Channel Estimation Techniques for Half-Duplex Cooperative Communication systems

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Abstract— **In this work we present efficient channel estimation algorithms for wideband detect-and-forward (DF) based relay networks. A generalization of a recently proposed transmission model is utilized and the analysis is completely performed in the frequency domain (FD). It is shown that all channels in the network from any node to the destination/receiver node can be blindly estimated up to a phase ambiguities vector, related to the source to destination channel frequency response. Hence, by utilizing a small number of pilot symbols, phase ambiguities can be effectively resolved. As verified by computer simulations, the proposed methods exhibit high estimation accuracy even for a short training sequence, and outperform direct training-based channel estimation. A performance study of the proposed schemes in high SNR conditions is also presented and verified through computer simulations.**

I. Introduction

Cooperation among nodes in a wireless network provides an effective means of improving spectral and power efficiency, as an alternative to multiple-antenna transmission schemes [1]. In [2], the use of conventional orthogonal space-time block coding (STBC) in a distributed fashion has been proposed for practical implementation of user cooperation schemes. Moreover, there have been recently several sporadic results reported on broadband cooperative transmission techniques for frequency selective channels. In [3], distributed STBC for regenerative relay networks is studied following a frequency domain (FD) approach. Performance analysis of a relay-assisted uplink OFDM-STBC scheme has been presented in [4]. Three broadband cooperative transmission methods for distributed STBC have been also proposed and analysed in [5]. A common assumption in all these works is that channel state information (CSI) is known at the receiver (i.e., destination node).

To the best of our knowledge, very few results have been published on channel estimation for broadband cooperative systems. Thus, acquisition of the CSI between a source node (S), the cooperative terminals (R_i) and a destination node (D) becomes a challenging and imperative task. In this work we propose efficient semi-blind channel estimation techniques for the general case of cooperative broadcast channels, where N regenerative relays cooperate with a single source. We

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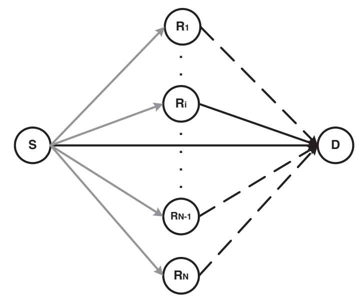

Fig. 1. Relay-assisted communication model

consider our techniques as semi-blind in the sense that they rely on a very short training sequence lying in only one of the links whereas all the others are estimated blindly. The transmission protocol that has been adopted is a generalization for multiple relays of the so-called protocol I described in [6]. The new methods are implemented in the FD, to exploit the diagonal structure of the channel matrices resulting due to block, cyclic prefixed (CP) transmission. First, it is shown that all required channels can be blindly estimated in the FD, up to multiple phase ambiguities, from the Cholesky factor of the received signal autocorrelation matrix. More importantly, these ambiguities are related exclusively to the $S \rightarrow D$ channel frequency response. To resolve phase ambiguities a training sequence can be employed in the $S \rightarrow D$ link only. As also verified by simulations, a very short training sequence results in accurate estimation of all channels in the network outperfoming a globally training based approach. Furthermore, experiments have shown that the presented techniques are robust to detection errors that may occur at the relay. A theoretical performance analysis of the proposed schemes in high SNR conditions is also presented and verified by extensive computer simulations.

Notation: With $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ we denote conjugate, transpose, and Hermitian transpose operations, respectively. We use $[\cdot]_{k,l}$ to denote the $(k,l)-th$ element of a matrix. Operator $E[\cdot]$ denotes expectation. The identity matrix and the all-zero matrix of size $M \times M$ are denoted by $\mathbf{I}_M$ and $\mathbf{0}_M$ respectively. $\mathbf{F}$ represents the $M \times M$ FFT matrix whose (k,l) element is given by $\mathbf{F}(l,k) = 1/\sqrt{M}e^{-j2\pi(l-1)(k-1)/M}$ with $1 \leq l,k \leq M$. Finally, vectors and matrices are denoted by bold lower case and bold upper case letters, respectively.

