A Robust Method for Estimating Multipath Channel Parameters in the Uplink of a DS-CDMA System

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***Abstract*— In this paper, the problem of estimating the multipath channel parameters of a new user entering the uplink of an asynchronous Direct Sequence - Code Division Multiple Access (DS-CDMA) system is addressed. The problem is described via a least squares (LS) cost function, which is non-linear with respect to the time delays and linear with respect to the gains of the multipath channel. This cost function is proved to be approximately decoupled in terms of the path delays, and thus an iterative procedure of one-dimensional searches turns out to be adequate for time delays estimation. The resulting method is computationally efficient and performs well even for a small number of training symbols. Simulation results show that the proposed technique offers a better estimation accuracy compared to existing related methods, and is robust to multiple access interference.**

I. Introduction

In this paper, we propose a new method for the estimation of multipath channel parameters in the uplink of a DS-CDMA system. Accurate channel estimates are highly desirable at the base station (BS), where advanced signal processing can be employed to mitigate multiple access interference (MAI) and multipath fading. Between the conventional tapped-delay-line (TDL) channel model and the parametric model, where channel impulse response (CIR) is characterized by the gains and delays of dominant paths, the parametric one is more effective, since fewer parameters are adequate for accurate channel representation. Our focus is on the uplink of a DS-CDMA system, which is usually asynchronous and hence estimation methods robust to MAI are required.

To combat MAI interference and multipath fading, joint multiuser detection and parametric channel estimation approaches have been developed [1], [2]. The increased complexity of these algorithms renders them impractical, and the channel estimation problem is usually treated separately from the detection one. Blind subspace-based channel estimation methods have been proposed in [3], [4], however they require long observation intervals, which limit their tracking capability in rapidly varying channels. Maximum Likelihood (ML) optimization has been also adopted for multipath channel parameter estimation. ML-based methods make use of training signals and model MAI as colored noise. In [5] and [6] channel estimates from MAI users are exploited during the estimation of a new user, but specific PN sequences are required. The only method that uses relatively few training symbols and does not require specific signals to be employed, is the one proposed in [7]. This method follows a ML-based approach and employs a deflation scheme originating from the SAGE (space-alternating generalized expectation-maximization) algorithm [8]: the optimization is performed with respect to a single path, and after this path has been estimated its contribution is subtracted from the received data. The deflation scheme applies similarly to the rest of the paths.

In this paper, the problem of channel parameters' estimation is described via a non-linear least squares (LS) cost function that is proven to be approximately decoupled with respect to the delay parameters. This allows for the development of an efficient search method for the estimation of the time delays. The new method constitutes an interesting alternative interpretation of the channel parameters' estimation problem. Moreover, simulations results show that the proposed method exhibits a lower mean squared estimation error than the method of [7], at the expense of a negligible increase of the computational complexity.

The outline of this paper is as follows. In Section II, the signal model is defined and the estimation problem is formulated. In Section III, the LS cost function is derived and the proposed algorithm is developed. Simulation results are presented in Section IV, and the paper is concluded in V.

II. Problem formulation

Let us consider the reverse link of a DS-CDMA system accommodating K simultaneously active users. If T is the symbol period, $\{b_k(i)\}$ the transmitted symbols, and $p_k(t)$ the spreading waveform of k_{th} user, then the baseband signal transmitted by this user can be expressed as

$$s_k(t) = \sum_i b_k(i) p_k(t - iT) \tag{1}$$

Let N be the spreading factor, $T_c = T/N$ the chip period, $\{c_k(n),\ n = 0, \ldots, N-1\}$ the chip sequence, and $g(t)$ the

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chip pulse. Then, the spreading waveform $p_k(t)$ is given by

$$p_k(t) = \sum_{n=0}^{N-1} c_k(n) g(t - nT_c) \quad (2)$$

The signal $s_k(t)$ of each user is transmitted over a multipath channel with P discrete paths having impulse response

$$h_k(t) = \sum_{p=1}^{P} a_{k,p} \delta(t - \tau_{k,p}) \quad (3)$$

where $a_{k,p}$ and $\tau_{k,p}$ are the gain and the delay of the p_{th} path, respectively and $\delta(\cdot)$ is the Dirac function. The received signal is the superposition of the signals from all users, i.e.

$$x(t) = \sum_{k=1}^{K} \sum_{p=1}^{P} a_{k,p} s_k(t - \tau_{k,p}) + w(t) \quad (4)$$

contaminated by additive, white, Gaussian noise $w(t)$ of power spectral density N_0. The received signal is oversampled by a factor of Q samples per chip period, while a raised cosine function is used as the chip pulse.

