Channel Estimation Techniques in Amplify and Forward Relay Networks

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***Abstract*— In this work we present efficient channel estimation algorithms for wideband amplify-and-forward (AF) based relay networks which utilize a recently proposed transmission model. It is shown that all channels in the network from the source through the relays to the destination/receiver node can be blindly estimated up to a phase ambiguities vector which contains the phases of the direct source to destination channel frequency response. Hence, by employing a small number of pilot symbols, phase ambiguities can be effectively resolved. As verified by computer simulations, the proposed methods exhibit high estimation accuracy even for a short training sequence, and outperform direct training-based channel estimation. A performance study of the proposed schemes in high SNR conditions is also presented and verified through computer simulations.**

I. Introduction

Cooperation among nodes in a wireless network provides an effective means of improving spectral and power efficiency, as an alternative to multiple-antenna transmission schemes [1]. In [2], the use of conventional orthogonal space-time block coding (STBC) in a distributed fashion has been proposed for practical implementation of user cooperation schemes. Moreover, there have been recently several sporadic results reported on broadband cooperative transmission techniques for frequency selective channels. In [3], distributed STBC for regenerative relay networks is studied following a frequency domain (FD) approach. Performance analysis of a relay-assisted uplink OFDM-STBC scheme has been presented in [4]. Three broadband cooperative transmission methods for distributed STBC have been also proposed and analysed in [5]. A common assumption in all these works is that channel state information (CSI) is known at the receiver (i.e., at the destination node for amplify and forward case (AF) transmission or at both the relay and the destination in decode and forward (DF) transmission).

To the best of our knowledge, very few results have been published on channel estimation for broadband relay networks. Thus, acquisition of the CSI between the different nodes, i.e., the source (S), the cooperative terminals (R_i) and the destination node (D) becomes a challenging and imperative task. In the DF case each channel can be estimated individually, either at the relay or at the destination. In AF based transmission, the direct channel from the source to destination as well as the overall channels from the source through the relays to the destination, need to be estimated.

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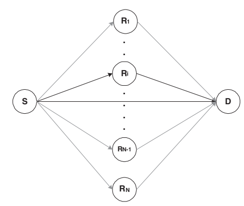

Fig. 1. Relay-assisted communication model

In this work we propose efficient channel estimation techniques for the general case of AF relay networks with N relays. Our technique might be considered as a semi-blind one in the sense that it relies on a very short training sequence lying in only one of the links and all the other links are estimated blindly. The transmission protocol that has been adopted was originally proposed in [6] and is a generalization of the so-called protocol I described in [7]. The new methods are implemented in the frequency domain, to exploit the diagonal structure of the channel matrices resulting due to block, cyclic prefixed (CP) transmission. First, it is shown that all required channels can be blindly estimated in the FD, up to multiple phase ambiguities, from the Cholesky factor of the received signal autocorrelation matrix. More importantly, these ambiguities are related exclusively to the $S \rightarrow D$ channel frequency response. These phase ambiguities may be resolved by employing a training sequence in the $S \rightarrow D$ link only. As also verified by simulations, a very short training sequence results in accurate estimation of all channels in the network outperforming a globally training based approach. A theoretical performance analysis of the proposed schemes in high SNR conditions is also presented and verified by extensive computer simulations.

II. System Model & Problem Formulation

We consider the relay based communication scenario (Fig. 1) with N relays R_i, $i = 1, 2, \ldots, N$, operating in an amplify and forward mode. The channels $S \rightarrow D$, $S \rightarrow R_i$ and $R_i \rightarrow D$, $i = 1, \ldots, N$, are assumed to be static, frequency selective and are given by $\mathbf{h}_{SD} = [h_{SD}(1), \ldots, h_{SD}(L_{SD})]$, $\mathbf{h}_{SR_i} = [h_{SR_i}(1), \ldots, h_{SR_i}(L_{SR_i})]$ and $\mathbf{h}_{R_iD} = [h_{R_iD}(1), \ldots, h_{R_iD}(L_{R_iD})]$ respectively. In this paper, we

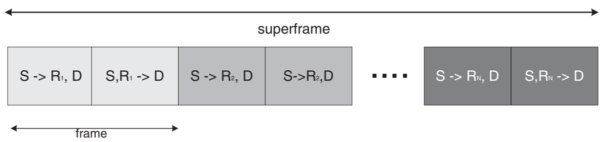

Fig. 2. Superframe structure

propose and analyze efficient frequency domain techniques for estimating both the $S \rightarrow D$ and the overall $S \rightarrow R_i \rightarrow D$ channels. As stated in [8], the receiver needs the overall $S \rightarrow R_i \rightarrow D$ channel and there is nothing to be gained by estimating $S \rightarrow R_i$ and $R_i \rightarrow D$ channels separately.