II. System Model & Problem Formulation

We consider the relay based communication scenario (Fig. 1) with N regenerative relays R_i, $i = 1, 2, \ldots, N$, operating in a detect and forward mode. The channels $S \rightarrow D$ and $R_i \rightarrow D$, $i = 1, \ldots, N$, are assumed to be static, frequency selective and are given by $\mathbf{h}_S = [h_S(1), \ldots, h_S(L_S)]^T$, and $\mathbf{h}_{R_i} = [h_{R_i}(1), \ldots, h_{R_i}(L_i)]^T$ respectively. The aim of this work is the development and performance analysis of efficient frequency domain techniques for estimating the frequency responses of the $S \rightarrow D$ and $R_i \rightarrow D$ links.

The transmission protocol that has been adopted is a generalization of the so-called protocol I described in [6]. A similar generalization for the amplify & forward (A&F) schenario has been recently proposed in [7]. First, we define a super-frame as a concatenation of N consecutive cooperation frames. Each frame consists of two signal intervals, the odd and the even. Within each frame, the source transmits in both the odd and the even intervals, while one relay R_i, $i = 1, \ldots, N$ listens to the source during the odd interval and then detects and transmits the encoded received signal to the destination during the even interval. Note that at each frame i only one relay R_i is involved, while the other relays either remain silent or possibly take part in another communication task. Without loss of generality, we have assumed that the relays are selected sequentially during the transmission of one superframe. The transmission in each interval, is done in blocks of M symbols, where $M > max\,(L_S, L_i)$, $\forall i = 1, \ldots, N$. To eliminate interblock interference each block of length M is appended with a length-l cyclic prefix which is discarded at the destination.

The signal received at the destination during the odd and even signal intervals can be expressed as follows:

$$\begin{array}{ll} \mathbf{y}_{2i-1} = \mathbf{H}_S \mathbf{x}_{2i-1} + \mathbf{w}_{2i-1}, & S \rightarrow R_i, D \\ \mathbf{y}_{2i} = \mathbf{H}_S \mathbf{x}_{2i} + \mathbf{H}_{R_i} \widetilde{\mathbf{x}}_{2i-1} + \mathbf{w}_{2i}, & S, R_i \rightarrow D \end{array} \quad (1)$$

where the $M \times 1$ vectors $\mathbf{x}_{2i-1}, \mathbf{x}_{2i}$, $i = 1, \ldots, N$ represent the transmitted blocks from the source and $\widetilde{\mathbf{x}}_{2i-1}$ is the block of decisions taken at the relay R_i. The $M \times 1$ vectors $\mathbf{y}_j$, and $\mathbf{w}_j$ $j = 1, , \ldots, 2N$ correspond to the received blocks at the destination and additive white Gaussian noise, respectively. Finally, $\mathbf{H}_S$, $\mathbf{H}_{R_i}$ are $M \times M$ circulant matrices with entries $[\mathbf{H}_S]_{k,l} = \mathbf{h}_S\left((k-l)\, modM\right)$ and $[\mathbf{H}_{R_i}]_{k,l} = \mathbf{h}_{R_i}\left((k-l)\, modM\right)$. Due to their circulant structure, they can be decomposed by using the fast fourier transform (FFT) matrix operator $\mathbf{F}$ as:

$$\mathbf{H}_S = \mathbf{F}^H \mathbf{\Lambda}_S \mathbf{F} \quad (2)$$

$$\mathbf{H}_{R_i} = \mathbf{F}^H \mathbf{\Lambda}_{R_i} \mathbf{F} \quad (3)$$

where $\mathbf{\Lambda}_S$ and $\mathbf{\Lambda}_{R_i}$ are diagonal matrices that contain the FFT coefficients of the zero padded $S \rightarrow D$, $R_i \rightarrow D$ channel impulse responses (CIRs).