The delay spread of the physical channel $h_k(t)$, usually encountered in the applications of interest, is restricted to a few chip periods [9]. Also, taking into account the asynchronous access of the k_{th} user to the channel, the first delay $\tau_{k,1}$ could appear anywhere in the interval $[0, NT_c)$ of the BS timing. Thus, a time support of two symbols can be adequate for the total CIR, which is the convolution of the physical channel, $h_k(t)$, with the chip sequence $\{c_k(n)\}$.

Our goal is the estimation of the physical channel parameters for one user assuming that the parameters of all other $(K-1)$ users have already been estimated. To this end and using the formulation presented above, the samples collected at the BS receiver over a period of M symbols can be written in vector form as

$$\mathbf{x} = \sum_{k=1}^{K} \mathbf{S}_k(\boldsymbol{\tau}_k)\mathbf{a}_k + \mathbf{w} \quad (5)$$

where $\mathbf{a}_k, \boldsymbol{\tau}_k$ are the vectors of delays and gains of user k, $\mathbf{w}$ is the $MQN \times 1$ noise vector and $\mathbf{S}_k(\boldsymbol{\tau}_k)$ is expressed as

$$\mathbf{S}_k(\boldsymbol{\tau}_k) = \left(\mathbf{B}_k^H \otimes \mathbf{I}_{QN}\right)\left(\mathbf{C}_k^H \otimes \mathbf{I}_Q\right)\mathbf{G}(\boldsymbol{\tau}_k) \quad (6)$$

$\mathbf{B}_k$ is a $2 \times M$ data matrix with Hankel structure, $\mathbf{C}_k$ is a $2N \times 2N$ convolution matrix with its first row containing the chip sequence as $\begin{bmatrix} \mathbf{c}_k^T & \mathbf{0}_N^T \end{bmatrix}$, $\mathbf{c}_k = [c_k(0), \ldots, c_k(N-1)]^T$ and $\mathbf{G}(\boldsymbol{\tau}_k)$ is a $2QN \times P$ matrix whose columns contain the oversampled delayed chip pulses denoted in vector form as $\mathbf{g}(\tau_{k,p}),\ p=1,\ldots,P$. Note that each column of $\mathbf{G}(\boldsymbol{\tau}_k)$ is a function of a single delay parameter only. Symbol $\otimes$ stands for the Kronecker product and $\mathbf{I}_Q$ is the $Q \times Q$ identity matrix.

Let us consider that a new user, called hereafter the desired user, is entering the system. Then, (5) can be rewritten as

$$\mathbf{x} = \mathbf{S}(\boldsymbol{\tau})\mathbf{a} + \boldsymbol{\eta} \quad (7)$$

where the user index has been dropped for simplicity and $\boldsymbol{\eta}$ comprises MAI and thermal noise.

We assume that the spreading sequences of all the users are known at the BS, while the desired user is in training mode and has been synchronized to the BS. Although the channel parameters of the interfering users have already been estimated, their symbol sequences have not been detected yet. Hence, MAI can be treated as a stochastic random process [7]. Specifically, MAI vector $\boldsymbol{\eta}$ can be modelled as a zero mean Gaussian vector with covariance matrix $\mathbf{R}_\eta = E[\boldsymbol{\eta}\boldsymbol{\eta}^H]$.