The transmission protocol that has been adopted was proposed in [6] and it constitutes a generalization of the so-called protocol I described in [7]. First, we define a super-frame as a concatenation of N consecutive cooperation frames. A schematic diagram of the structure of a superframe is given in Fig. 2. Each frame consists of two signal intervals, the odd and the even. Within each frame, the source transmits in both the odd and the even intervals, while one relay R_i, $i = 1, \ldots, N$ listens to the source during the odd interval and transmits the received signal to the destination during the even interval. Note that at each frame i only one relay R_i is involved, while the other relays either remain silent or possibly take part in another communication task. As mentioned in [6], there is nothing to be gained in terms of diversity-multiplexing trade-off, by having more than one relays transmitting the same symbol simultaneously. Without loss of generality, we have assumed that the relays are selected sequentially during the transmission of one superframe. The transmission in each interval, is done in blocks of M symbols, where $M > max\,(L_{SD}, L_{SR_i} + L_{R_iD} - 1)$, $\forall i = 1, \ldots, N$. To eliminate interblock interference each block of length M is appended with a length-l cyclic prefix which is discarded at the destination.

The signal received at the relay R_i during the odd interval of the $i-th$ frame, for $i = 1, \ldots, N$, can be written as

$$\mathbf{r}_{2i-1} = \mathbf{H}_{SR_i}\mathbf{x}_{2i-1} + \mathbf{w}^r_{2i-1} \tag{1}$$

and the respective signals received at the destination during the odd and even intervals of the $i-th$ frame , $i = 1, \ldots, N$, can be expressed as follows

$$\mathbf{y}_{2i-1} = \mathbf{H}_{SD}\mathbf{x}_{2i-1} + \mathbf{w}^d_{2i-1} \tag{2}$$

$$\mathbf{y}_{2i} = \mathbf{H}_{SD}\mathbf{x}_{2i} + a_i\mathbf{H}_{R_iD}\mathbf{r}_{2i-1} + \mathbf{w}^d_{2i}, \tag{3}$$

where the $M \times 1$ vectors $\mathbf{x}_{2i-1}, \mathbf{x}_{2i}$ represent the transmitted blocks from the source, $\mathbf{r}_{2i-1}$ is the received block at the relay during the odd time interval and a_i is a scale factor that maintains the average energy of the signal transmitted from the relay equal to σ_x^2. We assume that both the source and the relay transmit with equal power. The $M \times 1$ vectors $\mathbf{y}_j$, and $\mathbf{w}^d_j$, $j = 1, \ldots, 2N$, correspond to the received blocks and additive white Gaussian noise at the destination, respectively, while with $\mathbf{r}_{2i-1}$ and $\mathbf{w}^r_{2i-1}$ we denote the received blocks and additive white Gaussian noise at the relay. For convenience, all noise variances are assumed equal to σ_n^2. Finally, $\mathbf{H}_{SD}$, $\mathbf{H}_{SR_i}$ and $\mathbf{H}_{R_iD}$ are $M \times M$ circulant matrices, whose first columns correspond to $\mathbf{h}_{SD}$, $\mathbf{h}_{SR_i}$ and $\mathbf{h}_{R_iD}$ appended with $(M - L_{SD} - 1)$, $(M - L_{SR_i} - 1)$ and $(M - L_{R_iD} - 1)$ zeros, respectively. Due to their circulant structure, they can be decomposed by using the discrete fourier transform (DFT) matrix operator $\mathbf{F}$. Combining (1) and (3), we obtain

$$\mathbf{y}_{2i} = \mathbf{H}_{SD}\mathbf{x}_{2i} + a_i\mathbf{H}_{R_iD}\mathbf{H}_{SR_i}\mathbf{x}_{2i-1} + \tilde{\mathbf{w}}^d_{2i}, \tag{4}$$

where $\tilde{\mathbf{w}}^d_{2i}$ is given by

$$\tilde{\mathbf{w}}^d_{2i} = a_i\mathbf{H}_{R_iD}\mathbf{w}^r_{2i-1} + \mathbf{w}^d_{2i}, \tag{5}$$

