or further from the set of rank deficient matrices.

In *Protocol I*, *Scheme II* produces a $\boldsymbol{Y}^{\mathrm{a}}$ of better condition for all channels except the random one as compared to *Scheme I*. This is reasonable because in the random channels, *Scheme II* pairs subchannels which have common parts and, so, they are correlated, thus affecting the condition of $\boldsymbol{Y}^{\mathrm{a}}$. For the other two channel types, the behavior is reversed because, due to raised cosine pulse, the subchannels produced by the oversampling become correlated. This means that, in *Scheme II*, the subchannels tend to be uncorrelated because they are derived from different channels and, hence, $\boldsymbol{Y}^{\mathrm{a}}$ is further from the set of rank deficient matrices as opposed to the one of *Scheme I*.

Similar are the observations for *Protocol II*. However, in the case of the random channels, the condition of $\boldsymbol{Y}^{\mathrm{a}}$ is the same regardless the scheme. This is due to the fact that in *Protocol II*, the channels do not have common parts.

Finally, comparing the two protocols, we see that in the case of *Scheme I*, the conditions of $\boldsymbol{Y}^{\mathrm{a}}$ are similar. However, in the case of *Scheme II*, *Protocol II* is better than *Protocol I* because the involved channel pairs, as already mentioned, do not have common parts.

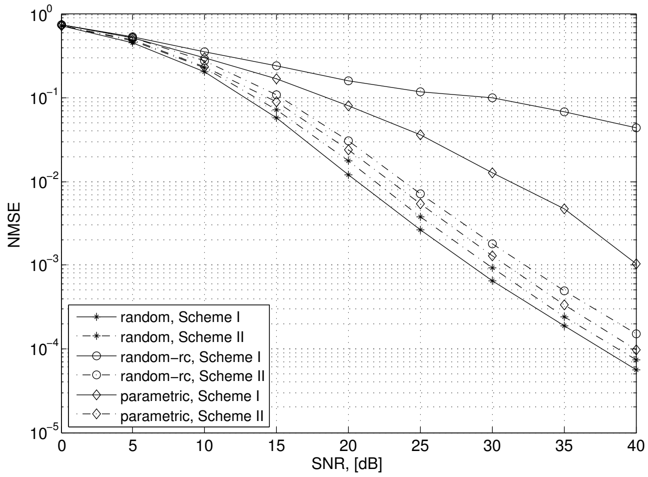

Fig. 3. The NMSE for Protocol I

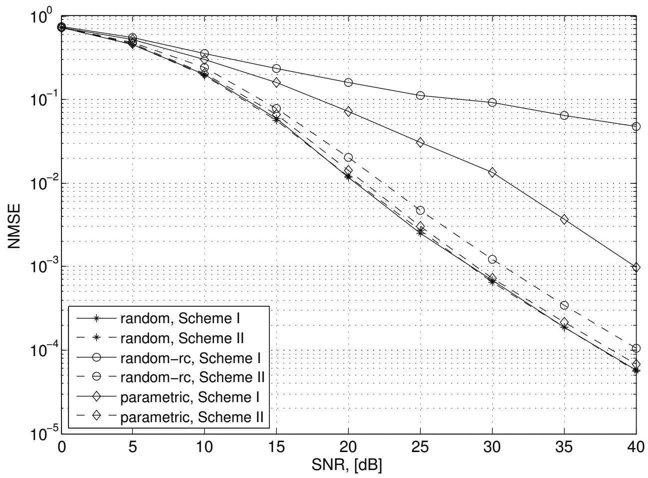

Fig. 4. The NMSE for Protocol II

C. Simulations

The normalized mean squared error $\|\boldsymbol{h} - \hat{\boldsymbol{h}}\|^2/\|\boldsymbol{h}\|^2$ (NMSE) is obtained through Monte-Carlo simulations. Here, $\boldsymbol{h}$ and the corresponding estimation $\hat{\boldsymbol{h}}$ incorporate all four subchannels. In Figs. 3, 4, the NMSE curves versus the signal to noise ratio (SNR) are plotted for *Protocol I* and *II*, respectively. We see in the figures that the ordering (worst to best) of the NMSE curves is the same as the ordering (small to large) of the minimum singular values. Also, it is evident that *Scheme II* outperforms *Scheme I* for the cases of the random-rc and parametric channels which are the types that are most frequently encountered in practice.

D. Impact of errors in the relay

The semi-analytic arguments and the experimental results that were presented in the previous sections are derived under the assumption that in node *R* the decoding process is error-free. However, this assumption may not be true in practice. In the present section, we provide some experiments assuming that errors do occur in the relay.

More specifically, we define a probability of error P_e for the symbol sequence that node *R* detects and retransmits during the second phase. By doing this, we hide the details of the detecting process at the relay and focus only on the performance of the two schemes. As we will see, the impact on the performance comes from the fact that in order the CR criterion in (5) to be valid, the input to the multichannel model (produced either by oversampling or by design) must be exactly the same.

We produce the NMSE curves versus the SNR for five values of P_e, i.e. $P_e = \{0, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}\}$. Here, the experimental results refer only to *Protocol II*. The results for *Protocol I* are similar especially for *Scheme II*. Comments will be made at the appropriate point for *Scheme I*.