III. Derivation of the New Algorithm

A. The New Cost Function

As can been seen from (7), the data available for the estimation of channel parameters are contaminated by colored noise $\boldsymbol{\eta}$ with covariance matrix $\mathbf{R}_\eta$. After prewhitening of the noise, the required channel parameters, i.e. $\boldsymbol{\tau}$ and $\mathbf{a}$, may be estimated by minimizing the least squares (LS) cost function

$$J(\boldsymbol{\tau}, \mathbf{a}) = \left\| \mathbf{R}_\eta^{-1/2}\mathbf{x} - \mathbf{R}_\eta^{-1/2}\mathbf{S}(\boldsymbol{\tau})\mathbf{a} \right\|^2 \quad (8)$$

where $\mathbf{R}_\eta^{-1/2}$ is a square root factor of $\mathbf{R}_\eta^{-1}$. This cost function is linear with respect to the path gains and nonlinear with respect to the delays. Since the two sets of parameters are independent, the optimization problem can be split up with respect to each set, as

$$\boldsymbol{\tau}_{opt} = \arg\max_{\boldsymbol{\tau}} \left\| \mathbf{R}_\eta^{-1/2}\mathbf{S}(\boldsymbol{\tau}) \left(\mathbf{R}_\eta^{-1/2}\mathbf{S}(\boldsymbol{\tau})\right)^{\dagger} \mathbf{R}_\eta^{-1/2}\mathbf{x} \right\|^2 \quad (9)$$

$$\mathbf{a}_{opt} = \left(\mathbf{R}_\eta^{-1/2}\mathbf{S}(\boldsymbol{\tau})\right)^{\dagger} \mathbf{R}_\eta^{-1/2}\mathbf{x} \quad (10)$$

where symbol $\dagger$ denotes the pseudoinverse of a matrix.

It is apparent that the most difficult part of the above optimization procedure is the maximization in (9). Commonly, such a nonlinear problem is treated either by performing a costly multidimensional search, or by applying an iterative Newton type method, which could be trapped in a local maximum. In the following, we show that the estimation of each delay parameter τ_p can be performed separately leading to a much more efficient estimation algorithm. We begin by rewriting the cost function in (9) as

$$F(\boldsymbol{\tau}) = \mathbf{y}^H(\boldsymbol{\tau})\mathbf{D}(\boldsymbol{\tau})\mathbf{y}(\boldsymbol{\tau}) \text{ , where} \quad (11)$$

$$\mathbf{y}(\boldsymbol{\tau}) = \mathbf{S}^H(\boldsymbol{\tau})\mathbf{R}_\eta^{-1}\mathbf{x} \quad \text{and} \quad \mathbf{D}(\boldsymbol{\tau}) = \left(\mathbf{S}^H(\boldsymbol{\tau})\mathbf{R}_\eta^{-1}\mathbf{S}(\boldsymbol{\tau})\right)^{-1}$$

It is readily seen from (6) that each column of $\mathbf{S}(\boldsymbol{\tau})$ depends on a single delay parameter. The same property holds for the elements of vector $\mathbf{y}(\boldsymbol{\tau})$ as well. Based on this observation, we deduce that the cost function $F(\boldsymbol{\tau})$ would be decoupled with respect to the delay parameters, if matrix $\mathbf{D}(\boldsymbol{\tau})$ were diagonal and each element $[\mathbf{D}(\boldsymbol{\tau})]_{i,i}$ were associated only to the corresponding delay parameter τ_i. Even though matrix $\mathbf{D}(\boldsymbol{\tau})$ is not exactly diagonal, we show that it is strongly diagonally dominant.

To this end, we invoke a proposition proved in [10]. According to that proposition: *Let a matrix* $\mathbf{A} \in \mathcal{C}^{n \times n}$ *and* r_A *the* *mean ratio of its off-diagonal and diagonal elements*[1]. *If this matrix is pre/post multiplied by a unitary matrix* $\mathbf{Q}\in\mathcal{C}^{n\times m}$ *and* $m\ll n$, *then the resulting matrix* $\mathbf{B}=\mathbf{Q}^H\mathbf{A}\mathbf{Q}$ *has smaller mean ratio compared to* $\mathbf{A}$, *upper bounded by* $\frac{m}{n}r_A$. Consequently, if matrix $\mathbf{A}$ has diagonal elements of much higher amplitude than the off-diagonal ones, and $m\ll n$, then matrix $\mathbf{B}$ is strongly diagonally dominant. It is easily shown, using Taylor series expansion, that the same holds for its $\mathbf{B}^{-1}$. To apply the aforementioned proposition in our problem, e.g. for matrix $\mathbf{D}(\boldsymbol{\tau})$ in (11), three conditions should be satisfied:

1) $P \ll MQN$, which always holds true.
2) Matrix $\mathbf{R}_\eta^{-1}=E[\boldsymbol{\eta}\boldsymbol{\eta}^H]$ should have a 'heavy' diagonal.
3) Matrix $\mathbf{S}(\boldsymbol{\tau})$ should possess a unitary structure.