Since we are interested in estimating the channel $\mathbf{h}_{R_i}$ of the overall link $S \rightarrow R_i \rightarrow D$, we define the matrix $\mathbf{H}_{R_i} \equiv a_i\mathbf{H}_{SR_i}\mathbf{H}_{R_iD} = a_i\mathbf{H}_{R_iD}\mathbf{H}_{SR_i}$. This matrix has a circulant structure and its first column corresponds to the convolution of the zero padded $\mathbf{h}_{SR_i}$ and $\mathbf{h}_{R_iD}$ channels scaled by the factor a_i that ensures that the average energy at the relays equals σ_x^2. Then from (2), (4), the signal received at the destination during frame i is expressed in matrix form in the FD as follows

$$\begin{bmatrix} \boldsymbol{\mathcal{Y}}_{2i-1} \\ \boldsymbol{\mathcal{Y}}_{2i} \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{\Lambda}_{SD} & \mathbf{0}_M \\ \boldsymbol{\Lambda}_{R_i} & \boldsymbol{\Lambda}_{SD} \end{bmatrix}}_{\boldsymbol{\Lambda}_i} \begin{bmatrix} \boldsymbol{\mathcal{X}}_{2i-1} \\ \boldsymbol{\mathcal{X}}_{2i} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\mathcal{W}}^d_{2i-1} \\ \tilde{\boldsymbol{\mathcal{W}}}^d_{2i} \end{bmatrix}, \tag{6}$$

where $\boldsymbol{\mathcal{Y}}_j = \mathbf{F}\mathbf{y}_j$, $\boldsymbol{\mathcal{X}}_j = \mathbf{F}\mathbf{x}_j$, $j = 1, 2, \ldots, 2N-1$, $\boldsymbol{\mathcal{W}}^d_j = \mathbf{F}\mathbf{w}^d_j$, $j = 1, 3, \ldots, 2N-1$, $\tilde{\boldsymbol{\mathcal{W}}}^d_j = \mathbf{F}\tilde{\mathbf{w}}^d_j$, $j = 2, 4, \ldots, 2N$, and $\boldsymbol{\Lambda}_{SD}$, $\boldsymbol{\Lambda}_{R_i}$ are diagonal matrices that contain the DFT coefficients of the zero padded $\mathbf{h}_{SD}$ and $\mathbf{h}_{R_i}$ channels. Notice that matrix $\boldsymbol{\Lambda}_i$, which is to be estimated possesses a special lower triangular structure, namely it contains the DFT coefficients of the $\mathbf{h}_{SD}$ channel on the main diagonal and the DFT coefficients of the $\mathbf{h}_{R_i}$ channel on the M-th lower subdiagonal.

III. Channel Identification Techniques

Assuming complex, zero-mean, and uncorrelated input and noise signals of variances σ_x^2 and σ_n^2 respectively, the autocorrelation matrix of the FD received vectors is expressed as

$$\boldsymbol{\Phi}_i = E\left[\begin{bmatrix} \boldsymbol{\mathcal{Y}}_{2i-1} \\ \boldsymbol{\mathcal{Y}}_{2i} \end{bmatrix} \begin{bmatrix} \boldsymbol{\mathcal{Y}}_{2i-1} \\ \boldsymbol{\mathcal{Y}}_{2i} \end{bmatrix}^H\right] = \left[\begin{array}{c|c} \boldsymbol{\mathcal{R}}_{2i-1} & \boldsymbol{\mathcal{R}}_{2i-1,2i} \\ \hline \boldsymbol{\mathcal{R}}^H_{2i-1,2i} & \boldsymbol{\mathcal{R}}_{2i} \end{array}\right] \tag{7}$$

where

$$\boldsymbol{\mathcal{R}}_{2i-1} = \sigma_x^2\boldsymbol{\Lambda}_{SD}\boldsymbol{\Lambda}^H_{SD} + \sigma_n^2\mathbf{I}_M \tag{8}$$

$$\boldsymbol{\mathcal{R}}_{2i} = \sigma_x^2\boldsymbol{\Lambda}_{SD}\boldsymbol{\Lambda}^H_{SD} + \sigma_x^2\boldsymbol{\Lambda}_{R_i}\boldsymbol{\Lambda}^H_{R_i} + \tilde{\sigma}_n^2\mathbf{I}_M \tag{9}$$

$$\boldsymbol{\mathcal{R}}_{2i-1,2i} = \sigma_x^2\boldsymbol{\Lambda}_{SD}\boldsymbol{\Lambda}^H_{R_i} \tag{10}$$

$$\begin{aligned} \tilde{\sigma}_n^2 &= E\left[\tilde{\boldsymbol{\mathcal{W}}}^d_j\left(\tilde{\boldsymbol{\mathcal{W}}}^d_j\right)^H\right] \\ &= a_i^2 E\boldsymbol{\Lambda}_{R_iD}\boldsymbol{\Lambda}^H_{R_iD}\sigma_n^2 + \sigma_n^2\mathbf{I}_M \end{aligned} \tag{11}$$