In Fig. 5, the NMSE curves for *Scheme I* are presented. As observed, the performance of this scheme is independent of the errors that occur in the relay. This comes from the fact that the subchannels that are estimated in each phase, have always a common input. More specifically, at the first phase, the channels $\boldsymbol{h}_{SD,1}$ and $\boldsymbol{h}_{SD,2}$ are estimated using (13) which is based on the multichannel model $y_{A,k} = \boldsymbol{h}_{SD,k}^H \boldsymbol{s}(n)$, $k = 1, 2$. At the second phase, the corresponding estimation uses (14) which is based on $y_{B,k} = \boldsymbol{h}_{RD,k}^H \boldsymbol{s}_e(n)$, $k = 1, 2$. Thus, at each separate phase, the corresponding subchannels have a common input regardless it is correct (i.e. $\boldsymbol{s}(n)$) or erroneous (i.e. $\boldsymbol{s}_e(n)$), and therefore, the CR criterion may be applied equally well. When *Protocol I* is used, the performance of the scheme is affected because, at the second phase, the channel is not the sum of $\boldsymbol{h}_{SD}$ and $\boldsymbol{h}_{RD}$ since the inputs to them are different. Hence, the multichannel model is only approximately valid which has an impact on the validity of the CR criterion.

In Fig. 6, the NMSE curves for *Scheme II* are presented. Here, the performance is affected by the P_e at the relay. This observation comes from the fact that the channels $\boldsymbol{h}_{SD}$ and $\boldsymbol{h}_{RD}$ are estimated together using (15). In this case, the outputs $y_A = \boldsymbol{h}_{SD}^H \boldsymbol{s}(n)$ and $y_B = \boldsymbol{h}_{RD}^H \boldsymbol{s}_e(n)$ that are used for the construction of (15), do not have a common input and, therefore, the performance is degraded.

In conclusion, when errors occur in the relay, the relative performance of the two schemes depends on the involved channels and the P_e at the relay.

VI. Conclusions

In this paper, two schemes have been described and evaluated for the construction of a semi-blind least-squares estimator for a cooperative system based on the CR criterion. When no errors occur at node *R*, *Scheme II* outperforms *Scheme I* according to the semi-analytic arguments and the experiments presented in sections V-B and V-C, respectively. The main reason for this stems from the fact that when *Scheme II* is used, the matrix $\boldsymbol{Y}$ is better conditioned because the matrices $\boldsymbol{Y}_1$, $\boldsymbol{Y}_2$ have uncorrelated columns with each other. This is not the case with *Scheme I* where the channels under consideration are closely related since they constitute different sampled versions of the same overall channel. However, when errors do occur in the relay, it is not clear which scheme is best and the choice depends on the channel that is involved and on the P_e at the relay.

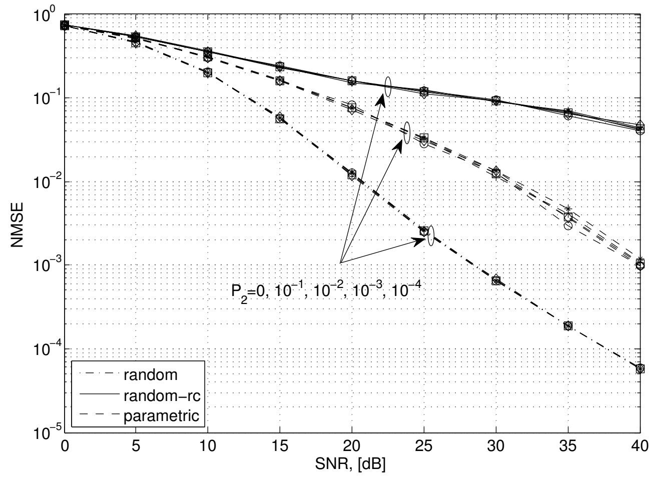

Fig. 5. The NMSE for Scheme I using Protocol II with errors in the relay

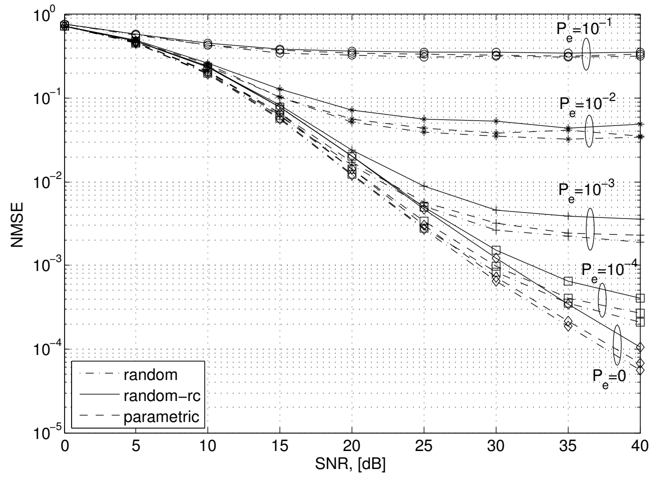

Fig. 6. The NMSE for Scheme II using Protocol II with errors in the relay

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