Concerning the second condition, we firstly observe that due to the i.i.d. property of the symbol sequences, the cross-user terms inside the expectation operator are equal to zero, and MAI covariance matrix can be expressed as (assuming the desired user is user 1)

$$\mathbf{R}_\eta = \sum_{k=2}^{K} E\left[(\mathbf{S}_k(\boldsymbol{\tau}_k)\mathbf{a}_k)(\mathbf{S}_k(\boldsymbol{\tau}_k)\mathbf{a}_k)^H\right] + \frac{N_0}{2}\mathbf{I}_{MQN} \quad (12)$$

From (5) and (6) the overall CIR of user k, can be written as

$$\mathbf{q}_k = \left(\mathbf{C}_k^T \otimes \mathbf{I}_Q\right)\mathbf{G}(\boldsymbol{\tau}_k)\mathbf{a}_k = \begin{bmatrix} \mathbf{q}_k^{(1)} \\ \mathbf{q}_k^{(2)} \end{bmatrix} \quad (13)$$

In the last equation, $\mathbf{q}_k$ is partitioned into two $QN\times 1$ blocks corresponding to one symbol period each. Hence, according to (6), the contribution of user k can be simplified as

$$\mathbf{S}_k(\boldsymbol{\tau}_k)\mathbf{a}_k = \begin{bmatrix} b_k^*(1)\mathbf{q}_k^{(1)} + b_k^*(2)\mathbf{q}_k^{(2)} \\ \vdots \\ b_k^*(M-1)\mathbf{q}_k^{(1)} + b_k^*(M)\mathbf{q}_k^{(2)} \end{bmatrix} \quad (14)$$

The covariance matrix in (12) can be partitioned similarly to (14) into blocks of dimension $QN\times QN$, namely $\{\mathbf{R}_\eta^{(i,j)}; i,j = 1\ldots M\}$. Since each of these blocks depends only on two consecutive symbols for each user, the blocks lying in other than the main and the sub/super diagonals will vanish, resulting in a block tridiagonal form for $\mathbf{R}_\eta$. The non-zero blocks of $\mathbf{R}_\eta$ can be expressed as follows

$$\mathbf{R}_\eta^{(i,i)}=\sum_{k=2}^{K}\left(\mathbf{q}_k^{(1)}\mathbf{q}_k^{(1)H} + \mathbf{q}_k^{(2)}\mathbf{q}_k^{(2)H}\right) + \frac{N_0}{2}\mathbf{I}_{QN} \quad (15)$$

$$\mathbf{R}_\eta^{(i,i+1)}=\sum_{k=2}^{K}\mathbf{q}_k^{(2)}\mathbf{q}_k^{(1)H}, \text{ and } \mathbf{R}_\eta^{(i,i-1)}=\left(\mathbf{R}_\eta^{(i,i+1)}\right)^H \quad (16)$$

Due to the orthogonality of the spreading codes, vectors $\mathbf{q}_k^{(j)}$ can be considered approximately orthogonal. Moreover, we may assume that the elements of these vectors are of the same order, which is quite reasonable according to (13). Thus, it is easily verified that the elements of the off-diagonal blocks are negligible compared to the main diagonal block of $\mathbf{R}_\eta$. Hence, the MAI covariance matrix $\mathbf{R}_\eta$ can be approximated as a block diagonal matrix (15). Note that such an approximation has already been adopted intuitively in the relevant literature [7]. Moving a step further, by applying the matrix inversion lemma to (15), it can be showed that the inverse MAI covariance matrix can be also approximated by a diagonal matrix.