III. Channel Identification Techniques

Initially, we make the assumption that the decisions taken at the relays are correct, i.e., $\widetilde{\mathbf{x}}_{2i-1} = \mathbf{x}_{2i-1}$, for $i = 1, \ldots, N$. Although this seems to be unrealistic, it greatly simplifies derivations. Furthermore, as it will be verified via simulations, detection errors that may occur at the relays even with a sufficiently high probability, affect slightly the performance of the proposed techniques. Thus, under the aforementioned hypothesis, the input-output relation of the system during two successive intervals is transformed in matrix form in the FD, by applying the FFT as follows:

$$\begin{bmatrix} \boldsymbol{\mathcal{Y}}_{2i-1} \\ \boldsymbol{\mathcal{Y}}_{2i} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{\Lambda}_S & \mathbf{0}_M \\ \mathbf{\Lambda}_{R_i} & \mathbf{\Lambda}_S \end{bmatrix}}_{\mathbf{\Lambda}_i} \begin{bmatrix} \boldsymbol{\mathcal{X}}_{2i-1} \\ \boldsymbol{\mathcal{X}}_{2i} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\mathcal{W}}_{2i-1} \\ \boldsymbol{\mathcal{W}}_{2i} \end{bmatrix}, \quad (4)$$

where $\boldsymbol{\mathcal{Y}}_j = \mathbf{F}\mathbf{y}_j$, $\boldsymbol{\mathcal{X}}_j = \mathbf{F}\mathbf{x}_j$, $\boldsymbol{\mathcal{W}}_j = \mathbf{F}\mathbf{w}_j$, $j = 1, \ldots, 2N$.

Assuming complex, zero-mean, and uncorrelated input and noise signals of variances σ_x^2 and σ_n^2 respectively, the autocorrelation matrix of the FD received vectors is expressed as

$$\mathbf{\Phi}_i = E\left[\begin{bmatrix} \boldsymbol{\mathcal{Y}}_{2i-1} \\ \boldsymbol{\mathcal{Y}}_{2i} \end{bmatrix} \begin{bmatrix} \boldsymbol{\mathcal{Y}}_{2i-1} \\ \boldsymbol{\mathcal{Y}}_{2i} \end{bmatrix}^H\right] = \left[\begin{array}{c|c} \boldsymbol{\mathcal{R}}_{2i-1} & \boldsymbol{\mathcal{R}}_{2i-1,2i} \\ \hline \boldsymbol{\mathcal{R}}_{2i-1,2i}^H & \boldsymbol{\mathcal{R}}_{2i} \end{array}\right] =$$

$$\left[\begin{array}{c|c} \sigma_x^2 \mathbf{\Lambda}_S \mathbf{\Lambda}_S^H + \sigma_n^2 \mathbf{I}_M & \sigma_x^2 \mathbf{\Lambda}_S \mathbf{\Lambda}_{R_i}^H \\ \hline \sigma_x^2 \mathbf{\Lambda}_S^H \mathbf{\Lambda}_{R_i} & \sigma_x^2 \mathbf{\Lambda}_S \mathbf{\Lambda}_S^H + \sigma_x^2 \mathbf{\Lambda}_{R_i} \mathbf{\Lambda}_{R_i}^H + \sigma_n^2 \mathbf{I}_M \end{array}\right] \quad (5)$$

Cholesky factorization of matrix $\mathbf{\Phi}_i$ can be easily obtained using Schur complements [8] as follows

$$\mathbf{\Phi}_i = \underbrace{\left[\begin{array}{c|c} \boldsymbol{\mathcal{R}}_{2i-1}^{1/2} & \mathbf{0}_M \\ \hline \boldsymbol{\mathcal{R}}_{2i-1}^{-1/2} \boldsymbol{\mathcal{R}}_{2i-1,2i}^H & \mathbf{\Delta}_R^{1/2} \end{array}\right]}_{\boldsymbol{\mathcal{G}}_i} \underbrace{\left[\begin{array}{c|c} \boldsymbol{\mathcal{R}}_{2i-1}^{1/2} & \boldsymbol{\mathcal{R}}_{2i-1}^{-1/2} \boldsymbol{\mathcal{R}}_{2i-1,2i} \\ \hline \mathbf{0}_M & \mathbf{\Delta}_R^{1/2} \end{array}\right]}_{\boldsymbol{\mathcal{G}}_i^H} \quad (6)$$

where

$$\mathbf{\Delta}_R = \boldsymbol{\mathcal{R}}_{2i} - \boldsymbol{\mathcal{R}}_{2i-1,2i}^H \boldsymbol{\mathcal{R}}_{2i-1}^{-1} \boldsymbol{\mathcal{R}}_{2i-1,2i} \quad (7)$$