Cholesky factorization of matrix $\boldsymbol{\Phi}_i$ can be easily obtained using Schur complements [9] as follows

$$\boldsymbol{\Phi}_i = \underbrace{\left[\begin{array}{c|c} \boldsymbol{\mathcal{R}}^{1/2}_{2i-1} & \mathbf{0}_M \\ \hline \boldsymbol{\mathcal{R}}^{-1/2}_{2i-1}\boldsymbol{\mathcal{R}}^H_{2i-1,2i} & \boldsymbol{\Delta}^{1/2}_R \end{array}\right]}_{\boldsymbol{\mathcal{G}}_i} \underbrace{\left[\begin{array}{c|c} \boldsymbol{\mathcal{R}}^{1/2}_{2i-1} & \boldsymbol{\mathcal{R}}^{-1/2}_{2i-1}\boldsymbol{\mathcal{R}}_{2i-1,2i} \\ \hline \mathbf{0}_M & \boldsymbol{\Delta}^{1/2}_R \end{array}\right]}_{\boldsymbol{\mathcal{G}}^H_i} \tag{12}$$

where

$$\mathbf{\Delta}_R = \boldsymbol{\mathcal{R}}_{2i} - \boldsymbol{\mathcal{R}}_{2i-1,2i}^H \boldsymbol{\mathcal{R}}_{2i-1}^{-1} \boldsymbol{\mathcal{R}}_{2i-1,2i} \tag{13}$$

is the Schur complement and $\boldsymbol{\mathcal{G}}_i$ is the Cholesky factor of $\mathbf{\Phi}_i$. By inspecting (8) - (11), it can be shown that in the absence of noise, i.e., $\sigma_n^2 \to 0$, (13) can be written as

$$\mathbf{\Delta}_R = \sigma_x^2 \mathbf{\Lambda}_{SD} \mathbf{\Lambda}_{SD}^H. \tag{14}$$

and

$$\mathbf{\Delta}_R^{1/2} = \boldsymbol{\mathcal{R}}_{2i-1}^{1/2} = \sigma_x \left|\mathbf{\Lambda}_{SD}\right| \tag{15}$$

where $|\mathbf{\Lambda}_{SD}|$ is a $M \times M$ diagonal matrix that contains the amplitudes $\left|[\mathbf{\Lambda}_{SD}]_{m,m}\right|$, $m = 1, \ldots, M$ of the $S \to D$ channel frequency response. We use $[\cdot]_{m,m}$ notation to denote the (m,m)-th entry of a matrix. The Cholesky factor of $\mathbf{\Phi}_i$ would then be equal to

$$\boldsymbol{\mathcal{G}}_i = \sigma_x \left[\begin{array}{c|c} |\mathbf{\Lambda}_{SD}| & \mathbf{0}_M \\ \hline (\angle\mathbf{\Lambda}_{SD}^H)\mathbf{\Lambda}_{R_i} & |\mathbf{\Lambda}_{SD}| \end{array}\right] \tag{16}$$

where $\angle\mathbf{\Lambda}_{SD}^H$ is a $M \times M$ diagonal matrix that contains the complex phases $\left[\mathbf{\Lambda}_{SD}^H\right]_{m,m} / \left|[\mathbf{\Lambda}_{SD}]_{m,m}\right|$, $m = 1, \ldots, M$ of the conjugate $S \to D$ frequency bins. Alternatively (16) can be rewritten as

$$\boldsymbol{\mathcal{G}}_i = \sigma_x \boldsymbol{\mathcal{Q}} \mathbf{\Lambda}_i \tag{17}$$

where

$$\boldsymbol{\mathcal{Q}} = \begin{bmatrix} \angle\mathbf{\Lambda}_{SD}^H & \mathbf{0}_M \\ \mathbf{0}_M & \angle\mathbf{\Lambda}_{SD}^H \end{bmatrix} \tag{18}$$