As far as the third condition is concerned, starting from (6) and after some algebra, we get

$$\mathbf{S}^H(\boldsymbol{\tau})\mathbf{S}(\boldsymbol{\tau})=\mathbf{G}^T(\boldsymbol{\tau})\left(\mathbf{C}\otimes\mathbf{I}_Q\right)\left(\mathbf{B}\mathbf{B}^H\otimes\mathbf{I}_{QN}\right)\left(\mathbf{C}^H\otimes\mathbf{I}_Q\right)\mathbf{G}(\boldsymbol{\tau})$$

The term $\mathbf{B}\mathbf{B}^H$ is the sample covariance matrix of the information symbols, and can be approximated asymptotically by $\mathbf{I}_2$. Moreover, the term $\mathbf{C}\mathbf{C}^H$ tends to the covariance matrix of a PN sequence, and thus can be approximated by $\mathbf{I}_{2N}$. Hence, $\mathbf{S}^H(\boldsymbol{\tau})\mathbf{S}(\boldsymbol{\tau})$ reduces to

$$\mathbf{S}^H(\boldsymbol{\tau})\mathbf{S}(\boldsymbol{\tau}) \simeq \mathbf{G}^T(\boldsymbol{\tau})\mathbf{G}(\boldsymbol{\tau}) \quad (17)$$

Recall that the columns of $\mathbf{G}(\boldsymbol{\tau})$ contain delayed versions of a raised cosine pulse shaping filter. The inner product of two columns of $\mathbf{G}(\boldsymbol{\tau})$, i.e. $\mathbf{g}(\tau_i)$ and $\mathbf{g}(\tau_j)$, depends on the delay difference $\Delta\tau=|\tau_i-\tau_j|$. If $\Delta\tau = 0$ the inner product takes its maximum value, whereas it decays rapidly as $\Delta\tau$ increases. Even for $\Delta\tau$ as small as a chip period, the inner product is one order of magnitude smaller than its maximum. Accordingly, $\mathbf{S}(\boldsymbol{\tau})$ has a structure very similar to a unitary matrix and thus the proposition can be applied to our problem.

B. Decomposed Form of the Cost Function

Next we consider a modification of the cost function (9) in order to derive an efficient estimation algorithm. To this end, matrix $\mathbf{S}(\boldsymbol{\tau})$ in (7) is partitioned as

$$\mathbf{S}(\boldsymbol{\tau}) = \begin{bmatrix} \mathbf{S}_{P-1} & \mathbf{s}_P \end{bmatrix} \quad (18)$$

where $\mathbf{S}_{P-1}$ corresponds to the first $(P-1)$ columns of $\mathbf{S}(\boldsymbol{\tau})$ and $\mathbf{s}_P$ is its last column. We define also matrix $\boldsymbol{\Phi}(\boldsymbol{\tau})$

$$\boldsymbol{\Phi}(\boldsymbol{\tau}) \equiv \mathbf{R}_\eta^{-1/2}\mathbf{S}(\boldsymbol{\tau}) = \begin{bmatrix} \boldsymbol{\Phi}_{P-1} & \boldsymbol{\phi}_P \end{bmatrix} \quad (19)$$

which is partitioned similarly to $\mathbf{S}(\boldsymbol{\tau})$. Hence, matrix $\mathbf{D}(\boldsymbol{\tau})$ in (11) can now be partitioned as

$$\mathbf{D}(\boldsymbol{\tau}) = \begin{bmatrix} \boldsymbol{\Phi}_{P-1}^H\boldsymbol{\Phi}_{P-1} & \boldsymbol{\Phi}_{P-1}^H\boldsymbol{\phi}_P \\ \boldsymbol{\phi}_P^H\boldsymbol{\Phi}_{P-1} & \boldsymbol{\phi}_P^H\boldsymbol{\phi}_P \end{bmatrix}^{-1} \quad (20)$$