is the Schur complement and $\boldsymbol{\mathcal{G}}_i$ is the Cholesky factor of $\mathbf{\Phi}_i$. By inspecting (5), it can be shown that in the absence of noise, i.e., $\sigma_n^2 \rightarrow 0$, (7) can be written as

$$\mathbf{\Delta}_R = \sigma_x^2 \mathbf{\Lambda}_S \mathbf{\Lambda}_S^H. \quad (8)$$

and

$$\mathbf{\Delta}_R^{1/2} = \boldsymbol{\mathcal{R}}_{2i-1}^{1/2} = \sigma_x \left|\mathbf{\Lambda}_S\right| \quad (9)$$

where $|\mathbf{\Lambda}_S|$ is a $M \times M$ diagonal matrix that contains the amplitudes $\left|[\mathbf{\Lambda}_S]_{m,m}\right|$, $m = 1, \ldots, M$ of the $S \rightarrow D$ channel frequency response. The Cholesky factor of $\mathbf{\Phi}_i$ would then be equal to

$$\boldsymbol{\mathcal{G}}_i = \sigma_x \left[\begin{array}{c|c} |\mathbf{\Lambda}_S| & \mathbf{0}_M \\ \hline (\angle \mathbf{\Lambda}_S^H) \mathbf{\Lambda}_{R_i} & |\mathbf{\Lambda}_S| \end{array}\right] \quad (10)$$

where $\angle \mathbf{\Lambda}_S^H$ is a $M \times M$ diagonal matrix that contains the conjugate phases $\left[\mathbf{\Lambda}_S^H\right]_{m,m} / \left|[\mathbf{\Lambda}_S]_{m,m}\right|$, $m = 1, \ldots, M$ of the $S \rightarrow D$ frequency bins. Alternatively (10) can be rewritten as

$$\boldsymbol{\mathcal{G}}_i = \sigma_x \boldsymbol{\mathcal{Q}} \mathbf{\Lambda}_i \quad (11)$$

where

$$\mathcal{Q} = \begin{bmatrix} \angle\mathbf{\Lambda}_S^H & \mathbf{0}_M \\ \mathbf{0}_M & \angle\mathbf{\Lambda}_S^H \end{bmatrix} \quad (12)$$

According to (11) and (12) one has to resolve M phase ambiguities in order to determine the frequency response of the $S \rightarrow D$ and $R_i \rightarrow D$ channels, which are contained in $\mathbf{\Lambda}_i$. More specifically, from (4), (6) and (11), these channels are expressed in terms of the elements of the Cholesky factor as follows

$$\hat{\mathbf{\Lambda}}_S = \frac{\angle\mathbf{\Lambda}_S}{\sigma_x}\mathcal{R}_{2i-1}^{1/2}, \quad (13)$$

$$\hat{\mathbf{\Lambda}}_{R_i} = \frac{\angle\mathbf{\Lambda}_S}{\sigma_x}\mathcal{R}_{2i-1}^{-1/2}\mathcal{R}_{2i-1,2i}^H \approx \frac{\mathbf{\Lambda}_S^{-H}}{\sigma_x^2}\mathcal{R}_{2i-1,2i}^H, \quad (14)$$

where $i = 1, \ldots, N$. Using a second order statistics based blind technique [9] for estimating $\mathbf{h}_S$ all $R_i \rightarrow D$ frequency responses for $i = 1, \ldots, N$ can be calculated up to a single scalar ambiguity. Alternatively, the phase ambiguities may be resolved by making use of pilot symbols in the $S \rightarrow D$ link only, as explained in the next subsection.

Resolving the Phase Ambiguities

As mentioned above, in order to fully identify the unknown frequency responses of the involved channels, the phases of the $S \rightarrow D$ frequency response need to be estimated. This can be achieved by transmitting training blocks at the odd intervals. In such case, we can estimate the $S \rightarrow D$ frequency bins as

$$\mathbf{\Lambda}_S = \mathcal{R}_{xx}^{-1}\mathcal{R}_{xy} \quad (15)$$

where $\mathcal{R}_{xx} = E\left[\mathcal{X}_{2i-1}\mathcal{X}_{2i-1}^H\right]$, $\mathcal{R}_{xy} = E\left[\mathcal{Y}_{2i-1}\mathcal{X}_{2i-1}^H\right]$.