According to (17) and (18) one has to resolve M phase ambiguities in order to determine the frequency response of the $S \to D$ and $S \to R_i \to D$ channels, which are contained in $\mathbf{\Lambda}_i$. More specifically, from (6), (12) and (17), these channels are expressed in terms of the elements of the Cholesky factor as follows

$$\begin{aligned} \hat{\mathbf{\Lambda}}_{SD} &= \frac{\angle\mathbf{\Lambda}_{SD}}{\sigma_x} \boldsymbol{\mathcal{R}}_{2i-1}^{1/2}, & (19) \\ \hat{\mathbf{\Lambda}}_{R_i} &= \frac{\angle\mathbf{\Lambda}_{SD}}{\sigma_x} \boldsymbol{\mathcal{R}}_{2i-1}^{-1/2} \boldsymbol{\mathcal{R}}_{2i-1,2i}^H \approx \frac{\mathbf{\Lambda}_{SD}^{-H}}{\sigma_x^2} \boldsymbol{\mathcal{R}}_{2i-1,2i}^H, & (20) \end{aligned}$$

where $i = 1, \ldots, N$. Using a second order statistics based blind technique [10] for estimating $\mathbf{h}_{SD}$ all $S \to R_i \to D$ frequency responses for $i = 1, \ldots, N$ can be calculated up to a single scalar ambiguity. Alternatively, the phase ambiguities may be resolved by making use of pilot symbols in the $S \to D$ link only, as explained in the next subsection.

Resolving the Phase Ambiguities

As mentioned above, in order to fully identify the unknown frequency responses of the involved channels, the phases of the $S \to D$ frequency response need to be estimated. This can be achieved by transmitting training blocks at the odd intervals. In such case, we can estimate the $S \to D$ frequency bins as

$$\left[\tilde{\mathbf{\Lambda}}_{SD}\right]_{m,m} = \left(\left[\hat{\boldsymbol{\mathcal{R}}}_{xx}\right]_{m,m} + \delta\right)^{-1} \left[\hat{\boldsymbol{\mathcal{R}}}_{xy}\right]_{m,m} \tag{21}$$

where matrices $\hat{\boldsymbol{\mathcal{R}}}_{xx}$, $\hat{\boldsymbol{\mathcal{R}}}_{xy}$ have been estimated from a finite number of training blocks (i.e. L) as

$$\begin{aligned} \hat{\boldsymbol{\mathcal{R}}}_{xx} &= \frac{1}{L} \sum_{l=1}^{L} \boldsymbol{\mathcal{X}}_{2i-1}(l)\, \boldsymbol{\mathcal{X}}_{2i-1}^H(l) & (22) \\ \hat{\boldsymbol{\mathcal{R}}}_{xy} &= \frac{1}{L} \sum_{l=1}^{L} \boldsymbol{\mathcal{X}}_{2i-1}(l)\, \boldsymbol{\mathcal{Y}}_{2i-1}^H(l) & (23) \end{aligned}$$

and δ is a small constant. At this point it should be mentioned that instead of using a whole block of M pilots we can fully identify the $S \to D$ FR with L_{SD} equipowered pilots placed in equispaced positions [11].

Since matrix $\hat{\boldsymbol{\mathcal{R}}}_{xx}$ contains real elements, the phases may be computed directly from the phases of $\hat{\boldsymbol{\mathcal{R}}}_{xy}$, i.e.,

$$\angle\hat{\mathbf{\Lambda}}_{SD} = \angle\hat{\boldsymbol{\mathcal{R}}}_{xy} \tag{24}$$

Thus, two alternative schemes for estimating the $S \to D$ frequency response can be employed. One can either estimate both the phases and the amplitudes of the $S \to D$ frequency response using (21) or by using (24) for the phases and (19) for the amplitudes. The overall $S \to R_i \to D$, $i = 1, \ldots, N$ frequency responses can then be estimated using (20). However, as it will be shown through simulations, the second approach, where only the phases are estimated via training blocks, succeeds in estimating all the $(N+1)M$ frequency bins even when only one training block is used, while the first approach fails.

IV. Performance Study

In this section the two alternative schemes for estimating the $S \to D$ frequency response as well as their influence on the estimation of the $S \to R_i \to D$ frequency responses will be investigated. The performance of the proposed schemes will be studied in high SNR conditions and in terms of the attained NMSE between the actual and estimated frequency responses.