By expressing vector $\mathbf{y}(\boldsymbol{\tau})$ as $\mathbf{y}(\boldsymbol{\tau})=\begin{bmatrix}\boldsymbol{\Phi}_{P-1}^H & \boldsymbol{\phi}_P^H\end{bmatrix}\mathbf{R}_\eta^{-1/2}\mathbf{x}$, and using the matrix inversion lemma for $\mathbf{D}^{-1}(\boldsymbol{\tau})$, the cost function (11) can be written as

$$F(\boldsymbol{\tau})=F_{P-1} + F_{P|P-1}, \text{ where} \quad (21)$$

$$F_{P-1}\equiv\mathbf{x}^H\mathbf{R}_\eta^{-1}\mathbf{S}_{P-1}\left(\mathbf{S}_{P-1}^H\mathbf{R}_\eta^{-1}\mathbf{S}_{P-1}\right)^{-1}\mathbf{S}_{P-1}^H\mathbf{R}_\eta^{-1}\mathbf{x}$$

$$F_{P|P-1}\equiv$$

$$\frac{\left\|\mathbf{s}_P^H\mathbf{R}_\eta^{-1}(\mathbf{I}_{P-1}-\mathbf{S}_{P-1}\left(\mathbf{S}_{P-1}^H\mathbf{R}_\eta^{-1}\mathbf{S}_{P-1}\right)^{-1}\mathbf{S}_{P-1}^H\mathbf{R}_\eta^{-1})\mathbf{x}\right\|^2}{\mathbf{s}_P^H\mathbf{R}_\eta^{-1}\left(\mathbf{I}_{P-1}-\mathbf{S}_{P-1}(\mathbf{S}_{P-1}^H\mathbf{R}_\eta^{-1}\mathbf{S}_{P-1})^{-1}\mathbf{S}_{P-1}^H\mathbf{R}_\eta^{-1}\right)\mathbf{s}_P}$$

[1]The mean ratio r_A of a matrix $\mathbf{A}$ is defined as $E\left[\sum_{j\neq i}|a_{i,j}|/|a_{i,i}|\right]$ where the expectation is applied over the rows of the matrix.

Notice that the cost function consists of two non-negative terms. The first term, F_{P-1} depends only on the first $(P-1)$ delays, and it is actually the initial cost function (11) of reduced order. The P_{th} path delay appears only in the second term. Provided that the cost function (11) is almost decoupled with respect to the delays, each path can be estimated separately. Let us now assume that $(P-1)$ path delays have already been acquired and their estimates $\hat{\boldsymbol{\tau}}_{P-1}$ are accurate enough. Then according to (21), the estimation of the last delay τ_P is reduced to the maximization of the second term $F_{P|P-1}$, while keeping the rest of the delays fixed. Some interesting comments on the cost function should be made here:

1) The form of the cost function in (21) holds true for any permutation on the path indices, or equivalently for any permutation on the columns of $\mathbf{S}(\boldsymbol{\tau})$. This implies that if any $(P-1)$ delays have been estimated, the remaining delay can be estimated through $F_{P|P-1}$.
2) The term $F_{P|P-1}$ can be further decomposed through the same procedure we applied to $F(\boldsymbol{\tau})$. It can be shown that $F(\boldsymbol{\tau})$ can be finally decomposed in P terms as

$$F(\boldsymbol{\tau})=F_1+\sum_{i=2}^{P}F_{i|i-1} \qquad (22)$$

Due to the above order-recursive structure of $F(\boldsymbol{\tau})$, we conclude that in case only $(i-1)$ path delays have been estimated, the estimation of the i_{th} delay can be achieved using the corresponding $F_{i|i-1}$ term of (22).

C. The New Algorithm

Having analysed the cost function, we present the new algorithm which is called hereafter Decoupled Parametric Estimation (DPE). DPE is organized in steps and cycles. At each *step*, one delay parameter is estimated using the information of already acquired delays. A *cycle* consists of P steps and at the end of a cycle all delays have been estimated. During the first cycle and while searching for τ_i, the optimization is performed based on the $F_{i|i-1}$ term of (22). During the next cycles, DPE uses the estimates of all other $(P-1)$ delays (either obtained in a previous, or the current cycle) for the estimation of a single delay. The maximization now is performed using $F_{P|P-1}$ term with the delays properly ordered. Concerning the number of cycles, simulations show that two cycles are adequate for the method to converge. After all cycles have been completed, path gains are extracted through (10).