Since matrix $\mathcal{R}_{xx}$ contains real elements, the phases may be computed directly from the phases of $\mathcal{R}_{xy}$, i.e.,

$$\angle\mathbf{\Lambda}_S = \angle\mathcal{R}_{xy} \quad (16)$$

Thus, two alternative schemes for estimating the $S \rightarrow D$ frequency response can be employed. One can either estimate both the phases and the amplitudes of the $S \rightarrow D$ frequency response using (15) or by using (16) for the phases and then (13). The $R_i \rightarrow D, i = 1, \ldots, N$ frequency responses can then be estimated using (14). However, as it will be shown through simulations, the second approach, where only the phases are estimated via training, succeeds in estimating all the $(N+1)M$ frequency bins even when only one training block is used, while the first approach fails.

IV. Performance Study

In this section the two alternative schemes for estimating the $S \rightarrow D$ frequency response as well as their influence on the estimation of the $R_i \rightarrow D$ frequency responses will be investigated. The performance of the proposed schemes will be studied in high SNR conditions and in terms of the attained normalized mean square error (NMSE) between the actual and estimated frequency responses.

Initially, we will evaluate the variance of estimating the $S \rightarrow D$ frequency bins. Let us consider that $2NK$ blocks of received data and L blocks of training (transmitted during the odd intervals) have been received at the destination. Then each diagonal element of $\mathbf{\Lambda}_S$ can be estimated by the L training blocks according to the following relation

$$\left[\tilde{\mathbf{\Lambda}}_S\right]_{m,m} = \left(\left[\hat{\mathcal{R}}_{xx}\right]_{m,m} + \delta\right)^{-1}\left[\hat{\mathcal{R}}_{xy}\right]_{m,m} \quad (17)$$

where $\hat{\mathcal{R}}_{xx} = \frac{1}{L}\sum_{l=1}^{L}\mathcal{X}_{2i-1}(l)\mathcal{X}_{2i-1}^H(l)$, $\hat{\mathcal{R}}_{xy} = \frac{1}{L}\sum_{l=1}^{L}\mathcal{X}_{2i-1}(l)\mathcal{Y}_{2i-1}^H(l)$ and δ is a small constant. In that case, it has been shown in [10] that the variance of the estimator would be

$$var\left(\left[\tilde{\mathbf{\Lambda}}_S\right]_{m,m}\right) = \frac{\sigma_n^2}{\sigma_x^2 L} + \delta^2. \quad (18)$$

Thus, the NMSE between the actual and the estimated frequency response would be given by

$$E\left[\left\|\boldsymbol{\lambda}_S - \tilde{\boldsymbol{\lambda}}_S\right\|^2 / \left\|\boldsymbol{\lambda}_S\right\|^2\right] = \frac{\sigma_n^2}{\sigma_x^2 L} + \delta^2 \quad (19)$$

where $\boldsymbol{\lambda}_S$ and $\tilde{\boldsymbol{\lambda}}_S$ are $M \times 1$ vectors containing the diagonal elements of $\mathbf{\Lambda}_S$ and $\tilde{\mathbf{\Lambda}}_S$ respectively.

It has been mentioned that the $S \rightarrow D$ frequency bins can be alternatively computed by first estimating only the phases from the training blocks as

$$\left[\angle\hat{\mathbf{\Lambda}}_S\right]_{m,m} = \left[\hat{\mathcal{R}}_{xy}\right]_{m,m} / \left|\left[\hat{\mathcal{R}}_{xy}\right]_{m,m}\right| \quad (20)$$

and subsequently the amplitudes using (13). However, we can avoid using (13) which suffers from possible numerical inaccuracies of Cholesky factorization and instead estimate the required amplitudes based on the autocorrelation matrix of the blocks received at the odd intervals, as follows

$$\left|\left[\hat{\mathbf{\Lambda}}_S\right]_{m,m}\right| = \sqrt{\frac{1}{\sigma_x^2 N}\sum_{i=1}^{N}\left[\hat{\mathcal{R}}_{2i-1}(K)\right]_{m,m}}, \ m = 1, \ldots, M \quad (21)$$

where $\hat{\mathcal{R}}_{2i-1}(K)$ is estimated from K received blocks as

$$\hat{\mathcal{R}}_{2i-1}(K) = \frac{1}{K}\sum_{k=1}^{K}\mathcal{Y}_{2i-1}(k)\mathcal{Y}_{2i-1}^H(k) \quad (22)$$