Initially, we will evaluate the variance of estimating the $S \to D$ frequency bins. Let us consider that $2NK$ blocks of received data and L blocks of training (transmitted during the odd intervals) have been received at the destination. Then each element of $\mathbf{\Lambda}_{SD}$ can be estimated by the L training blocks according to eq. (21). In that case, it has been shown in [12] that the variance of the estimator would be

$$var\left(\left[\tilde{\mathbf{\Lambda}}_{SD}\right]_{m,m}\right) = \frac{\sigma_n^2}{\sigma_x^2 L} + \delta^2. \tag{25}$$

Thus, the NMSE between the actual and the estimated frequency response would be given by

$$E\left[\left\|\boldsymbol{\lambda}_{SD} - \tilde{\boldsymbol{\lambda}}_{SD}\right\|^2 / \left\|\boldsymbol{\lambda}_{SD}\right\|^2\right] = \frac{\sigma_n^2}{\sigma_x^2 L} + \delta^2 \tag{26}$$

where $\boldsymbol{\lambda}_{SD}$ and $\tilde{\boldsymbol{\lambda}}_{SD}$ are $M \times 1$ vectors containing the diagonal elements of $\mathbf{\Lambda}_{SD}$ and $\tilde{\mathbf{\Lambda}}_{SD}$ respectively.

It has been mentioned that the $S \to D$ frequency bins can be alternatively computed by first estimating only the phases from the training blocks as

$$\left[\angle\hat{\mathbf{\Lambda}}_{SD}\right]_{m,m} = \left[\hat{\boldsymbol{\mathcal{R}}}_{xy}\right]_{m,m} / \left|\left[\hat{\boldsymbol{\mathcal{R}}}_{xy}\right]_{m,m}\right| \tag{27}$$

and subsequently the amplitudes using (19). However, we can avoid using (19) which suffers from possible numerical inaccuracies of Cholesky factorization and instead estimate the required amplitudes based on (8), as follows

$$\left|\left[\hat{\mathbf{\Lambda}}_{SD}\right]_{m,m}\right| = \sqrt{\frac{1}{\sigma_x^2 N}\sum_{i=1}^{N}\left[\hat{\boldsymbol{\mathcal{R}}}_{2i-1}(K)\right]_{m,m}},\ m=1,\ldots,M \tag{28}$$

where $\hat{\boldsymbol{\mathcal{R}}}_{2i-1}(K)$ is estimated from K received blocks as

$$\hat{\boldsymbol{\mathcal{R}}}_{2i-1}(K)=\frac{1}{K}\sum_{k=1}^{K}\boldsymbol{\mathcal{Y}}_{2i-1}(k)\boldsymbol{\mathcal{Y}}_{2i-1}^H(k) \tag{29}$$

From the central limit theorem, $\boldsymbol{\mathcal{X}}_{2i-1}$, $\boldsymbol{\mathcal{X}}_{2i}$ can be considered as complex normal circular symmetric random vectors, for sufficiently large block length M. Then the mean value of the autocorrelation estimator is $E[\hat{\boldsymbol{\mathcal{R}}}_{2i-1}(K)] = \boldsymbol{\mathcal{R}}_{2i-1}$, while its variance can be computed by following standard arguments [12] as

$$var\left(\hat{\boldsymbol{\mathcal{R}}}_{2i-1}(K)\right) \approx \frac{\sigma_x^4}{K}\left(\mathbf{\Lambda}_{SD}\mathbf{\Lambda}_{SD}^H\right)^2 + \frac{\sigma_n^4}{K}\mathbf{I}_M \tag{30}$$

We may proceed to the computation of the variance of the estimators in Eqs. (28) by employing the so-called delta method [13]. This method employs second-order Taylor expansions to approximate the variance of a function of one or more RVs. Let x be a RV with $E[x] = \mu_x$ and $var(x) = \sigma_x^2$. Then the approximate variance of a function of one variable is given by

$$var\left(f\left(x\right)\right) \approx \left(\frac{\partial}{\partial x}f(x)\Big|_{\mu_x}\right)^2 \sigma_x^2 \tag{31}$$

Thus, it can be easily shown that the variance of the estimator in (28) is approximated by

$$var\left(\left|\left[\hat{\mathbf{\Lambda}}_{SD}\right]_{m,m}\right|\right) \approx \frac{1}{4NK\sigma_x^2}\frac{\left|\left[\hat{\mathbf{\Lambda}}_{SD}\right]_{m,m}\right|^4\sigma_x^4+\sigma_n^4}{\left|\left[\hat{\mathbf{\Lambda}}_{SD}\right]_{m,m}\right|^2\sigma_x^2+\sigma_n^2} \tag{32}$$