In any cycle, the maximization of the corresponding cost function is performed by a line search. Since the desired user has been synchronized with the BS and the delay spread of the physical channel is restricted to some chip periods, it is sufficient to scan the delay range $[0, NT_c/4)$ with a linear step size δ. Obviously, the value of δ affects the estimation accuracy of the maximization procedure. In any case, the estimates obtained through line search over the grid are not optimum. A further refinement of the estimates can be achieved running some Gauss-Newton iterations or an interpolation method.

Among all methods proposed so far for the estimation of channel parameters in a CDMA system, the one that is more relevant to DPE is the method presented in [7]. The algorithm presented there (Whitening Sliding Correlator with Cancellation, called hereafter WSCC) stems from a ML cost function, while the subtraction of each estimated path from the received data comes as a natural application of the SAGE algorithm. On the other hand, our method depends on a nonlinear LS cost function, which is proven to be almost decoupled with respect to the delay parameters. The deflation procedure (i.e. extracting the contribution of already resolved paths) is encapsulated naturally in the cost function, yielding better estimation results. As will be shown by simulation, DPE exhibits a lower estimation error, at the expense of a slight increase in computational complexity compared to WSCC.

More specifically, the computational complexity of both algorithms per step is $O((MQN)^2)$. The extra computational cost of DPE is related to the computation of matrix $\mathbf{R}_\eta^{-1}(\mathbf{I}_{i-1}-\mathbf{S}_{i-1}\left(\mathbf{S}_{i-1}^H\mathbf{R}_\eta^{-1}\mathbf{S}_{i-1})^{-1}\mathbf{S}_{i-1}^H\mathbf{R}_\eta^{-1}\right)$ at the beginning of each step. This computation can be performed recursively with $O((MQN)^2)$ operations, and thus the extra cost can be considered insignificant.

IV. Simulation results

In this section, we investigate the performance of the new algorithm through computer simulations. Most of the system parameters used in the simulations were in agreement with the UMTS specifications for FDD (Frequency Division Duplexing) [9]. Specifically, the scrambling codes were of length N=256, the modulation used was BPSK, the chip pulse was a raised cosine function with roll-off equal to 0.22, the oversampling factor Q was equal to 2, and the pilot signal consisted of 5 symbols.

The multipath channel consisted of four paths. The path gains were random variables following a zero mean Gaussian distribution with variances $[0, -1, -9, -10]$dB. The path delays of the desired user were fixed to the values $[0,\ 1.19,\ 2.72,\ 4.18]\,T_c$ (ITU - Vehicular Channel Model A). Considering the asynchronous nature of the system, the delays of the interfering users were modelled as random variables. The first delay of k_{th} user, $\tau_{k,1}$, followed a uniform distribution in the interval $[0, NT_c)$, while the remaining three delays were uniformly distributed in the interval $[\tau_{k,1},\ \tau_{k,1}+10T_c]$.

The estimation accuracy of the proposed algorithm was evaluated in terms of the *Normalized Mean Squared channel estimation Error* (NMSE) between actual and estimated total CIR, $NMSE{=}E\left[\|\mathbf{h}_{tot}-\hat{\mathbf{h}}_{tot}\|^2/\|\mathbf{h}_{tot}\|^2\right]$. The results presented in this section were obtained through 1000 Monte Carlo simulation runs. Comparisons are made with the WSCC algorithm. The asymptotic CRB is also presented. Notice here that the parameter estimates $\hat{\boldsymbol{\tau}},\ \hat{\mathbf{a}}$, were obtained by running the basic versions of the two algorithms, i.e. without any further refinement by Gauss-Newton iterations or interpolation. The step size used for both algorithms was set to δ=$0.125T_c$, and two estimation cycles were performed.

In Fig. 1, the NMSE versus E_b/N_0 is presented for a pilot signal of M=5. E_b is defined as the received bit energy for the desired user. There were K=64 active users and the Signal

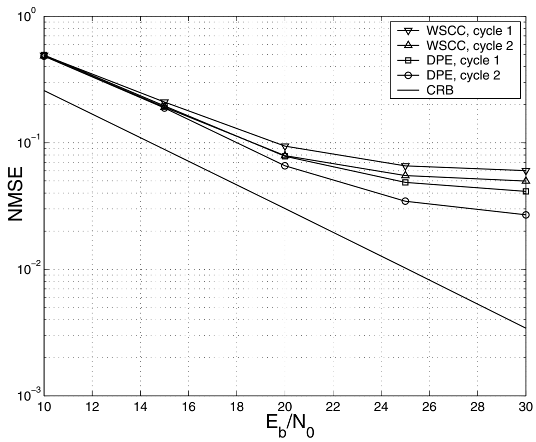

Fig. 1. NMSE vs SNR for M=5 training symbols, K=64 users, and SIR=0dB.