From the central limit theorem, $\mathcal{X}_{2i-1}$, $\mathcal{X}_{2i}$ can be considered as complex normal circular symmetric random vectors, for sufficiently large block length M. Then the mean value of the autocorrelation estimator is $E[\hat{\mathcal{R}}_{2i-1}(K)] = \mathcal{R}_{2i-1}$, while its variance can be computed by following standard arguments [10] as

$$var\left(\hat{\mathcal{R}}_{2i-1}(K)\right) \approx \frac{\sigma_x^4}{K}\left(\mathbf{\Lambda}_S\mathbf{\Lambda}_S^H\right)^2 + \frac{\sigma_n^4}{K}\mathbf{I}_M \quad (23)$$

We may proceed to the computation of the variance of the estimators in Eqs. (21) by employing the so-called delta method [11]. This method employs second-order Taylor expansions to approximate the variance of a function of one or more RVs.

Let x be a RV with $E[x] = \mu_x$ and $var(x) = \sigma_x^2$. Then the approximate variance of a function of one variable is given by

$$var(f(x)) \approx \left(\frac{\partial}{\partial x} f(x) \Big|_{\mu_x} \right)^2 \sigma_x^2 \qquad (24)$$

Thus, it can be easily shown that the variance of the estimator in (21) is approximated by

$$var\left(\left| \left[\hat{\mathbf{\Lambda}}_S \right]_{m,m} \right| \right) \approx \frac{1}{4NK\sigma_x^2} \frac{\left| \left[\hat{\mathbf{\Lambda}}_S \right]_{m,m} \right|^4 \sigma_x^4 + \sigma_n^4}{\left| \left[\hat{\mathbf{\Lambda}}_S \right]_{m,m} \right|^2 \sigma_x^2 + \sigma_n^2} \quad (25)$$

Based on a generalization of the delta method for functions of two random variables it can be shown that, in high SNR conditions, the variance of estimating the $S \rightarrow D$ frequency response is dominated by the variance of estimating the amplitudes. More specifically, when we estimate the phases of the $S \rightarrow D$ frequency response from (16) and the amplitudes from (21), then the NMSE between the actual and estimated responses exhibits a floor as the SNR increases. This floor depends on the variance of the estimator given in (25) and is a function of the number of relays and the number of blocks that have been used for the estimation of the autocorrelation matrix, i.e.,

$$\lim_{\sigma_n^2 \rightarrow 0} E\left[\left\| \boldsymbol{\lambda}_S - \hat{\boldsymbol{\lambda}}_S \right\|^2 / \left\| \boldsymbol{\lambda}_S \right\|^2 \right] \approx \frac{1}{4NK}. \qquad (26)$$

Having estimated the amplitudes of the $S \rightarrow D$ frequency response we can proceed to the estimation of all the $R_i \rightarrow D$ frequency responses, by using (14), which can be written alternatively as

$$\left[\hat{\mathbf{\Lambda}}_{R_i} \right]_{m,m} = \frac{\left[\hat{\boldsymbol{\mathcal{R}}}_{2i-1,2i}^H(K) \right]_{m,m}}{\sigma_x^2 \left[\hat{\mathbf{\Lambda}}_S^H \right]_{m,m}} \qquad (27)$$

where $\left[\hat{\mathbf{\Lambda}}_S \right]_{m,m}$ is the estimated m-th frequency bin of the $S \rightarrow D$ frequency response and matrix $\boldsymbol{\mathcal{R}}_{2i-1,2i}(K)$ can be estimated from K blocks received at the odd and K blocks received at the even intervals as follows

$$\hat{\boldsymbol{\mathcal{R}}}_{2i-1,2i}(K) = \frac{1}{K} \sum_{k=1}^{K} \boldsymbol{\mathcal{Y}}_{2i}(k) \boldsymbol{\mathcal{Y}}_{2i-1}^H(k). \qquad (28)$$