Based on a generalization of the delta method for functions of two random variables it can be shown that, in high SNR conditions, the variance of estimating the $S \rightarrow D$ frequency response is dominated by the variance of estimating the amplitudes. More specifically, when we estimate the phases of the $S \rightarrow D$ frequency response from (24) and the amplitudes from (28), then the NMSE between the actual and estimated responses exhibits a floor as the SNR increases. This floor depends on the variance of the estimator given in (32) and is a function of the number of relays and the number of blocks that have been used for the estimation of the autocorrelation matrix, i.e.,

$$\lim_{\sigma_n^2\rightarrow 0} E\left[\left\|\boldsymbol{\lambda}_{SD}-\hat{\boldsymbol{\lambda}}_{SD}\right\|^2 / \left\|\boldsymbol{\lambda}_{SD}\right\|^2\right] \approx \frac{1}{4NK} \tag{33}$$

where $\hat{\boldsymbol{\lambda}}_{SD}$ are $M \times 1$ vector containing the $\hat{\boldsymbol{\lambda}}_{SD}(m) = \left[\angle\hat{\mathbf{\Lambda}}_{SD}\right]_{m,m}\left|\left[\hat{\mathbf{\Lambda}}_{SD}\right]_{m,m}\right|$, $m=1,\ldots,M$.

Having estimated the amplitudes of the $S \rightarrow D$ frequency response we can proceed to the estimation of all the $S \rightarrow R_i \rightarrow D$ frequency responses, by using (20), which can be written alternatively as

$$\left[\hat{\mathbf{\Lambda}}_{R_i}\right]_{m,m} = \frac{\left[\hat{\boldsymbol{\mathcal{R}}}_{2i-1,2i}^H(K)\right]_{m,m}}{\sigma_x^2\left[\hat{\mathbf{\Lambda}}_{SD}^H\right]_{m,m}} \tag{34}$$

where $\left[\hat{\mathbf{\Lambda}}_{SD}\right]_{m,m}$ is the estimated m-th frequency bin of the $S \rightarrow D$ frequency response and matrix $\boldsymbol{\mathcal{R}}_{2i-1,2i}(K)$ can be estimated from K blocks received at the odd and K blocks received at the even intervals as follows

$$\hat{\boldsymbol{\mathcal{R}}}_{2i-1,2i}(K)=\frac{1}{K}\sum_{k=1}^{K}\boldsymbol{\mathcal{Y}}_{2i}(k)\boldsymbol{\mathcal{Y}}_{2i-1}^H(k). \tag{35}$$

The mean value of the above estimator is $E[\hat{\boldsymbol{\mathcal{R}}}_{2i-1}(K)] = \boldsymbol{\mathcal{R}}_{2i-1}$ and its variance can be calculated as

$$var\left(\left[\hat{\boldsymbol{\mathcal{R}}}_{2i-1,2i}(K)\right]_{m,m}\right) \approx \frac{\sigma_x^4}{K}\left|\left[\mathbf{\Lambda}_{SD}\mathbf{\Lambda}_{R_i}^H\right]_{m,m}\right|^2 \tag{36}$$

By employing the delta method for estimating the variance of a function of two random variables, it can be shown that the variance of the estimator in (34) seems to be dominated by the variance of the estimator in (36). Thus, independently of the way that the $S \rightarrow D$ frequency bins are estimated, the variance of the estimator of the $S \rightarrow R_i \rightarrow D$ frequency bins and the corresponding NMSE exhibits a floor as $SNR \rightarrow \infty$ which depends only on the number of blocks that have been used for the estimation of $\hat{\boldsymbol{\mathcal{R}}}_{2i-1,2i}(K)$. This floor can be approximated by

$$\lim_{\sigma_n^2\rightarrow 0} E\left[\left\|\boldsymbol{\lambda}_{R_i}-\hat{\boldsymbol{\lambda}}_{R_i}\right\|^2 / \left\|\boldsymbol{\lambda}_{R_i}\right\|^2\right] \approx \frac{2}{K} \tag{37}$$

where $\boldsymbol{\lambda}_{R_i}$ and $\hat{\boldsymbol{\lambda}}_{R_i}$ are $M \times 1$ vectors containing the diagonal elements of $\mathbf{\Lambda}_{R_i}$ and $\hat{\mathbf{\Lambda}}_{R_i}$ respectively. The above expressions have been also verified through simulations, as it will be shown in the section that follows.