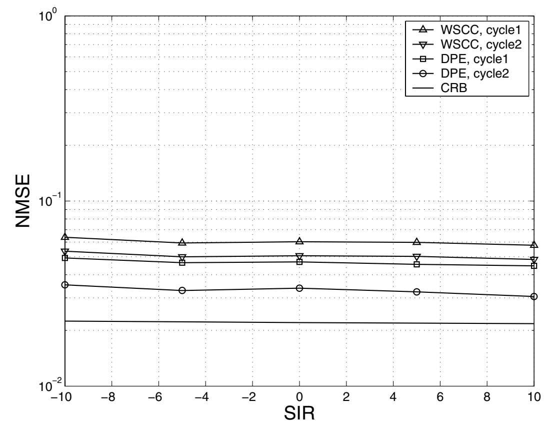

Fig. 3. NMSE vs SIR for M=5 training symbols, K=16 users, and SNR=20dB

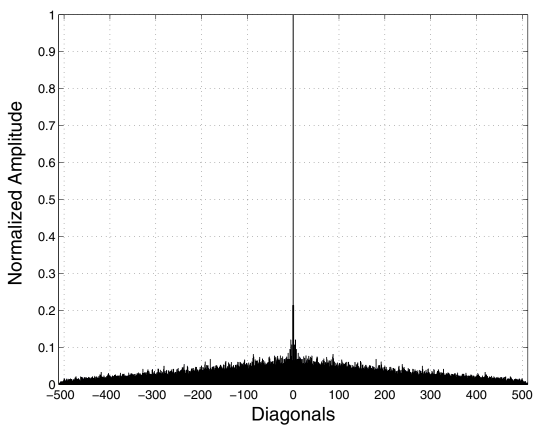

Fig. 2. Normalized amplitude of the diagonal block elements of $\mathbf{R}_\eta^{-1}$ for M=5 training symbols, K=64 users, SNR=20dB, and SIR=0dB.

to Interference Ratio (SIR), was set to 0dB. It can be seen that the two algorithms at the low SNR region (below 15dB) exhibit similar behaviour. But in the medium to high SNR region DPE outperforms WSCC. Specifically, near 20dB, each cycle of DPE has a 2dB gain in NMSE compared to the corresponding cycle of WSCC. Moreover, the first cycle of DPE attains the same NMSE as the second cycle of WSCC. The gain in estimation error is higher for increasing SNR.

It is clear that the decoupling of the delay parameters is based on the strong diagonal dominance property of matrix $\mathbf{R}_\eta^{-1}$. To verify our theoretical analysis, we plot in Fig. 2 the normalized amplitude of the elements of the main diagonal block of $\mathbf{R}_\eta^{-1}$ by properly projecting a 3-D mesh plot on two dimensions. As can be seen, its off-diagonal elements are at least one order of magnitude smaller than the diagonal ones.

Finally, we investigated the robustness of the two algorithms to the near-far-problem. The system accommodated K=16 users having a SIR ranging from -10 to 10dB. As shown in Fig. 3 both algorithms are robust to MAI, since their accuracy remained almost constant for all tested SIR values. DPE algorithm exhibits again superior performance.

V. Conclusions

A new method for estimating the multipath channel parameters of a single user in the uplink of a DS-CDMA system has been proposed. The new method is based on a LS approach. An approximate decoupling of the non-linear cost function with respect to the delay parameters leads to an iterative procedure of one-dimensional optimizations. At each step of the algorithm, a single delay is estimated while the rest are kept fixed. Additional cycles of the algorithm allow for further improvement of the estimates. The suggested method does not require any specific form of pilot signal and performs well for a short training interval. Simulation results have shown its robustness to MAI, as well as its higher estimation accuracy compared to an existing method, at the expense of an insignificant increase in computational complexity.

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