The mean value of the above estimator is $E[\hat{\boldsymbol{\mathcal{R}}}_{2i-1}(K)] = \boldsymbol{\mathcal{R}}_{2i-1}$ and its variance can be calculated as

$$var\left(\left[\hat{\boldsymbol{\mathcal{R}}}_{2i-1,2i}(K) \right]_{m,m} \right) \approx \frac{\sigma_x^4}{K} \left| \left[\mathbf{\Lambda}_S \mathbf{\Lambda}_{R_i}^H \right]_{m,m} \right|^2 \qquad (29)$$

By employing the delta method for estimating the variance of a function of two random variables, it can be shown that the variance of the estimator in (27) seems to be dominated by the variance of the estimator in (29). Thus, independently of the way that the $S \rightarrow D$ frequency bins are estimated, the variance of the estimator of the $R_i \rightarrow D$ frequency bins and the corresponding NMSE exhibits a floor as $SNR \rightarrow \infty$ which depends only on the number of blocks that have been used for the estimation of $\hat{\boldsymbol{\mathcal{R}}}_{2i-1,2i}(K)$. This floor can be approximated by

$$\lim_{\sigma_n^2 \rightarrow 0} E\left[\left\| \boldsymbol{\lambda}_{R_i} - \hat{\boldsymbol{\lambda}}_{R_i} \right\|^2 / \left\| \boldsymbol{\lambda}_{R_i} \right\|^2 \right] \approx \frac{2}{K} \qquad (30)$$

where $\boldsymbol{\lambda}_{R_i}$ and $\hat{\boldsymbol{\lambda}}_{R_i}$ are $M \times 1$ vectors containing the diagonal elements of $\mathbf{\Lambda}_{R_i}$ and $\hat{\mathbf{\Lambda}}_{R_i}$ respectively. The above expressions have been also verified through simulations, as it will be shown in the section that follows.

V. Simulation Results

The performance of the new techniques was evaluated through computer simulations. We assume Rayleigh fading channels and QPSK modulation. We consider 2 relays cooperating with the source according to the protocol presented in Section II. All the $S \rightarrow D$, $R_i \rightarrow D$, $(i = 1, 2)$ links are modeled as frequency selective channels with memory lengths $L_S = L_1 = L_2 = 6$. The power profile has been considered to be uniform. The transmission is done in blocks of $M = 32$ QPSK symbols. The channels remain static during the transmission of K superframes.

Initially, to study the effect of the training length, a configuration as the one described above operating at different SNRs was simulated. The SNR was defined as the expected SNR per bit (over the ensemble of channel realizations) at the destination. Three different schemes were tested. In the first scheme (Algorithm 1), the frequency response of the $S \rightarrow D$ channel is estimated through training according to (17) and then used for the estimation of the $R_i \rightarrow D$ channels from (27). In the second scheme (Algorithm 2), only the phases of the $S \rightarrow D$ channel frequency response are computed through training from (20), while the corresponding amplitudes are obtained blindly from the output autocorrelation matrix using (21). The $R_i \rightarrow D$ frequency response is then estimated by (27). For both algorithms 1 and 2, training symbols are transmitted during the odd time intervals only. The third scheme is a training-based (TB) algorithm where all the channels are estimated from training blocks sent not only at the odd but also at the even intervals. The $S \rightarrow D$ frequency response is estimated from the symbols transmitted during the odd intervals, as in Algorithm 1. This estimation along with the training blocks transmitted at the even intervals are used for computing the $R_i \rightarrow D$ frequency responses. In Fig. 2 the NMSE averaged over 1000 independent runs, is plotted. The superior performance of Algorithm 2 is obvious from the figure. Furthermore, Algorithm 2 identifies successfully all the channels even when only one block is used for training, while the other algorithms fail.

The accuracy of the derived theoretical expressions presented in section IV was also tested. In Fig. 3 the NMSE between the actual and estimated $S \rightarrow D$ frequency responses along with the theoretical expressions given in (19), (26) are plotted. Finally, the NMSE between the actual and estimated $R_i \rightarrow D$ frequency responses averaged over all relays, along with the theoretical expression given in (30) are plotted in Fig.