V. Simulation Results

The performance of the described techniques was evaluated through computer simulations. We consider 2 relays cooperating with the source according to the protocol proposed in [6]. The $S \rightarrow D$, $S \rightarrow R_i$, $R_i \rightarrow D$, $(i = 1, 2)$ links are modeled as frequency selective channels with memory lengths $L_{SD} = L_{SR_i} = L_{R_iD} = 6$, $\forall i = 1, 2$. All channel taps are assumed Rayleigh faded and the transmission is done in blocks of $M = 32$ QPSK symbols. The power profile has been considered to be uniform and the transmitted symbol energy σ_x^2 has been selected to be equal to one. The proposed algorithms were evaluated in terms of the normalized mean square error (NMSE), between actual and estimated frequency responses.

Initially, to study the effect of the training length, a configuration as the one described above operating at different

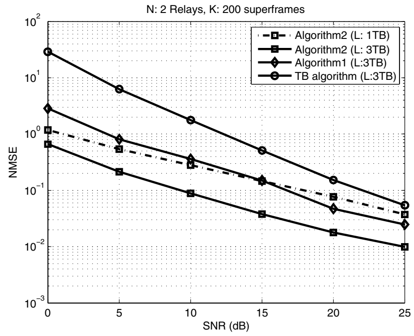

Fig. 3. NMSE of the estimated frequency responses

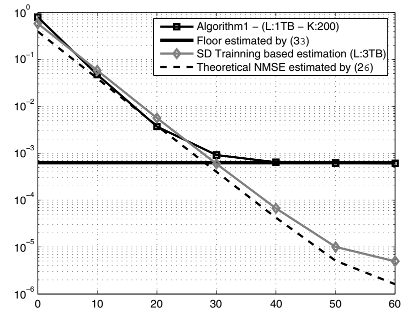

Fig. 4. NMSE of the $S \rightarrow D$ frequency responses.

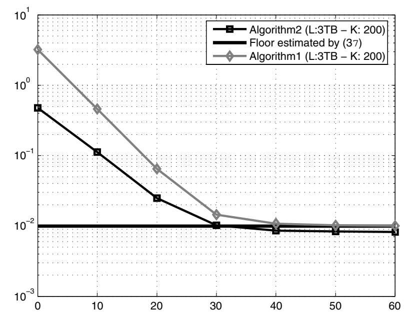

Fig. 5. NMSE of the $S \rightarrow R_i \rightarrow D$ frequency responses.

SNRs was simulated. The SNR was defined as the expected SNR per bit (over the ensemble of channel realizations) at the destination. Three different schemes were tested. In the first scheme (Algorithm 1), the frequency response of the $S \rightarrow D$ channel is estimated through training according to (21) and then used for the estimation of the $S \rightarrow R_i \rightarrow D$ channels from (34). In the second scheme (Algorithm 2), only the phases of the $S \rightarrow D$ channel frequency response are computed through training from (27), while the corresponding amplitudes are obtained blindly from the output autocorrelation matrix using (28). The $S \rightarrow R_i \rightarrow D$ frequency response is then estimated by (34). For both algorithms 1 and 2, training symbols are transmitted during the odd time intervals only. The third scheme is a training-based (TB) algorithm where all the channels are estimated from training blocks sent not only at the odd but also at the even intervals. The $S \rightarrow D$ frequency response is estimated from the symbols transmitted during the odd intervals, as in Algorithm 1. This estimation along with the training blocks transmitted at the even intervals are used for computing the $S \rightarrow R_i \rightarrow D$ frequency responses. In Fig. 3 the NMSE averaged over 1000 independent runs, is plotted. The superior performance of Algorithm 2 is obvious from the figure. Furthermore, Algorithm 2 identifies successfully all the channels even when only one block is used for training, while the other algorithms fail.

The accuracy of the derived theoretical expressions presented in section IV was also tested. In Fig. 4 the NMSE between the actual and estimated $S \rightarrow D$ frequency responses along with the theoretical expressions given in (26), (33) are plotted. Finally, the NMSE between the actual and estimated $S \rightarrow R_i \rightarrow D$ frequency responses averaged over all relays, along with the theoretical expression given in (37) are plotted in Fig. 5. It is clear from the figures that simulation results completely verify the presented theoretical analysis.

VI. Conclusion

Efficient channel estimation techniques for wideband cooperative systems with multiple relays operating in AF mode have been derived. It has been shown that all the channels from the source to the destination in a cooperative network consisting of N relays can be obtained from the elements of the Cholesky factor of the received signal autocorrelation matrix. By using a few pilot symbols in the $S \rightarrow D$ link all channels of the network can be efficiently estimated. The proposed methods have been analyzed theoretically and their excellent performance even when compared with direct training based methods, has been verified via extensive simulations.

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