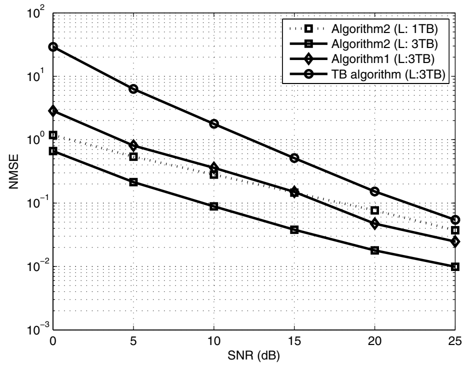

Fig. 2. NMSE of the estimated frequency responses.

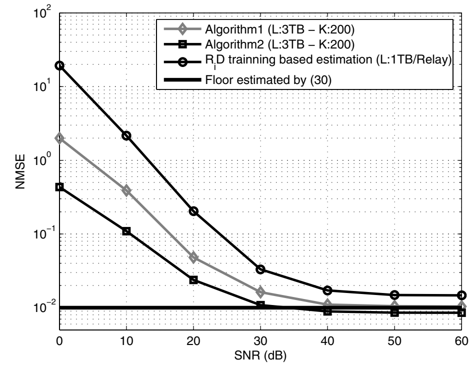

Fig. 4. NMSE of the $R_i \rightarrow D$ frequency responses.

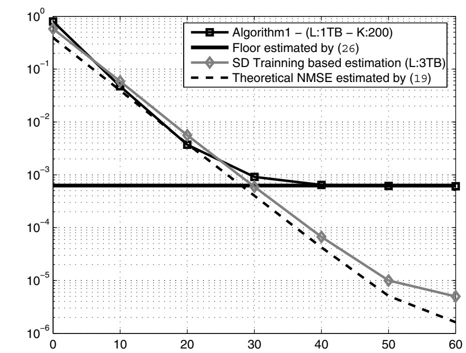

Fig. 3. NMSE of the $S \rightarrow D$ frequency response.

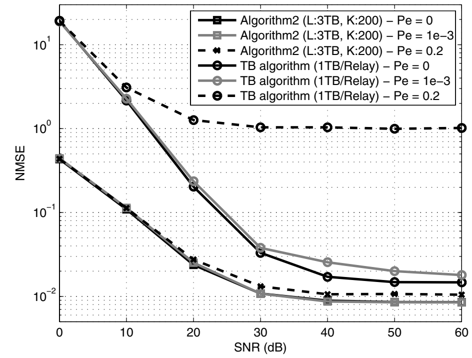

Fig. 5. NMSE of the $R_i \rightarrow D$ frequency responses. Each symbol is correctly detected at any relay with a probability $1 - P_e$.

4. It is clear from the figures that simulation results completely verify the presented theoretical analysis.

Finally, we investigated the effect of detection errors at the relays, on the performance of the proposed techniques. We assumed that each symbol is correctly detected at any relay with a probability $1 - P_e$. Furthermore, it was assumed that errors occur only between adjacent symbols. Thus, in a case of a wrong decision the relay transmits with probability $1/2$ any of the two adjacent to the correct one QPSK symbols. In Fig. 5 the NMSE between the actual and estimated $R_i \rightarrow D$ frequency responses, averaged over all relays, is plotted for Algorithm 2 and the TB algorithm, and for two different detection error probabilities P_e at the relays. The NMSE curve for the error free case is also presented for comparison purposes. It can be observed that contrary to the TB algorithm the performance of Algorithm 2 is only slightly affected from possible wrong decisions that are taken at the relays. The performance of Algorithm 1 is also slightly affected by detection errors (abeit not shown in the figure). Theoretical verification of the immunity of the proposed algorithms to detection errors is currently under investigation.

VI. Conclusion

Efficient channel estimation techniques for wideband cooperative systems with multiple regenerative relays have been derived. It has been shown that with the use of a few pilot symbols in the $S \rightarrow D$ link we can successfully identify all the CIRs between $S \rightarrow D$ and $R_i \rightarrow D$, by exploiting the special structure of the autocorrelation matrix. The proposed methods have been analyzed theoretically and their excellent performance even when compared with direct training based methods, has been verified via extensive simulations.

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